Welcome to
Biology 427
Biomechanics 2004

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Course web page:
http://faculty.washington.edu/danielt/BiomechanicsWeb/Bio427.html

Lecture 1: An introduction to Biomechanics: Jumping right in.

- What's the course about?
- How is the course organized?
- What physics basics need I review?
- Jumping right into it: the mechanics of ballistic bodies.

Physical principles underlying biological processes and mechanisms (movement, design, architecture, materials, transport).

Many levels of biological organization:

molecular → cellular → tissue → organism → population

Course Syllabus

COURSE ORGANIZATION

- 3 lectures per week
- Real and CD text books with assigned reading
- Handouts
- Weekly problem sets (physics and math don't get easliy or reasonably memorized)
- Critical reviews of the scientific literautre

50%

- Discussion (review problem sets, panel discussion of papers)
- One course project (in which you develop your own physical analysis of biological problems)
- No exams

50%

Some basics for Biomechanics: A simple problem with a physics review

Rule 1: Equations must be dimensionally correct!

Mass, Length and Time (we commonly use S.I. units*)

Describe physical quantities

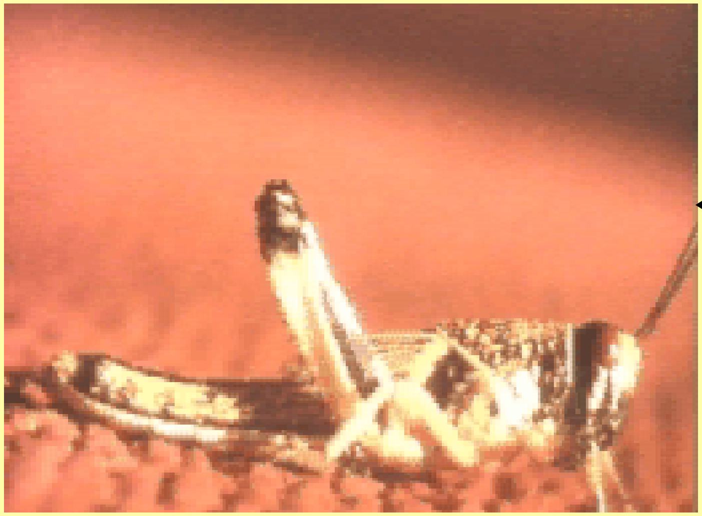
distance	$\mathbf{x}$	L	m
Velocity	$\mathbf{v}$,d$\mathbf{x}$/dt	L T^{-1}	m s^{-1}
Acceleration	$\mathbf{a}$,d$\mathbf{v}$/dt,$d^2\mathbf{x}/dt^2$	L T^{-2}	m s^{-2}
Momentum	$\mathbf{M}$, m $\mathbf{v}$	M L T^{-1}	kg m s^{-1}
Force	F, d(mv)/dt	M L T^{-2}	Newton, kg m s^{-2}
Work	E, F.x (if constant F)	M L^2 T^{-2}	Joule, kg m^2 s^{-2}
Power	P, dE/dt	M L^2 T^{-3}	Watt, kg m^2 s^{-3}

*Systemme Internationale

Some basics for Biomechanics: A simple problem with a physics review

Humans and parts of them

A locust



Distance?

Velocity?

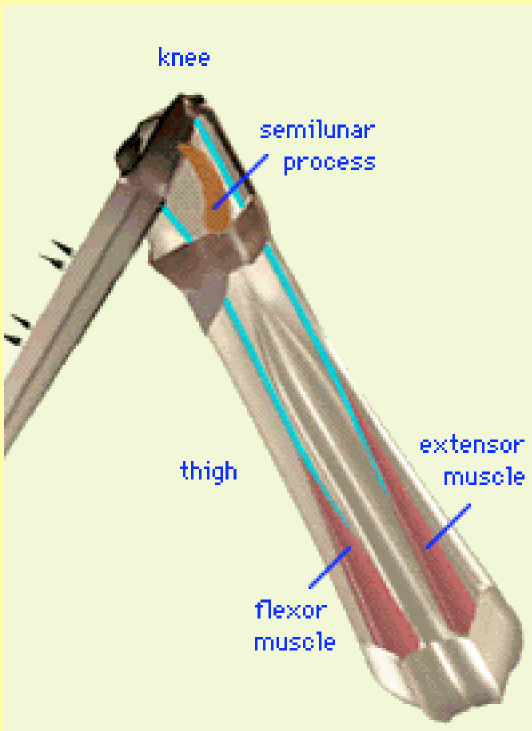
Acceleration?

Force?

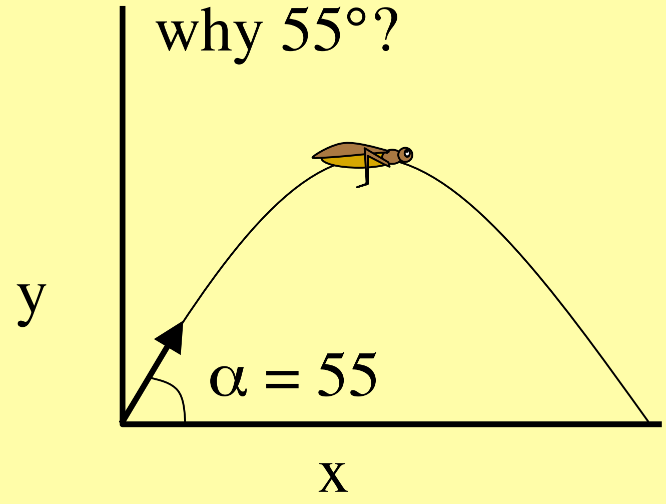
Work?

Power?

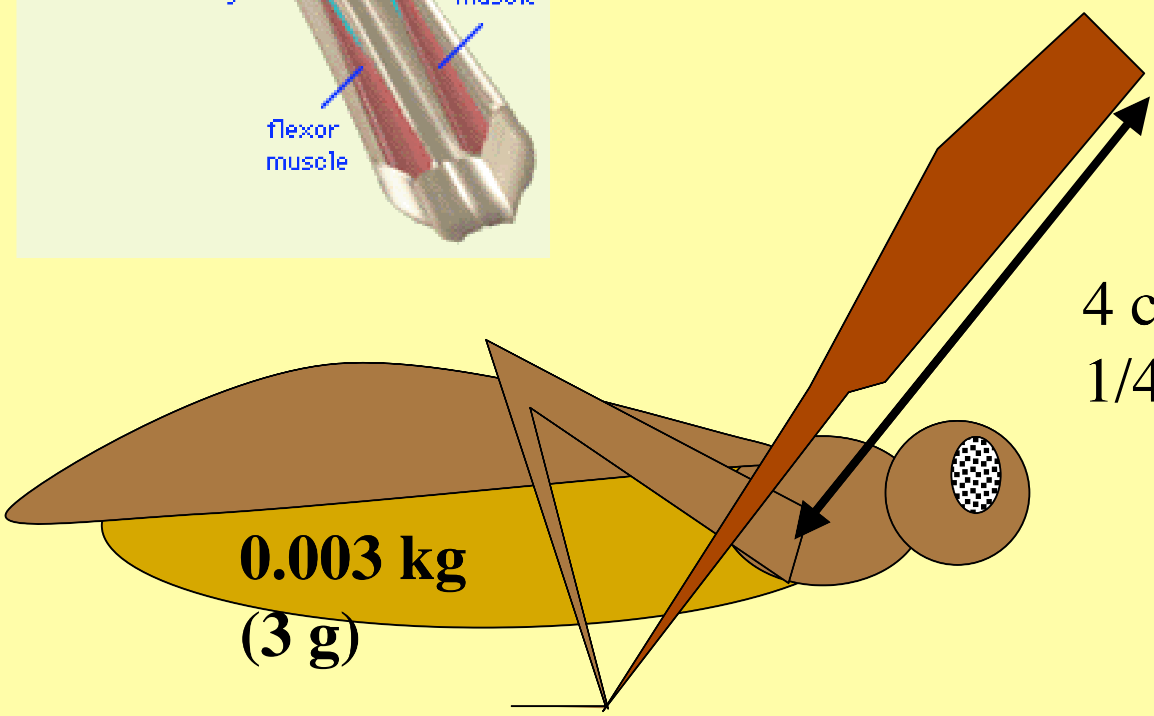
Some morphology and data



this is a parabola….
why 55°?



$u_o = 3.4$ m/s



4 cm (0.04 m)
1/40 s (0.025 s)

Force? Energy? Power?

Need the initial acceleration

$2\ a\ d = u_o^2$

$F = m\ a = 0.43$ N (both legs)

0.21 N/leg

a = 145 m s-2

muscle force ~10 foot force

$|u_o| = 3.4$ m/s

4 cm (0.04 m)

1/40 s (0.025 s)

Muscle Work = F Δx

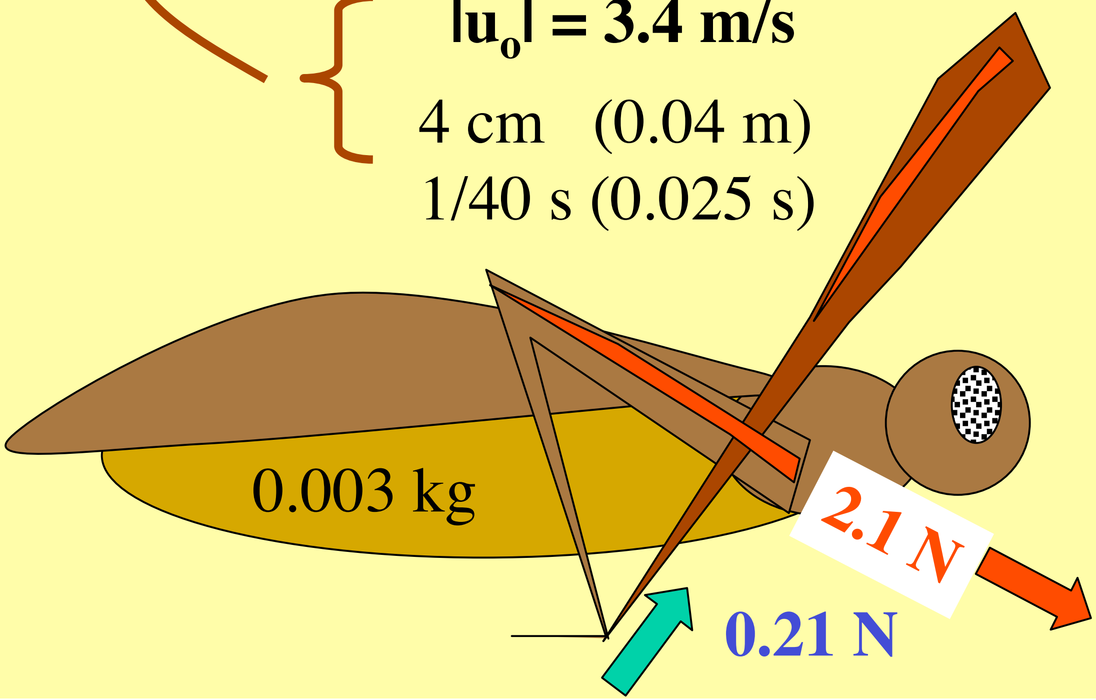
= 2.1 x 3 10^{-3} N m

= 6.3 mJ

Power = W/time

= 6.3/0.025

252 mW



Power = W/time

= 6.3/0.025

252 mW

5% mass = leg muscle

~ 1.5 10^{-4} kg

1680 W/kg muscle

Human on a bicycle ergometer ~ 40 W/kg

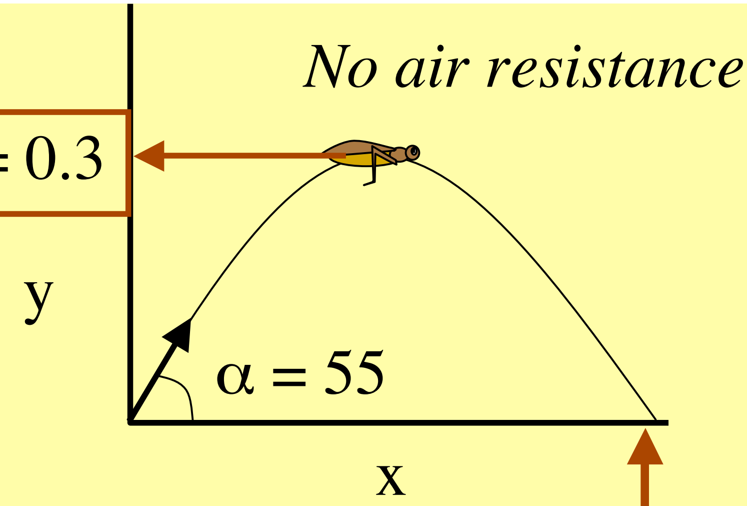
Maximum single twitch in vertebrate muscle ~400 W/kg

$m = 3\ 10^{-3}$ kg

$|u_o| = 3.4$ m/s

$u_o = [u_o \cos\alpha, u_o \sin\alpha]$

$y_{max} = (u_o \sin\alpha)^2/2g = 0.3$



After take-off forces sum to zero

$\Sigma F_x = m\, du_x/dt = 0$

$\Sigma F_y = m\, du_y/dt + mg = 0$

How high?

$m\, du_y/dt = -mg$

$du_y/dt = -g$

$u_y = -gt + u_o \sin\alpha$

$y = \int u_y\, dt = u_o \sin\alpha\ t - gt^2/2$

Max y when dy/dt = 0;

How far?

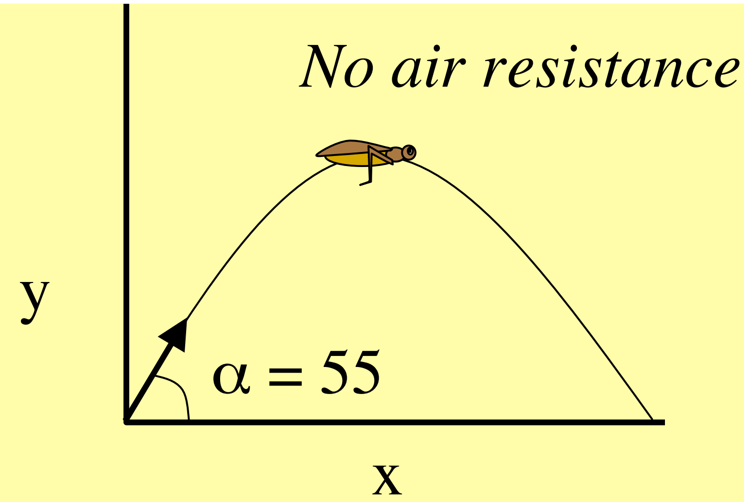
$x = u_o \cos\alpha\ t$

Find t when y is zero!

$t = 2\, u_o \sin\alpha/g$

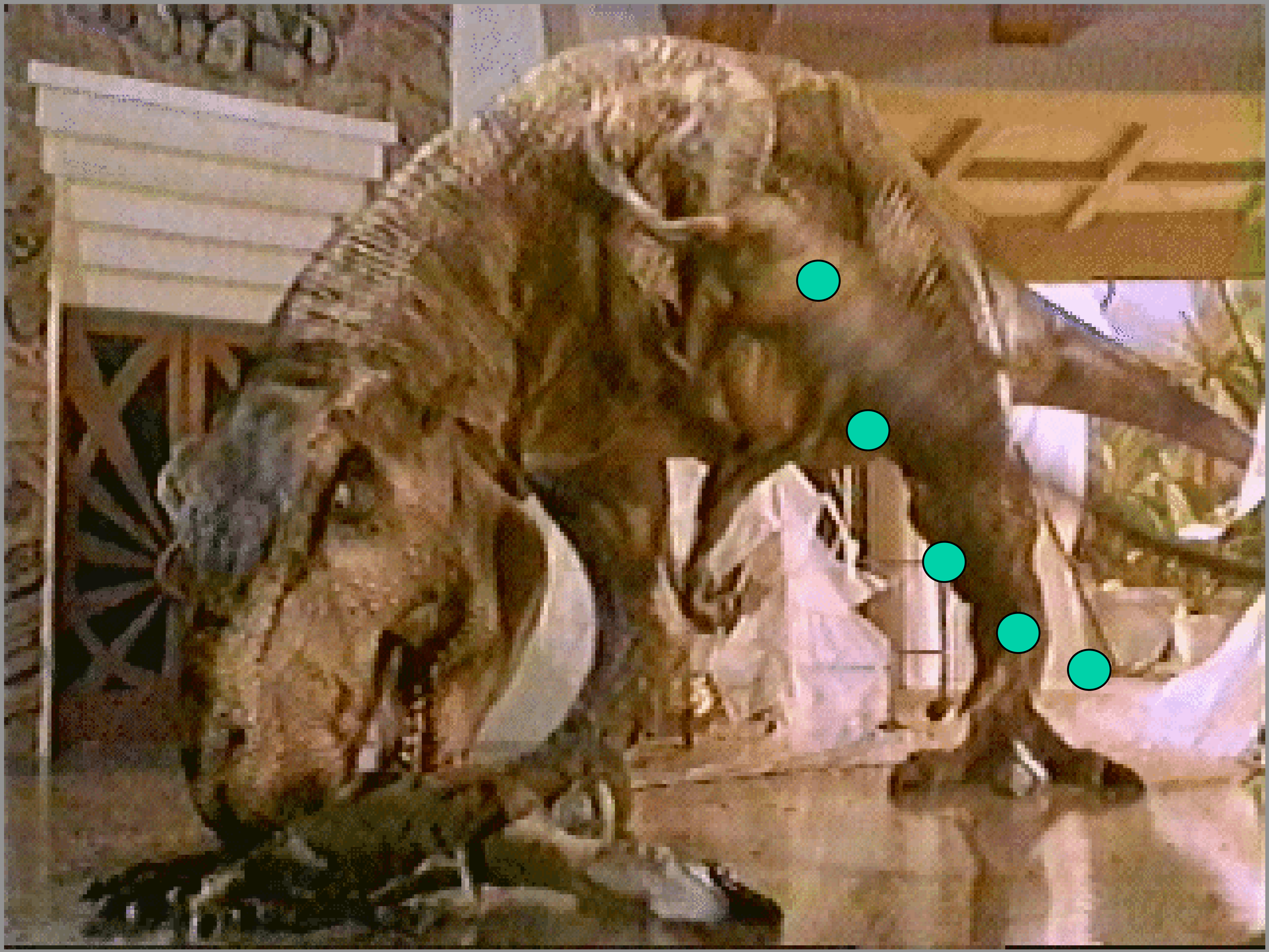
$x_{max} = 2\, u_o^2 \cos\alpha \sin\alpha/g$

$$y = x \sin\alpha \cos\alpha - g(x/u_o \cos\alpha)^2/2$$



$$x = u_o \cos\alpha \; t$$

$$y = \int u_y \, dt = u_o \sin\alpha \; t - gt^2/2$$



*Lecture 2: Muscle and molecular mechanics**

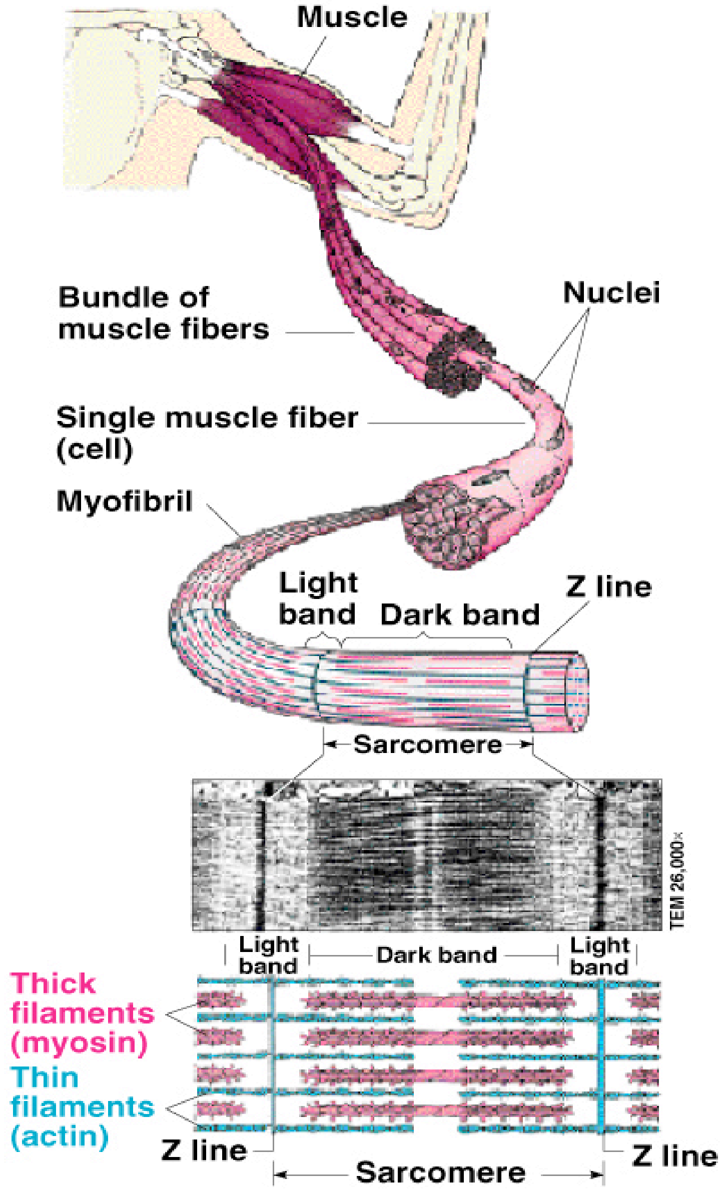
- *Recap*
- *Molecular basis of force generation: ATP hydrolysis expands an elastic molecule*
- *Release of 'elastic strain energy' is manifest as force.*
- *Three key experiments: old and new.*
- *New biomechanical analyses.*

***Read Chapter 1 "Machinery of Movement" on CD**

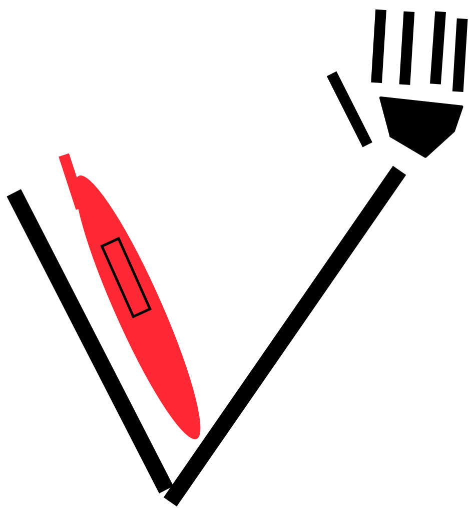
Course web page:

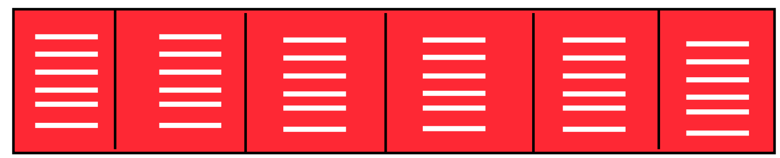
http://faculty.washington.edu/danielt/BiomechanicsWEB/Bio427.html

What are the molecular determinants of force ?

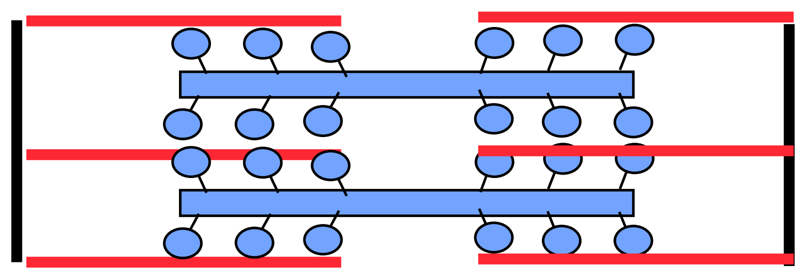


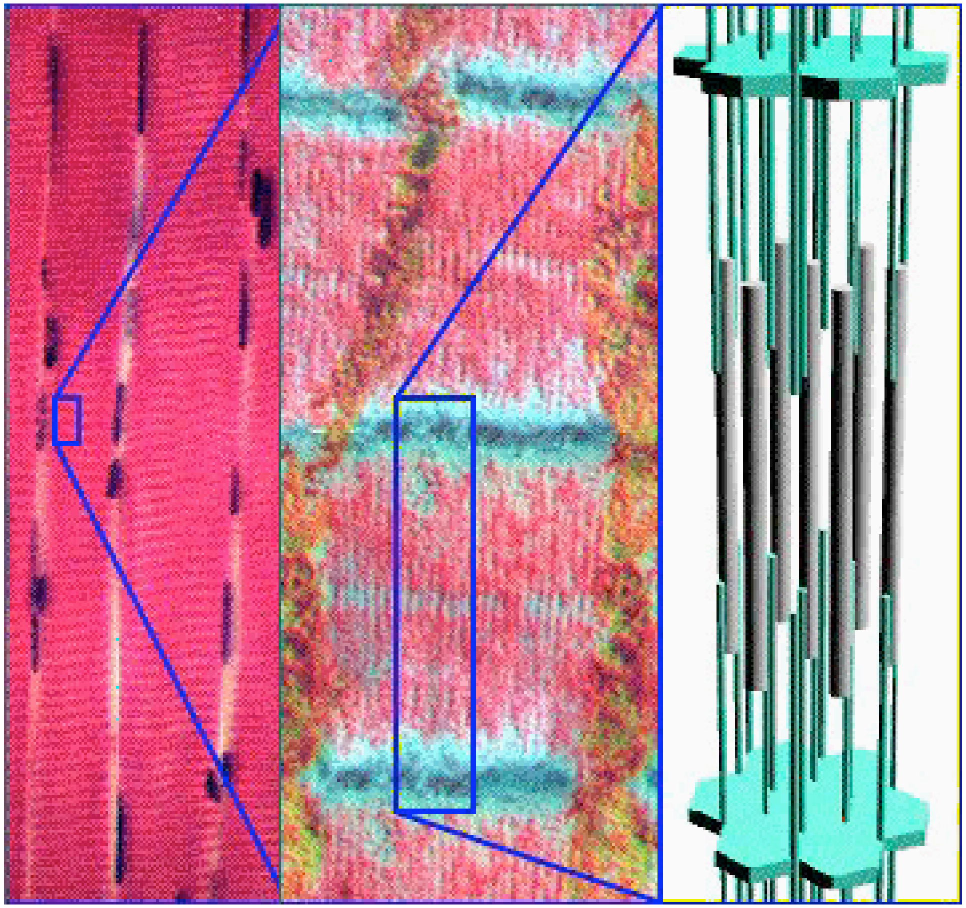
The Geometry of Muscle

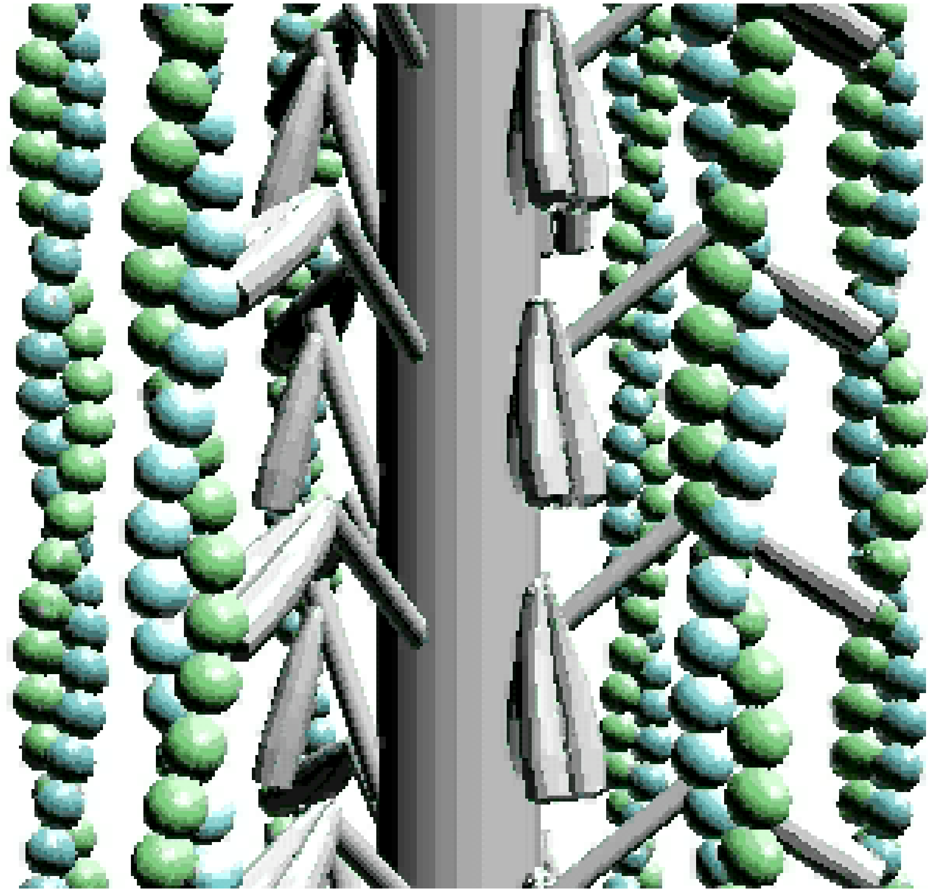


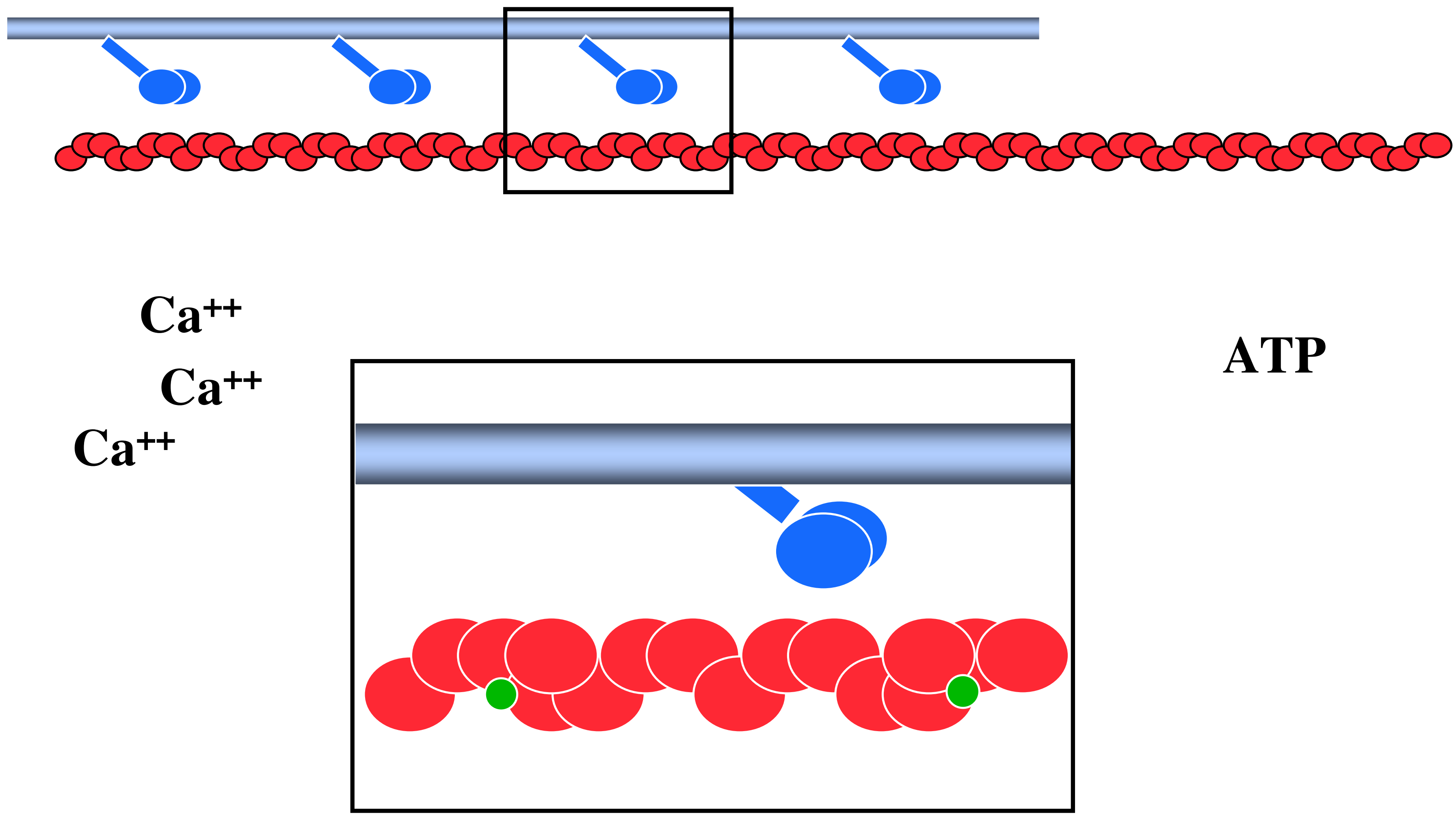


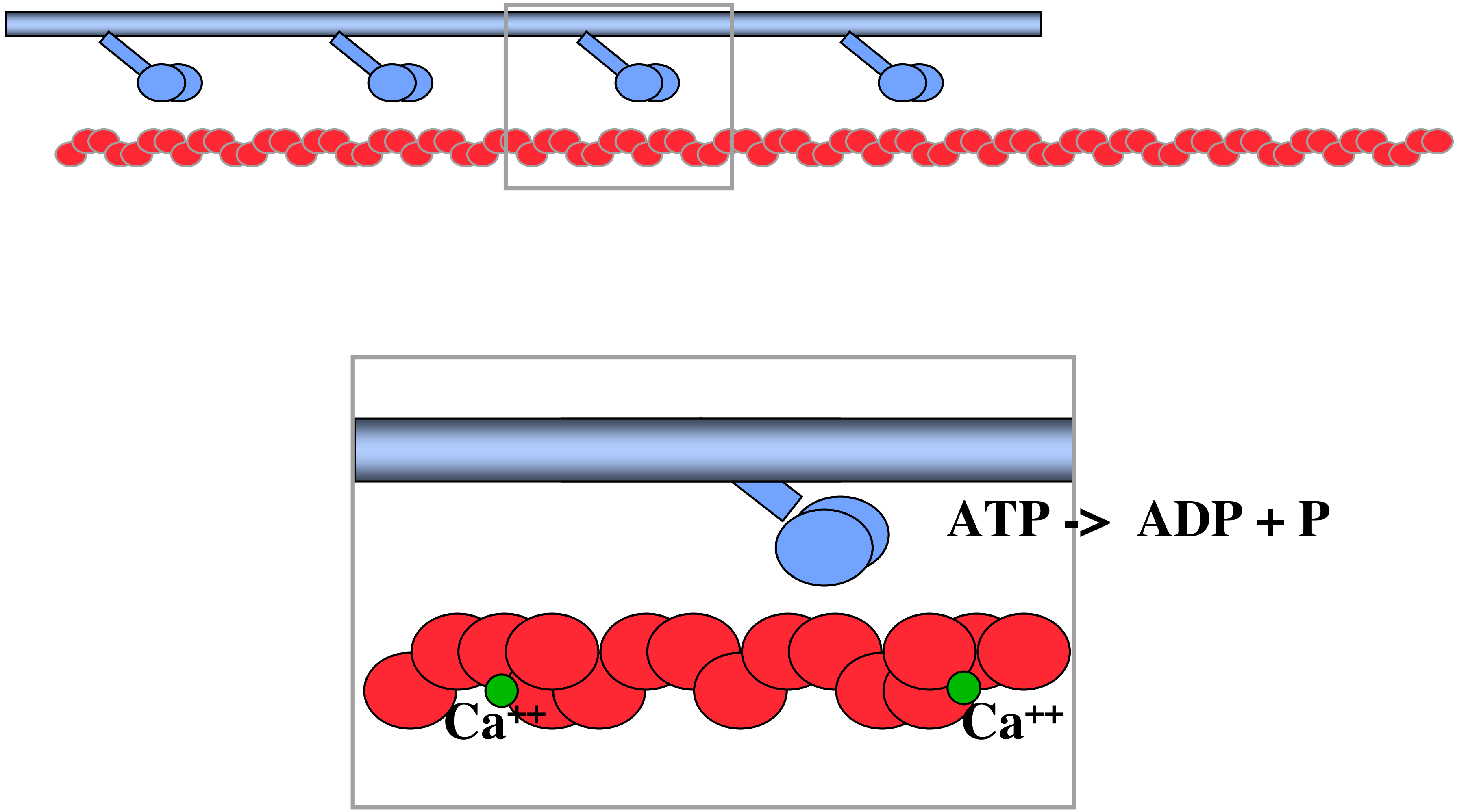
Sarcomere



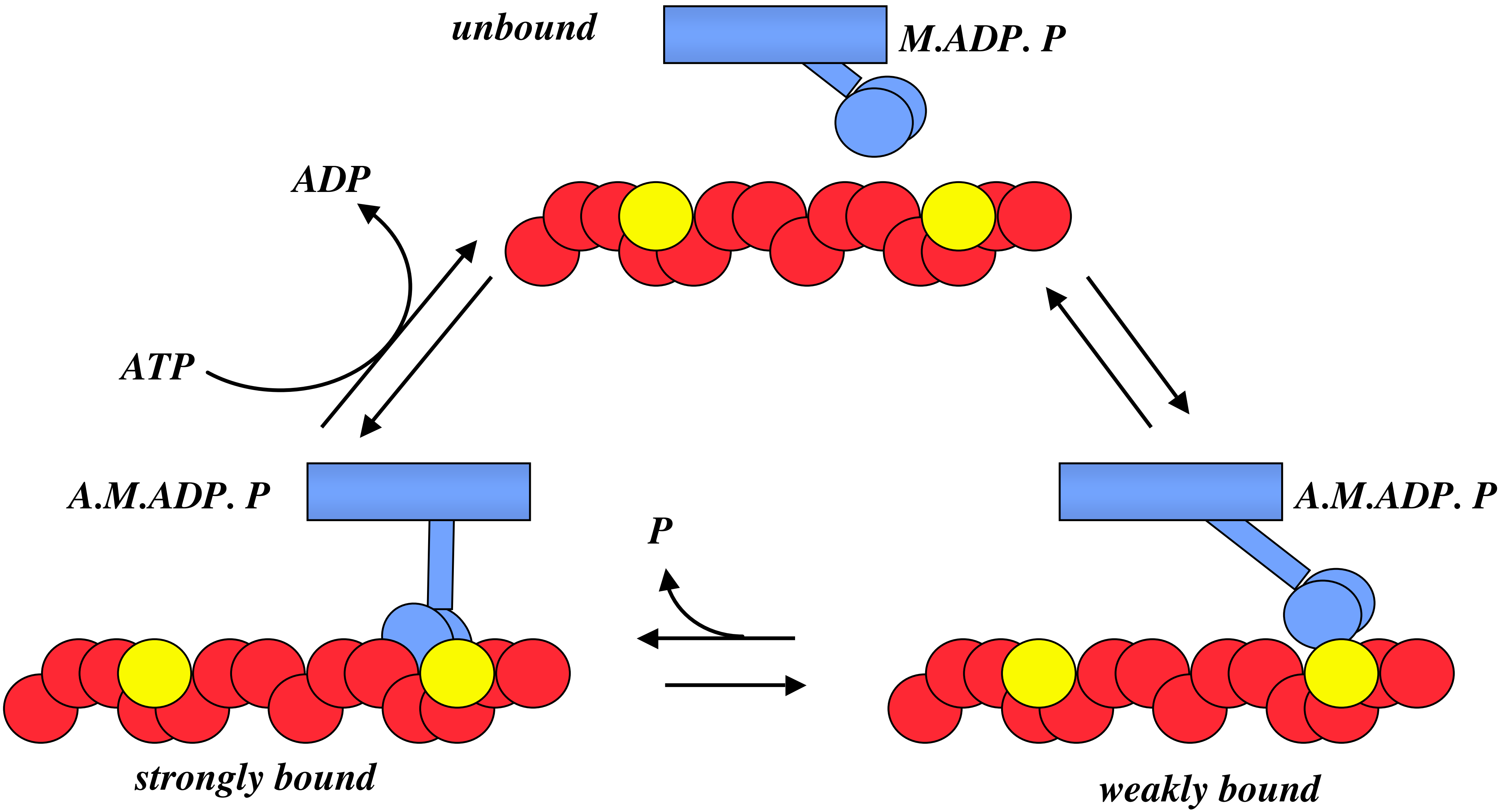




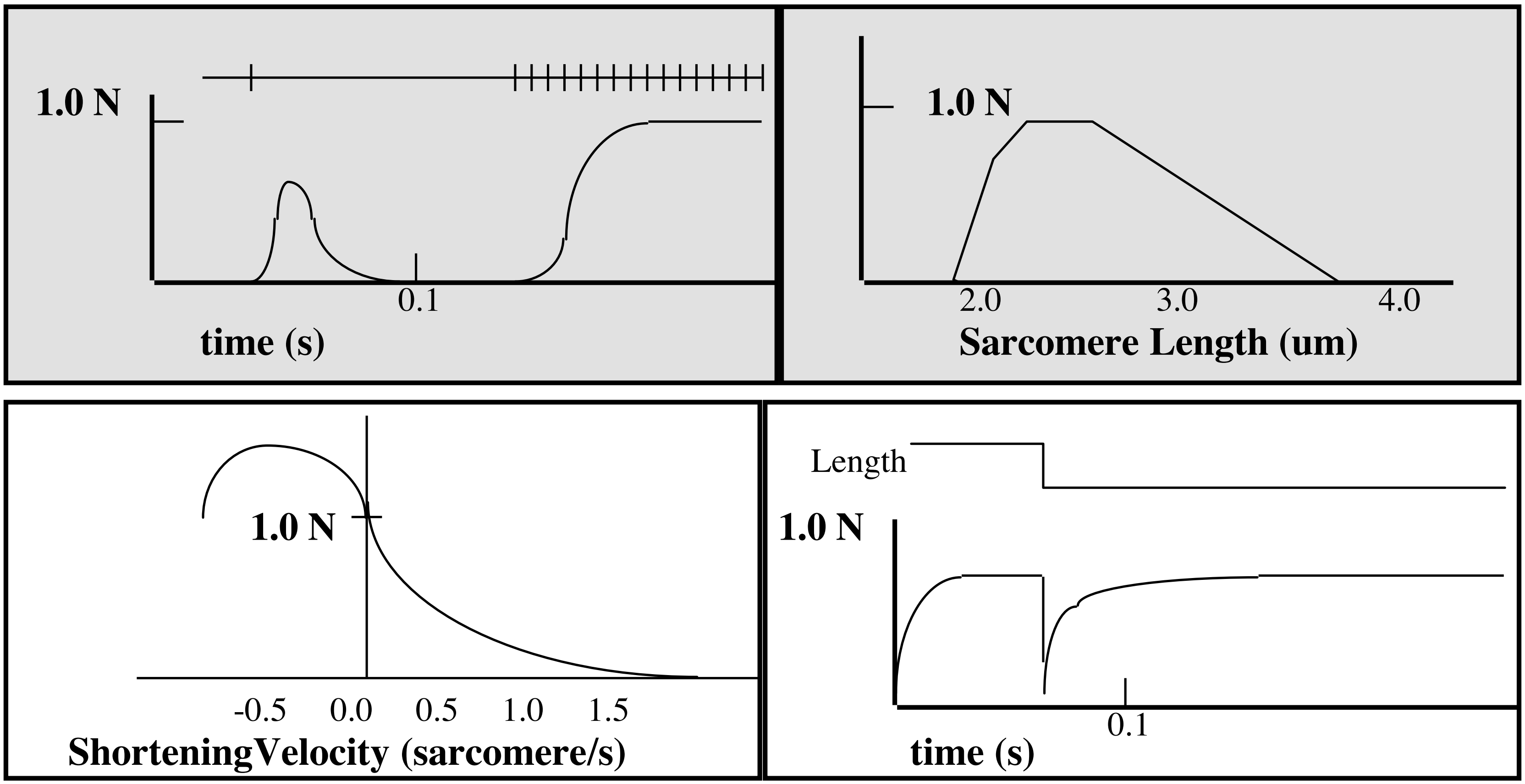

Ca++
Ca++
Ca++
ATP


ATP -> ADP + P
Ca++
Ca++

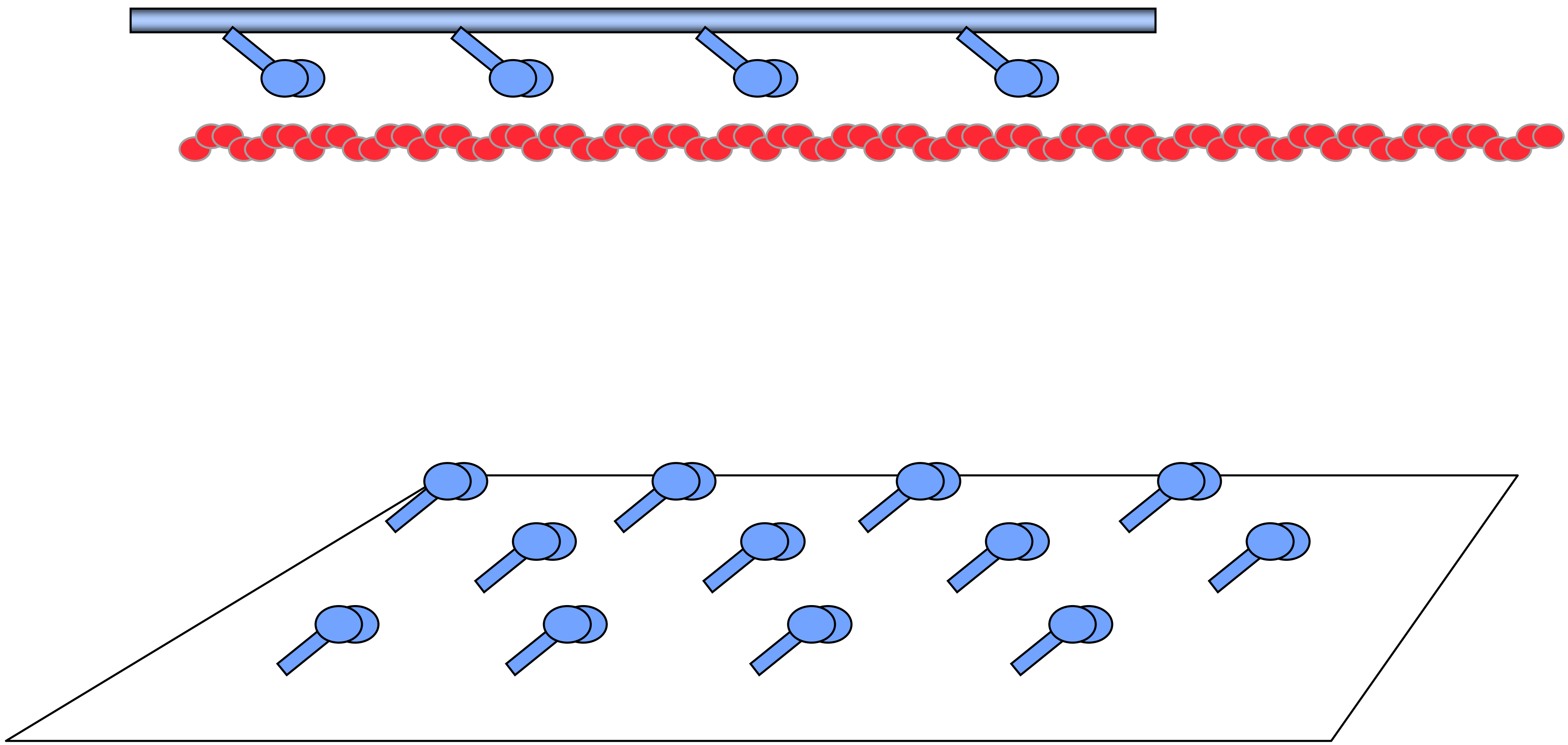
State Transitions



Experimental Mechanical Evidence?



In vitro motility *

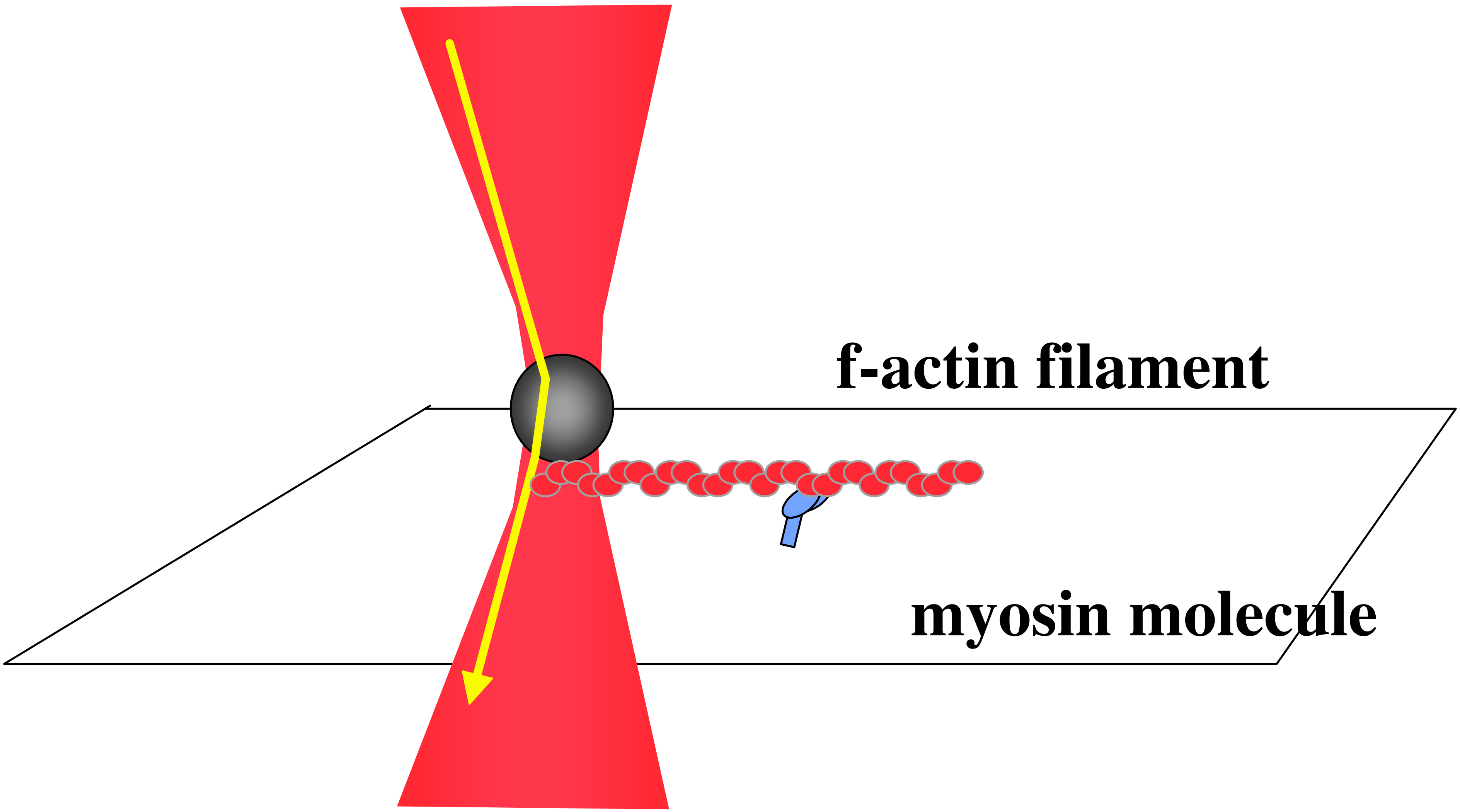


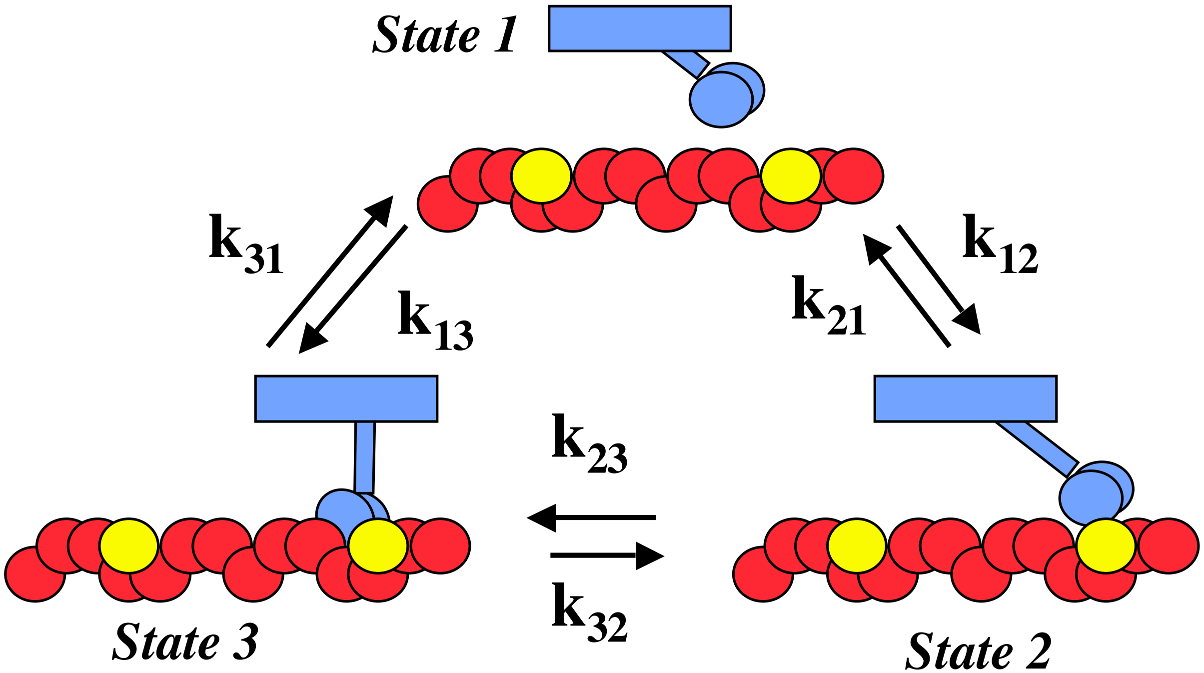
*Dr. Bryant Chase and Kristi Kulin

Optical Tweezers

Photons have momentum (but no mass)

Photon flux ~ momentum flux ~ force

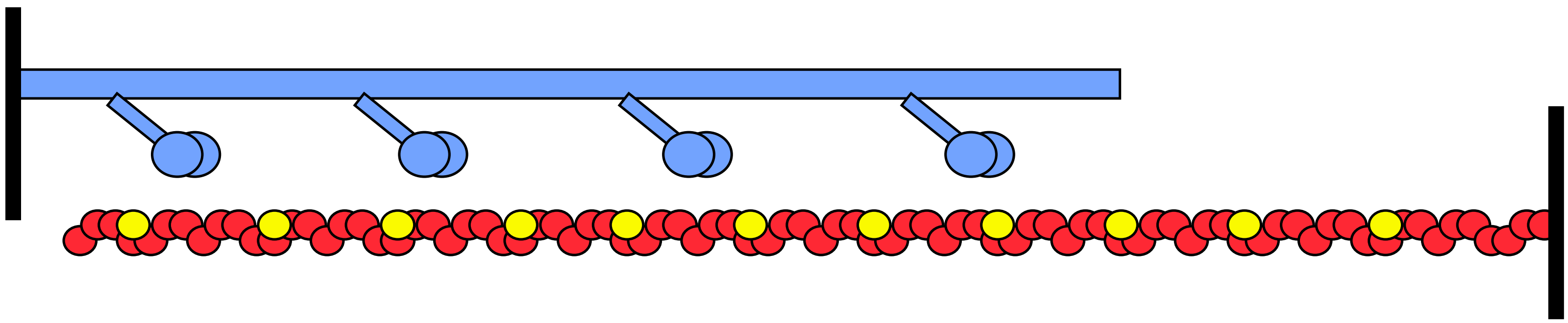




Forward transition rates depend on:

- **distance to a binding site**
- **distortion of a cross-bridge**
- **Reverse transition rates calculated from equilibrium thermodynamics (exp(ΔG) dynamics).**

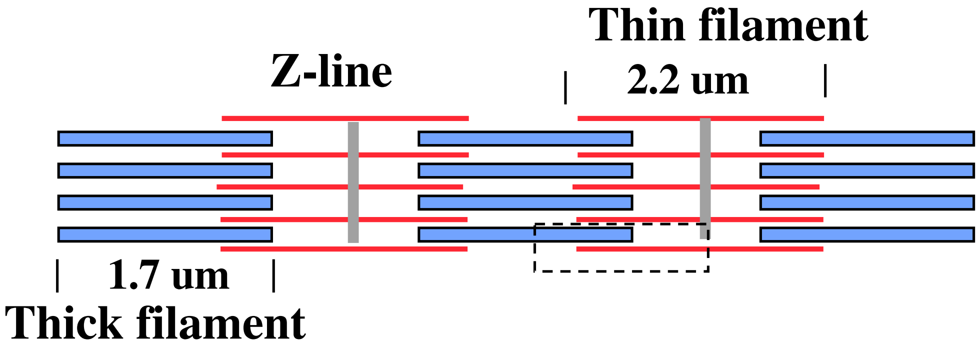
If filaments are not deformable, there is no interaction between cross-bridges and mass-action models can be used



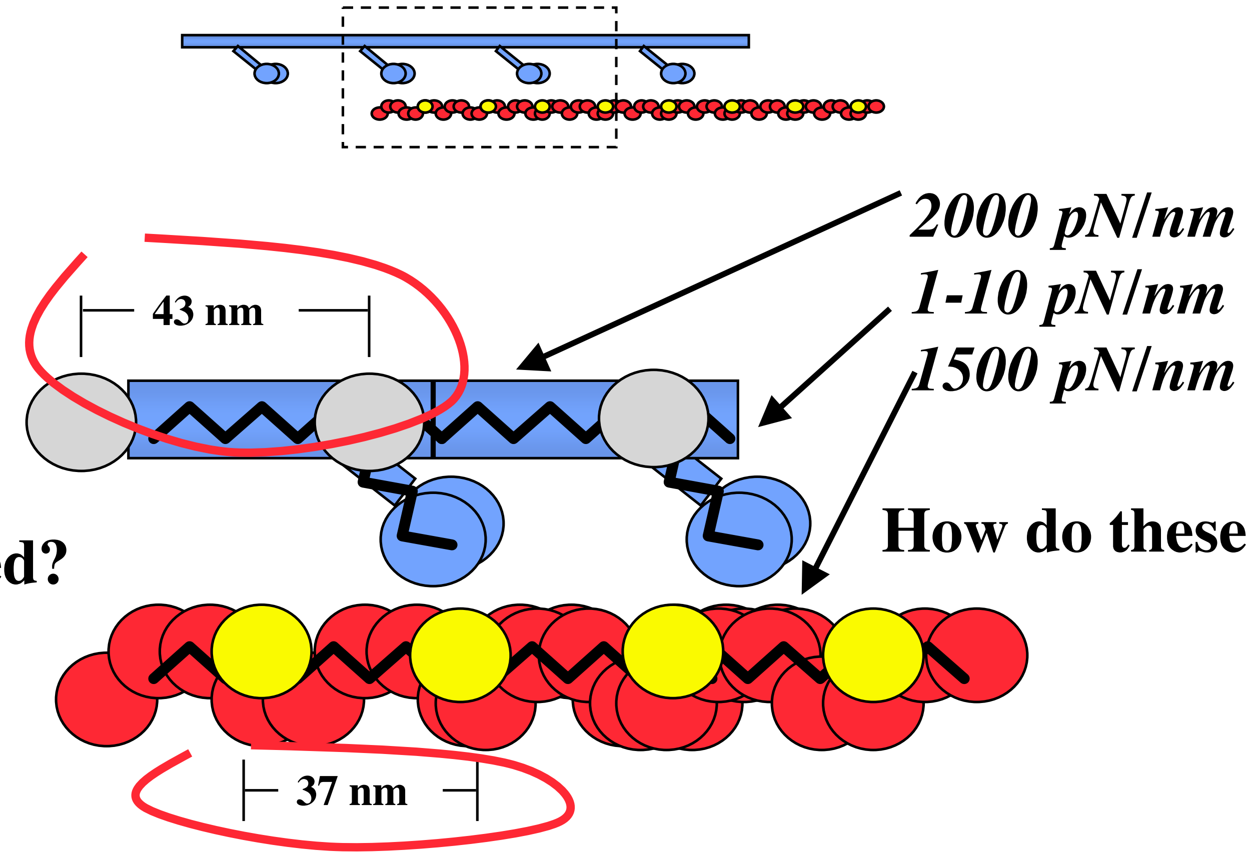
If deformations result from cross-bridge forces, then this is a coupled system -- spatially explicit models are then needed*.

***Huxley, Wakabayashi, Isambert, and many others**

Geometry*: Two interacting compliant filaments



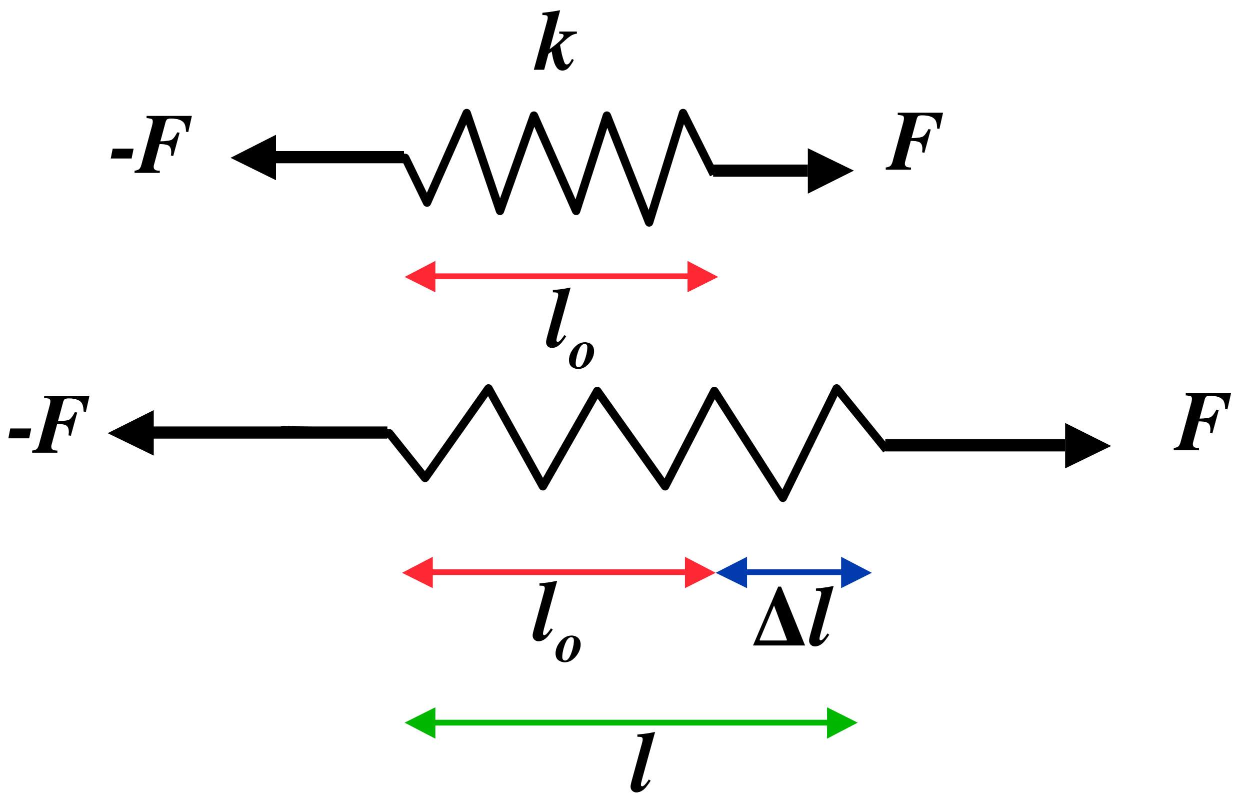
*About 20 cross-bridges and 30 binding sites in each half sarcomere.



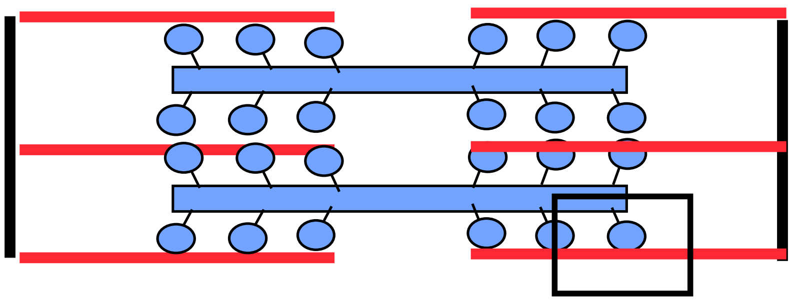
How are forces distributed?

How do these cycle?

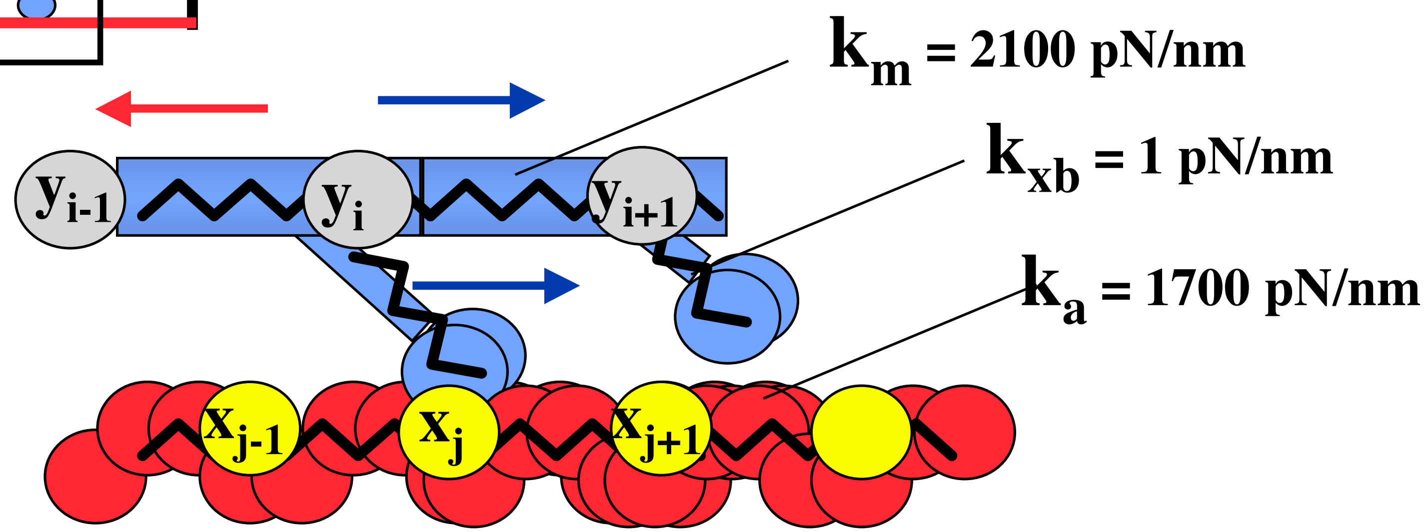
Recall Hooke's Law



$$F = -k\,(l - l_o)$$



Mechanics



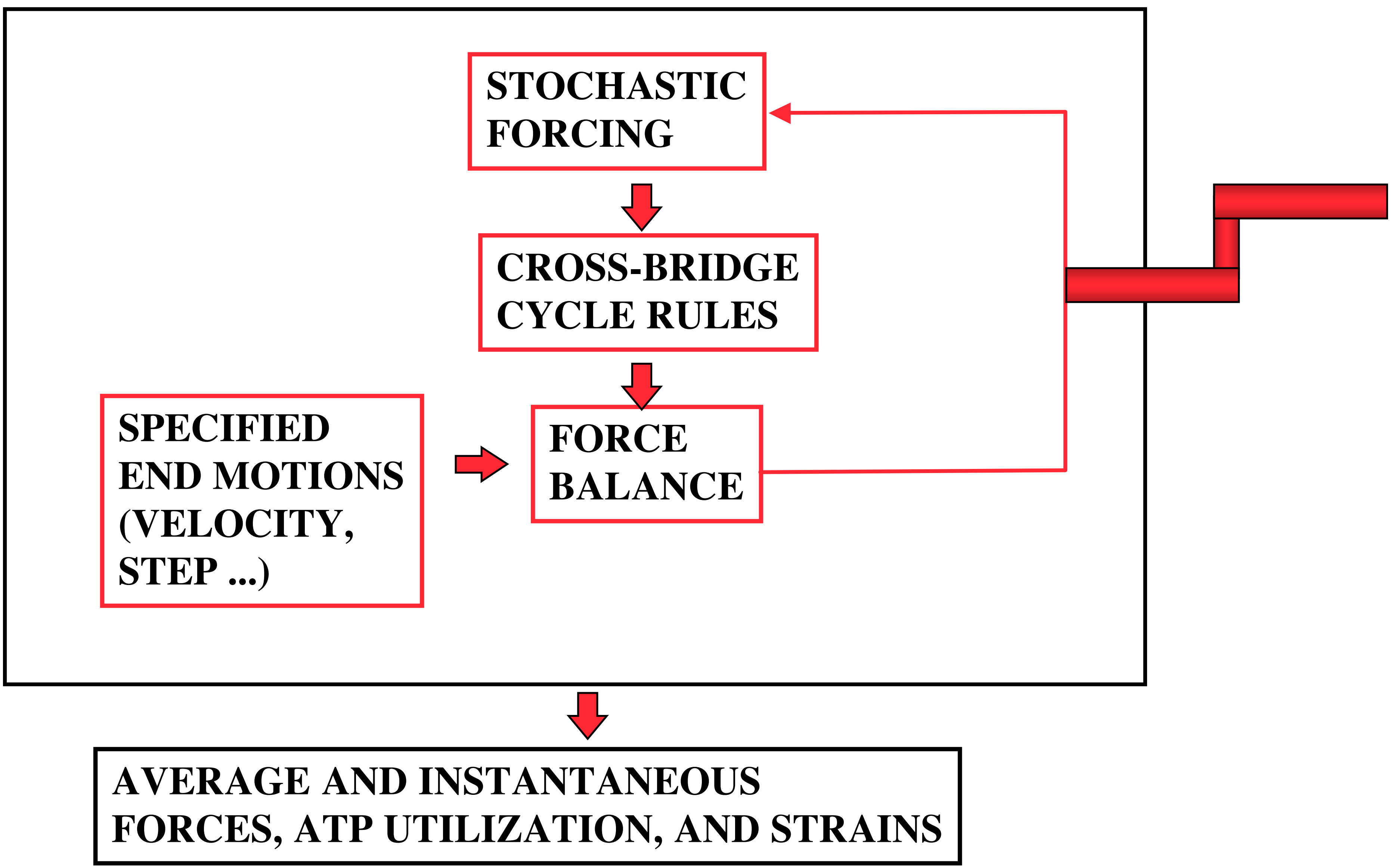
For the i th bound cross-bridge

$$k_m(y_{i+1}- y_i - y_o) + k_{xb} (x_j - y_i -x_{bo}) - k_m(y_i - y_{i-1} - y_o) = 0$$

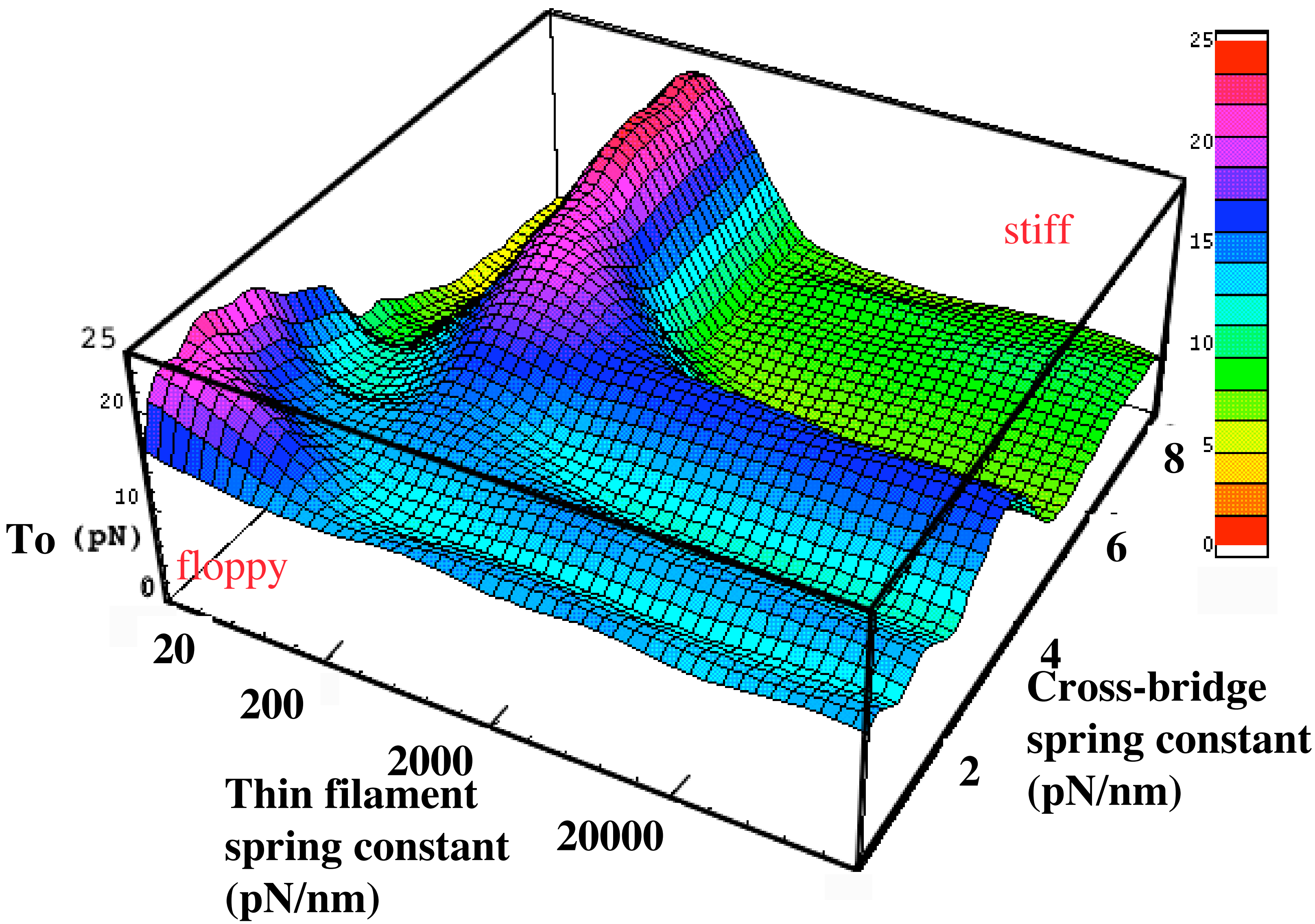
$$k_a (x_{j+1}- x_j- x_o) - k_{xb} ((x_j - y_i- x_{bo}) - k_a(x_i- x_{i-1}- x_o) = 0$$

Computational Approach: Monte-Carlo Simulation

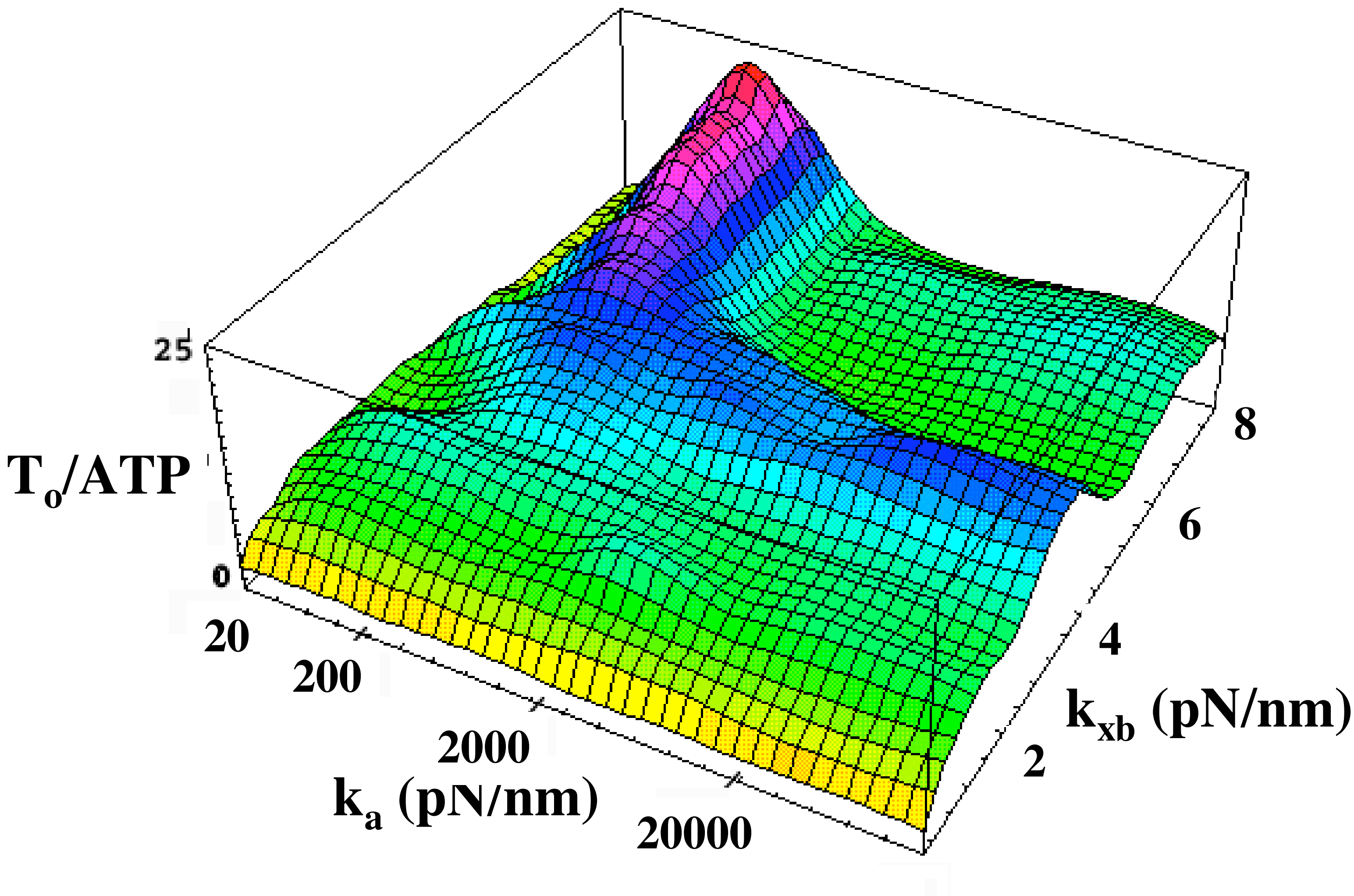
All cross-bridges initially unbound (state 1)



MECHANICAL TUNING EMERGES



"EFFICIENCY" CAN BE TUNED TOO

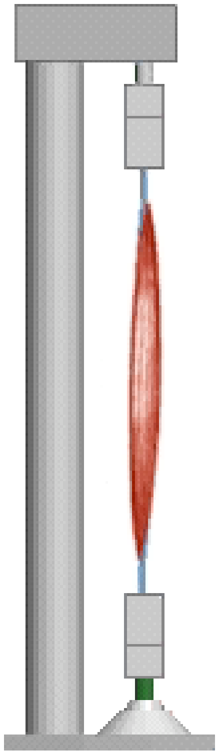


*Lecture 3: Muscle and physiological mechanics**

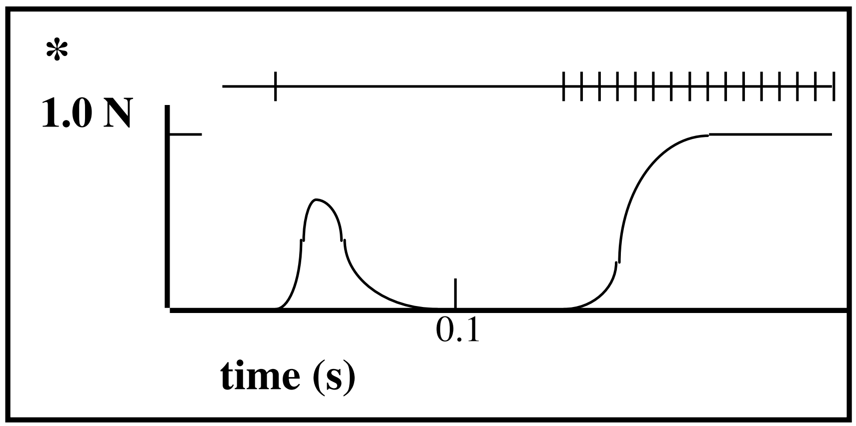
- *Recap*
- *Isometric versus isotonic experiments*
- *Relations between force and time, length and velocity*
- *The work-loop method: physiologically relevant mechanics.*

***Read Chapter 1 "Machinery of Movement" on CD**

What are the determinants of force ?

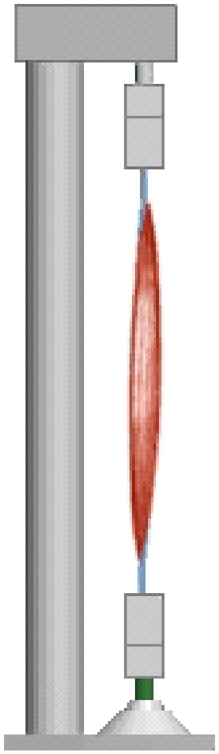


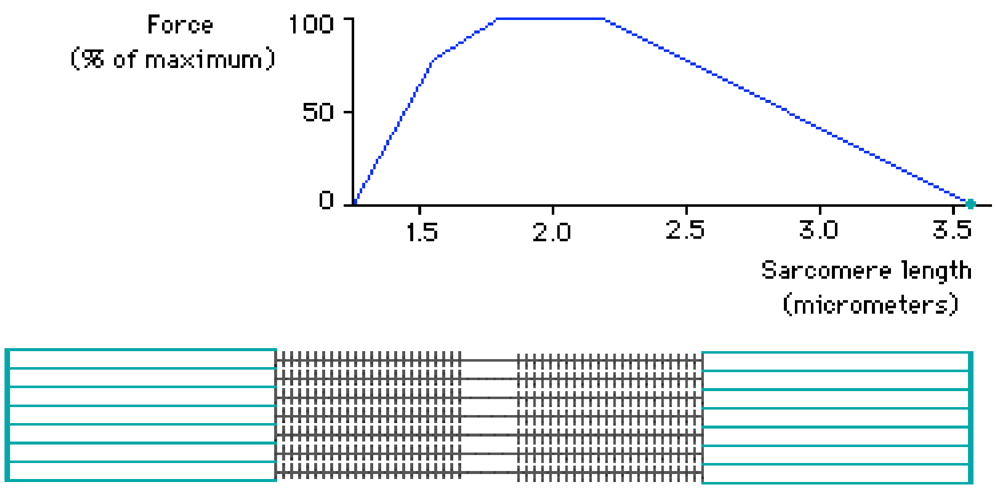
Temporal characteristics of activation



What are the determinants of force ?

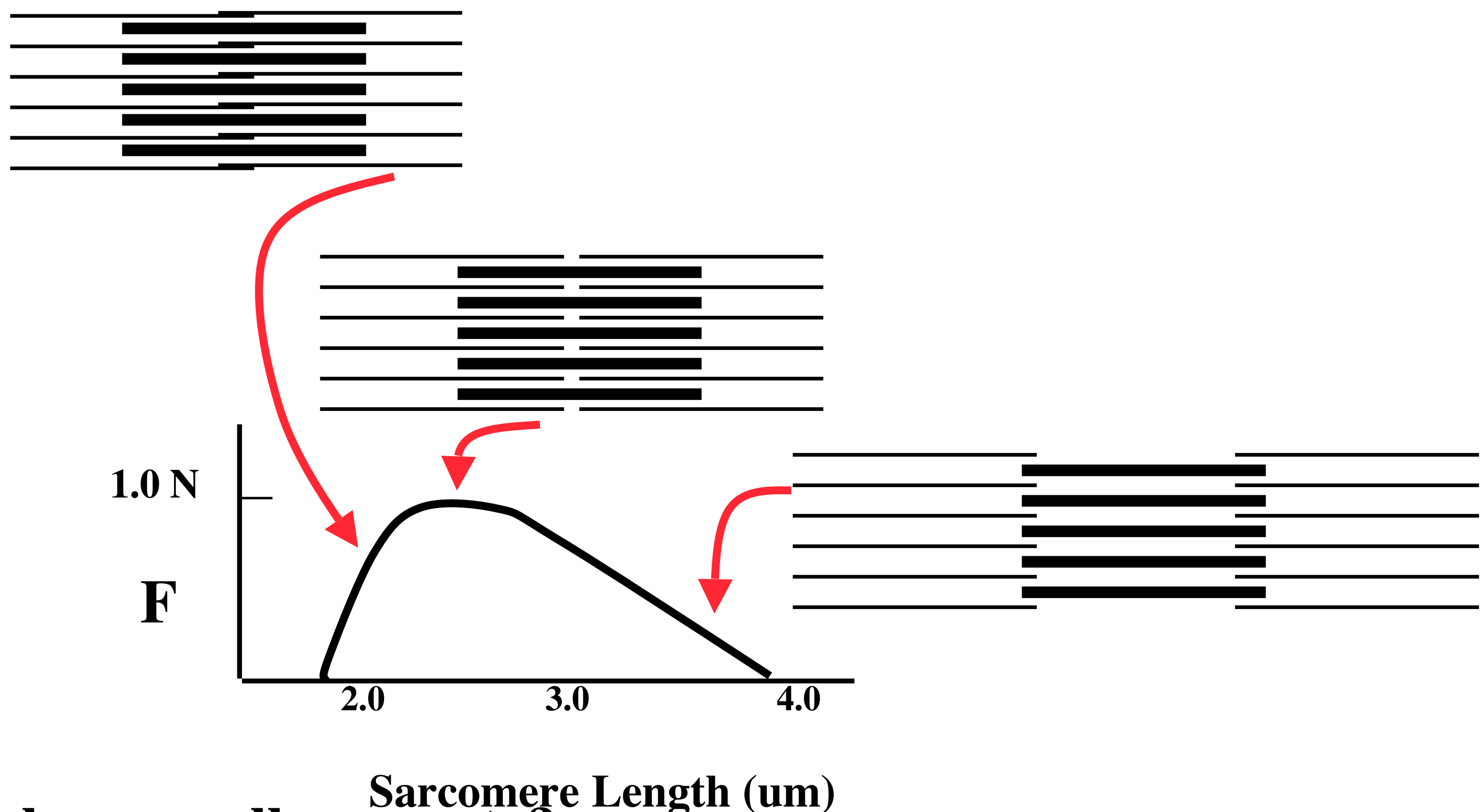
Length of muscle





Gordon, Huxley and Julian 1966

Reminders about Length Tension curves



But where do animals normally operate?
Who cares?
What are the mechanical consequences of dynamic length changes over different parts of this business?

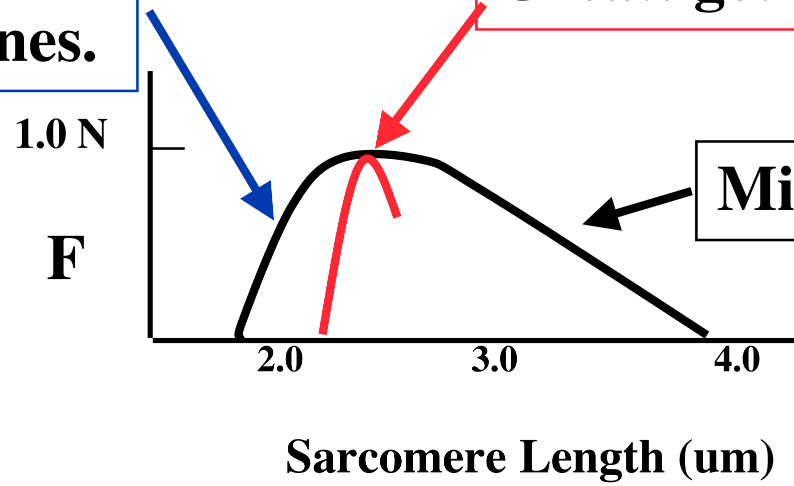
Reminders about Length Tension curves

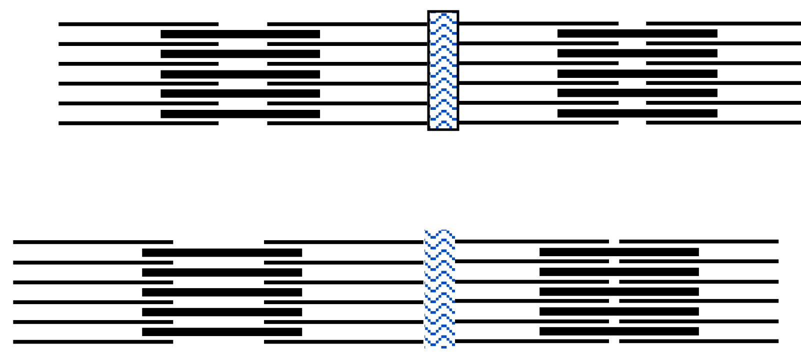
A region with hidden binding sites.
Faster force declines.

Great: get all the force but…. ????

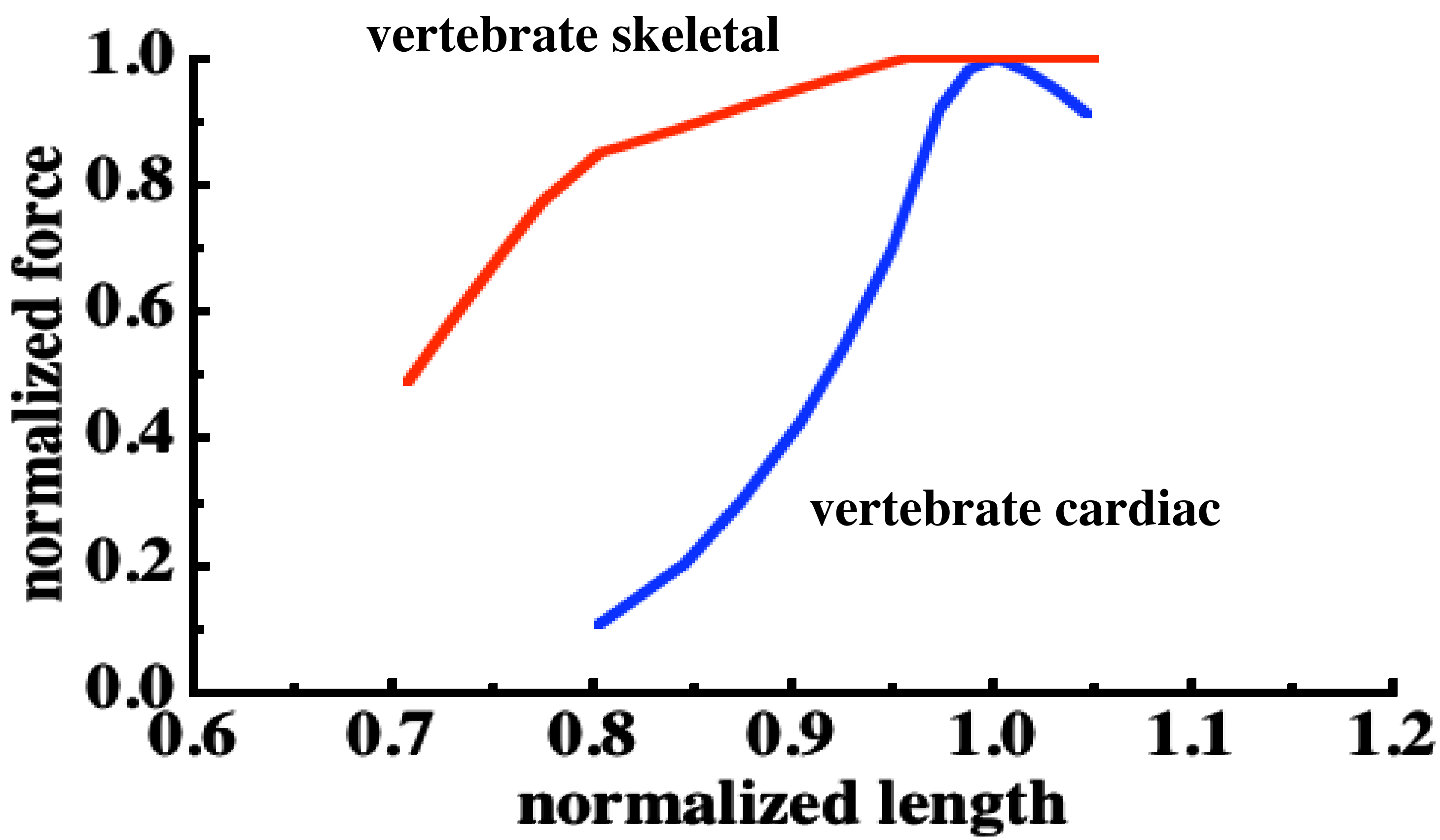
Might get an instability

e.g. hearts;

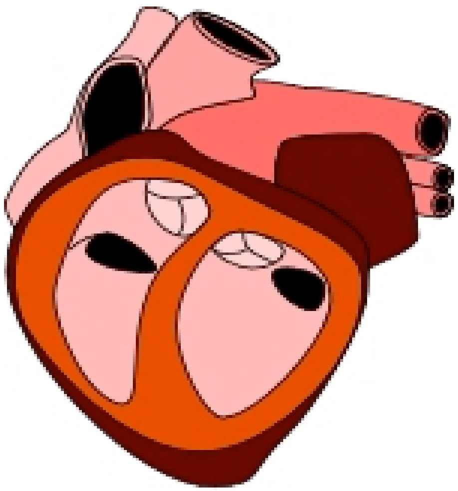




Length - Tension relationships of skeletal and cardiac muscle differ considerably



Regulation of Cardiac Output

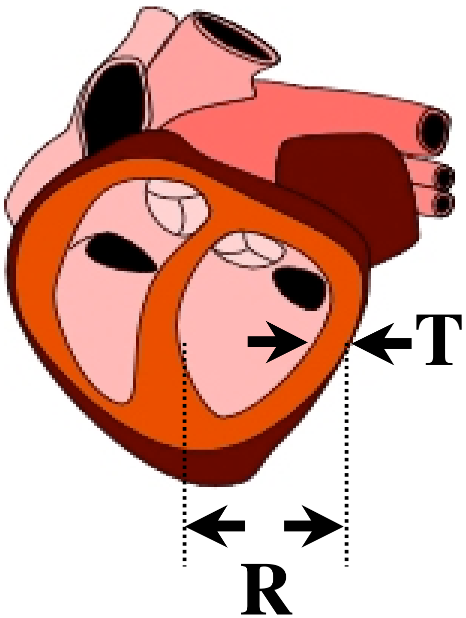


Frank-Starling Law:

Stroke volume $\propto$ End-diastolic volume

(increased cardiac output due to increased ventricular filling)

Frank-Starling Law: Mechanical Consequences



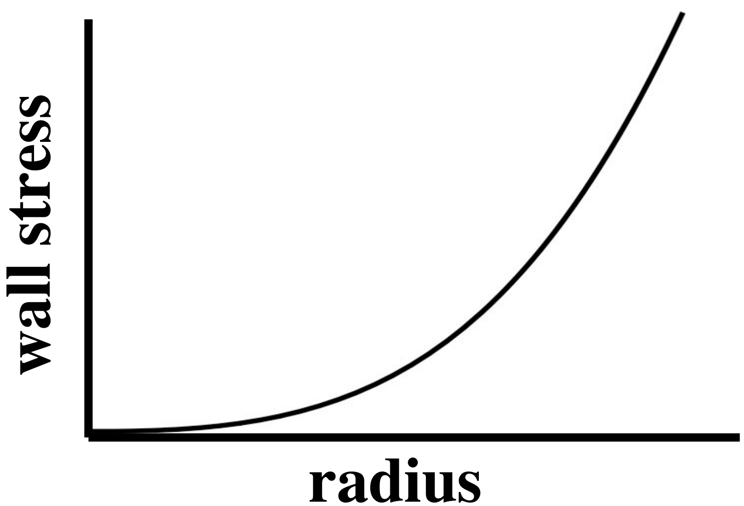
Volume of Wall: $V_w = 4/3\pi(R^3 - (R-T)^3)$

$$V_w \propto R^2T$$

With constant muscle volume: $T \propto \dfrac{1}{R^2}$

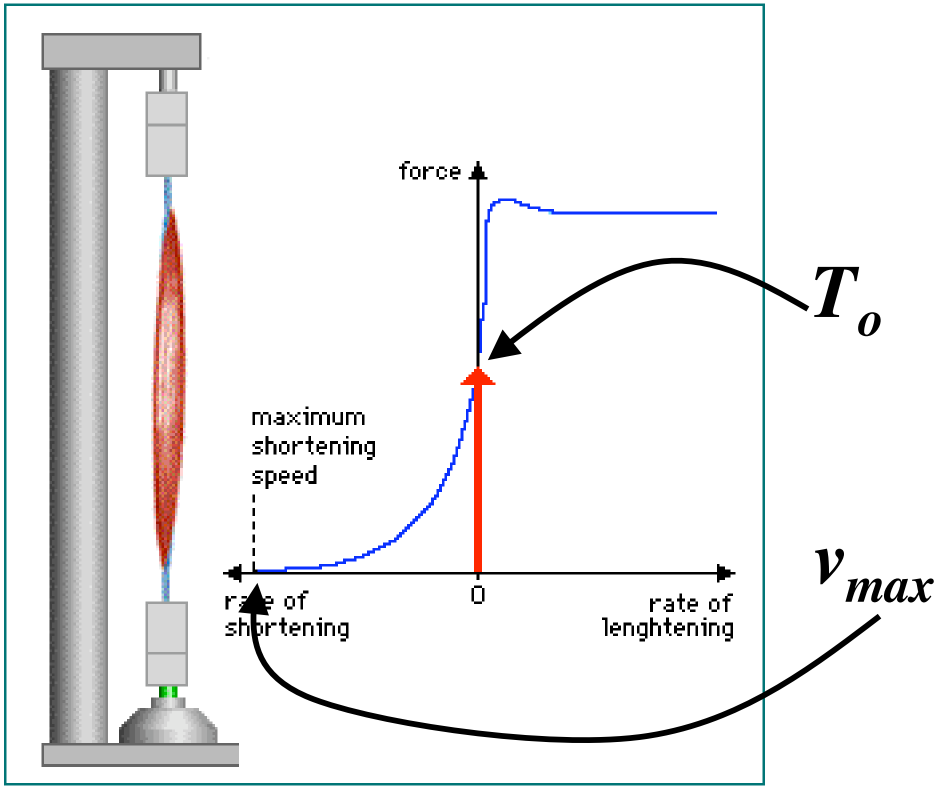
Stress in wall: $\sigma = \dfrac{PR}{T}$

$$\sigma \propto PR^3$$



What are the determinants of force ?

Speed of muscle shortening



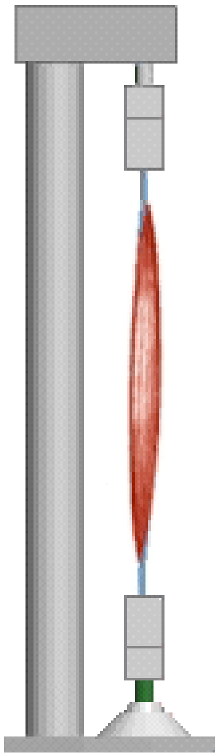
Hill's equation

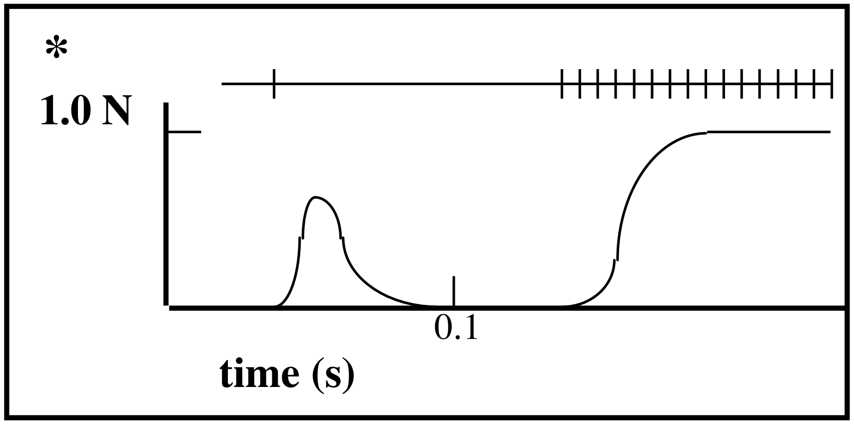
$$T = \frac{b\,T_o - a\,v}{v + b}$$

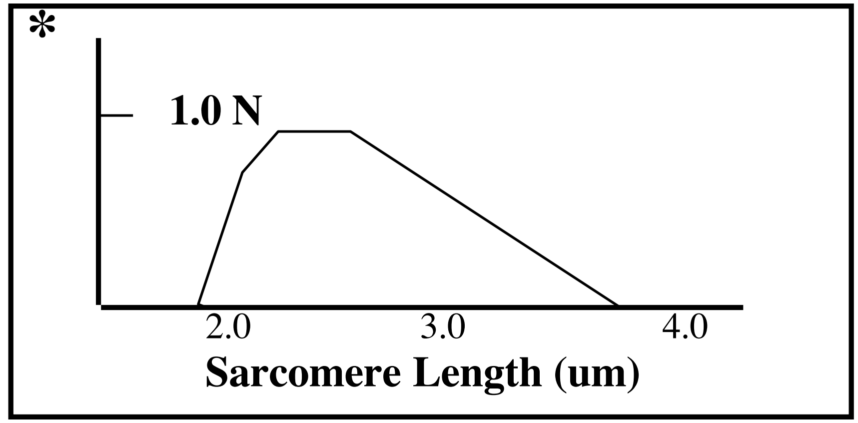
$$b/v_{max} \approx a/T_o \approx 1/4$$

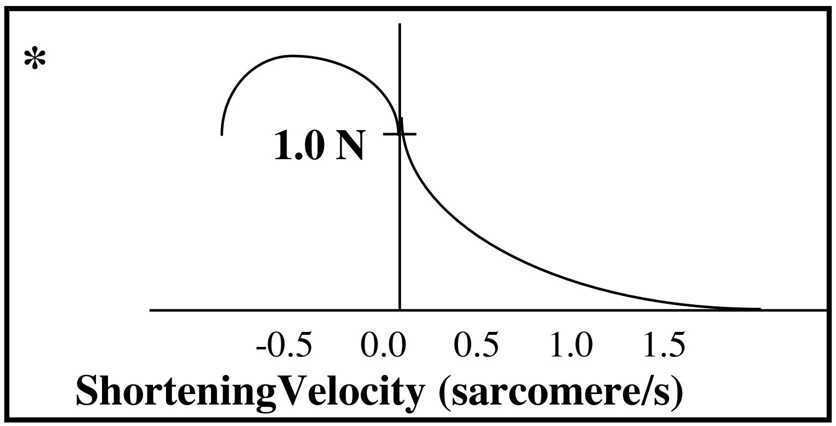
$$\frac{T}{T_o} = \frac{v_{max} - v}{4v + v_{max/4}}$$

Force = *f*(*t*,*l*,*v*)



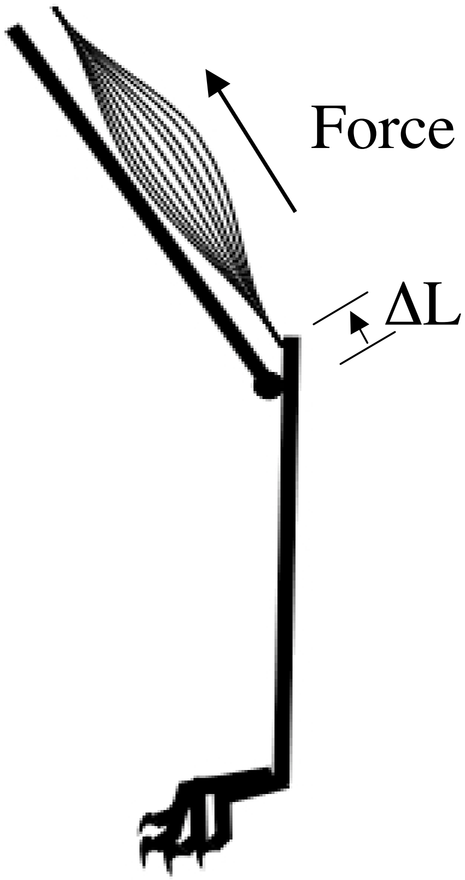






Isometric and isotonic conditions are unlikely in physiologically relevant situations.

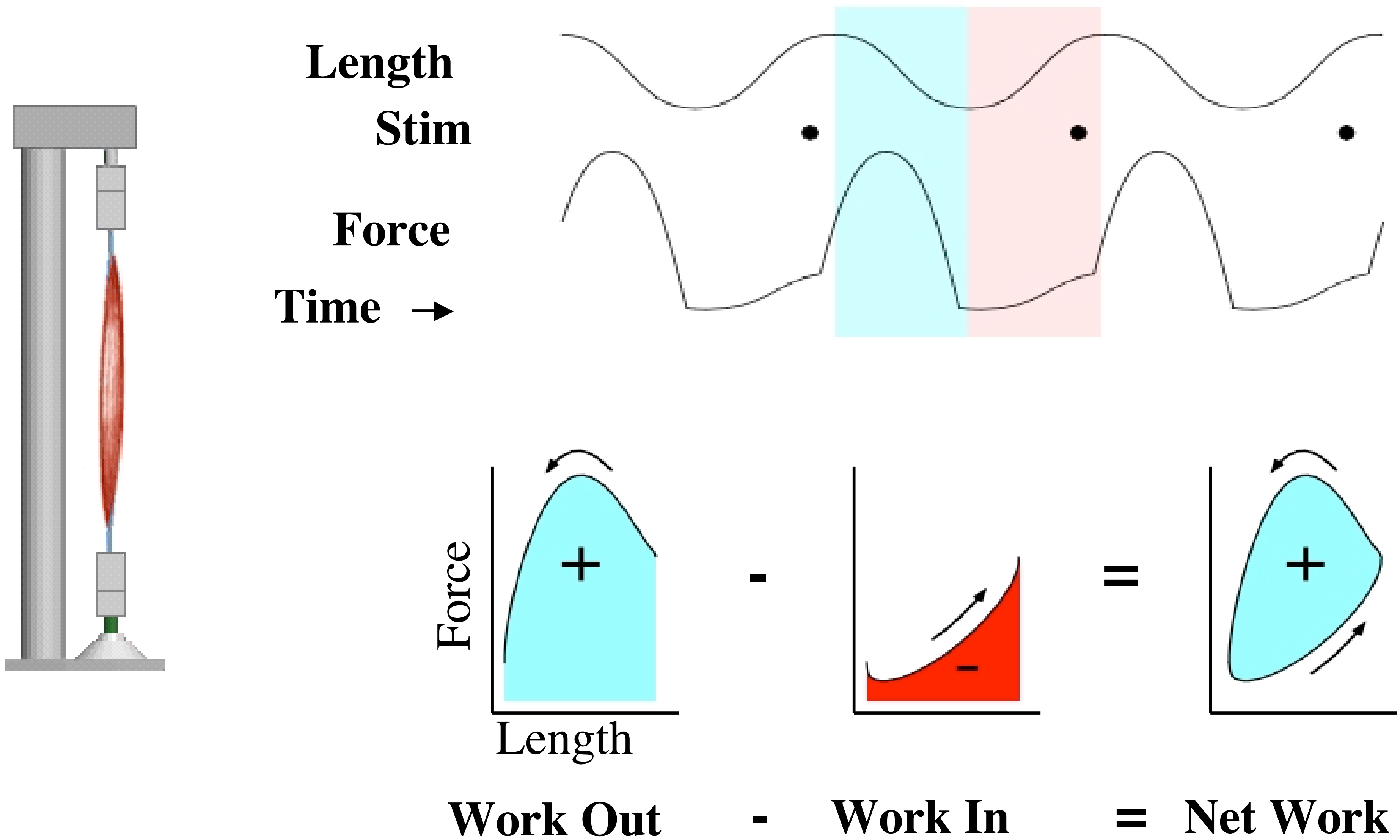
Muscle Function: Generate mechanical power



Work = F x ΔL (Joules)

Power = Work x Frequency
(Joules/s = Watts)

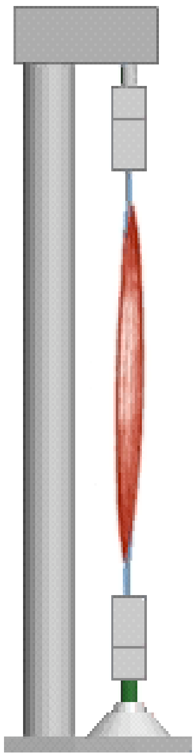
Workloop Methods

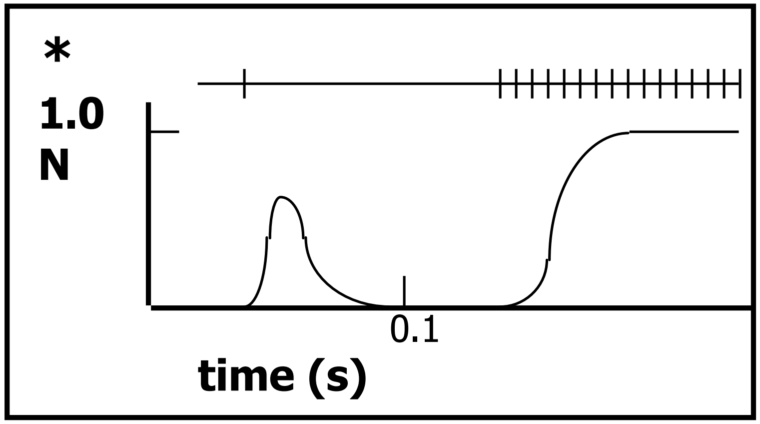


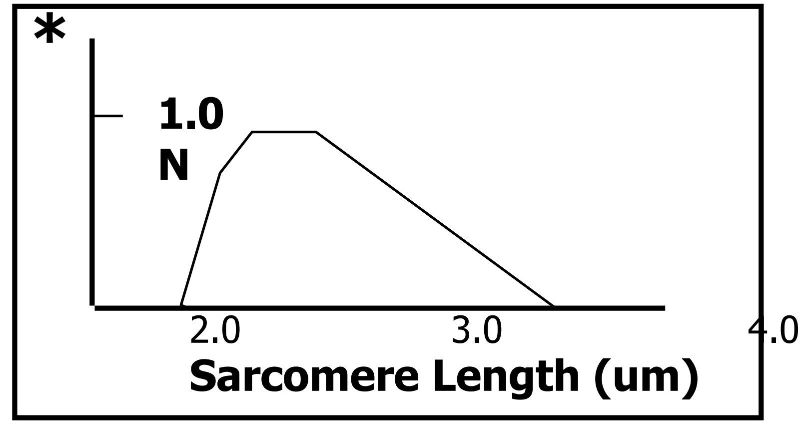
Lecture 4: Terrestrial Locomotion I Simple Analyses of Ballistic Movement.

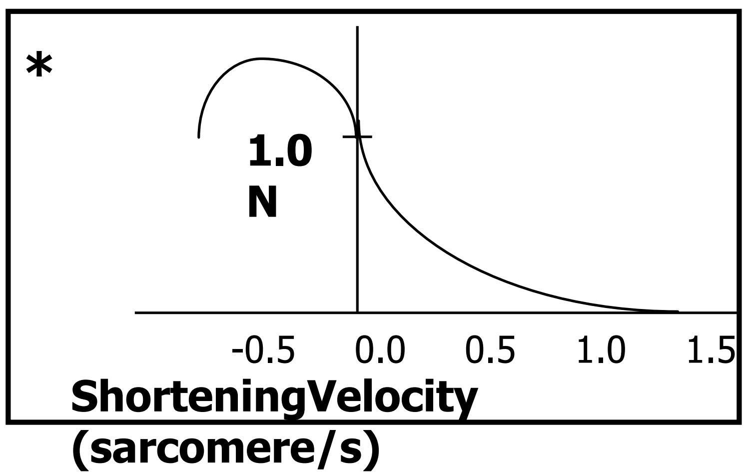
- Recap: projectiles and muscle
- Current approaches for analyzing terrestrial locomotion.
- Gaits and patterns of limb motion
- Ballistic walking and the inverted pendulum

Force = f(t,l,v)



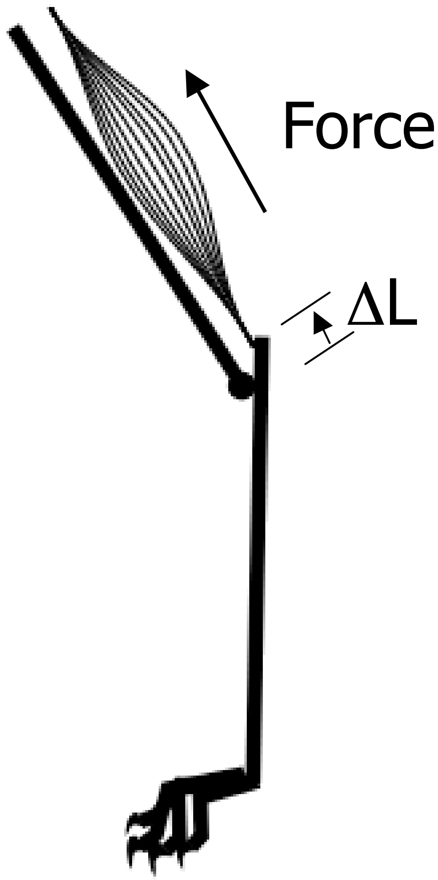






Isometric and isotonic conditions are unlikely in physiologically relevant situations.

Muscle Function: Generate mechanical power



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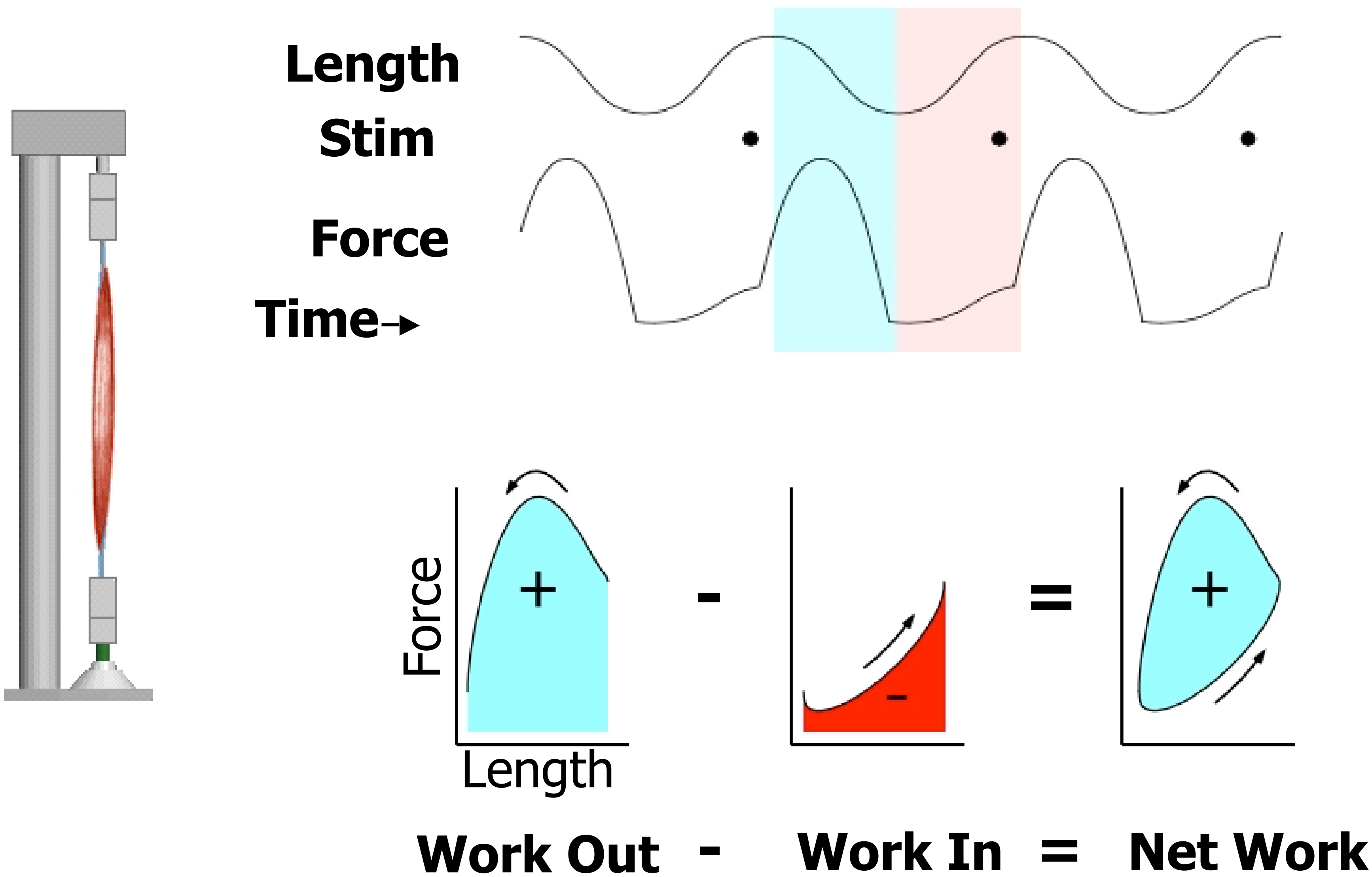
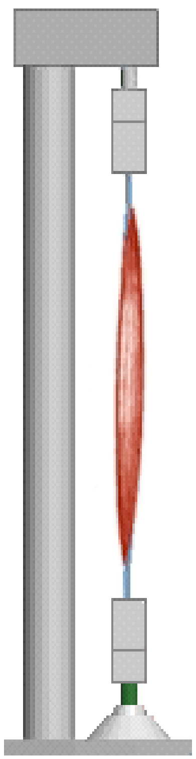
(Joules/s = Watts)

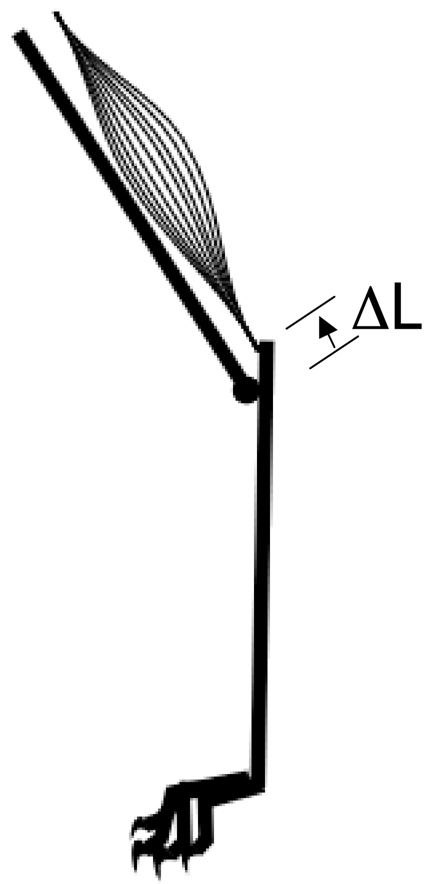
Workloop Methods



What are the determinants of force ?

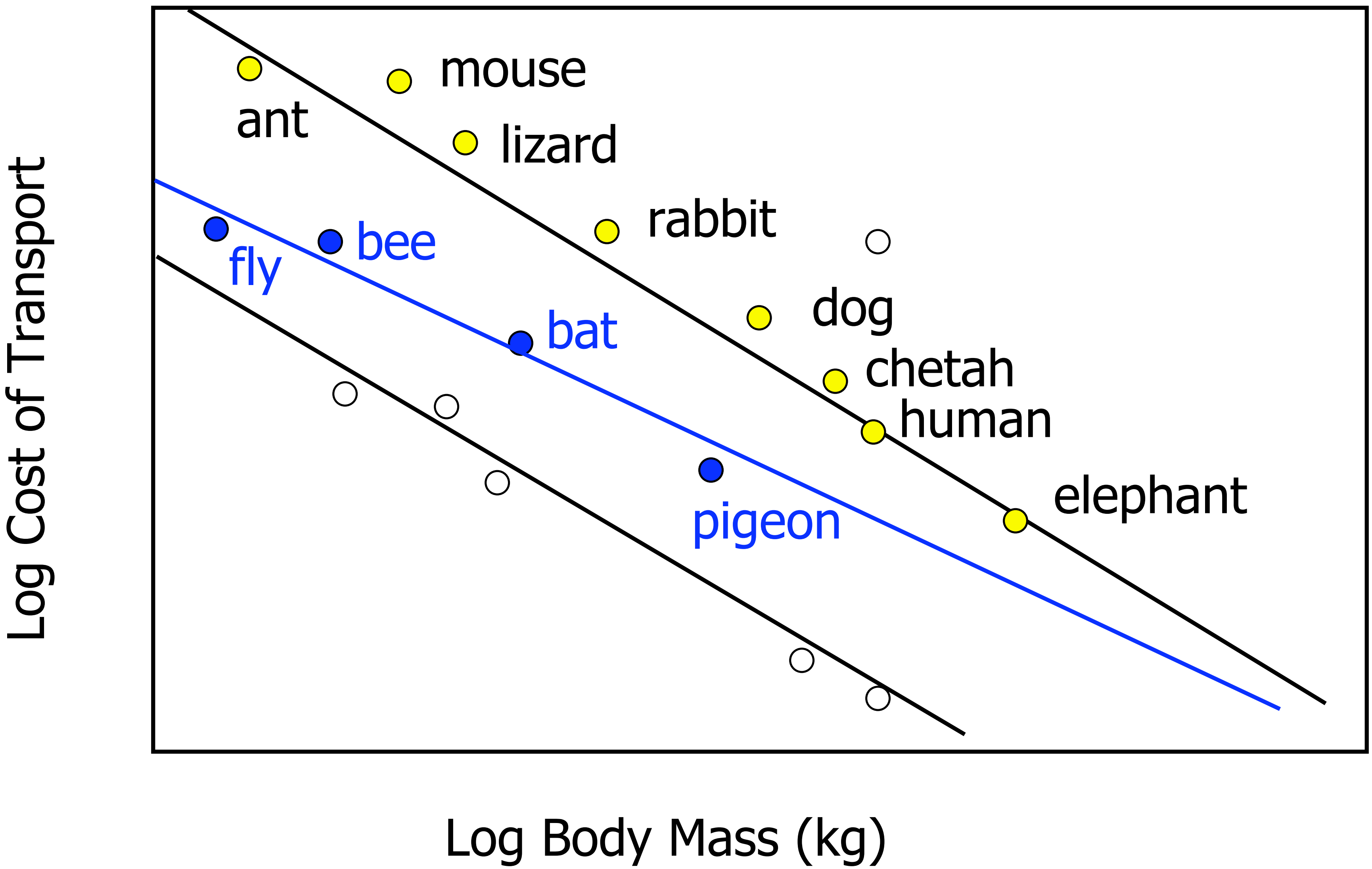
Run the "work loop" program on page 21 in the chapter on muscle in the CD





$$Md^2x/dt^2 = F(x,dx/dt,t)$$

Cost of transport declines with body size



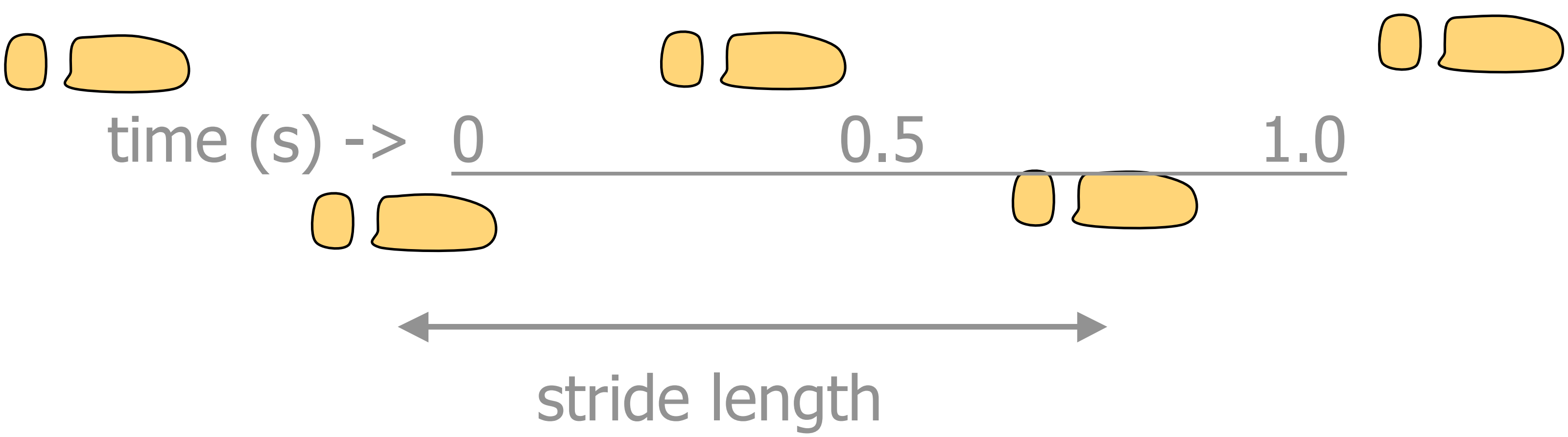
Movement Biomechanics

Patterns in time and space (Kinematics)

Forces

Dynamics (control and stability)

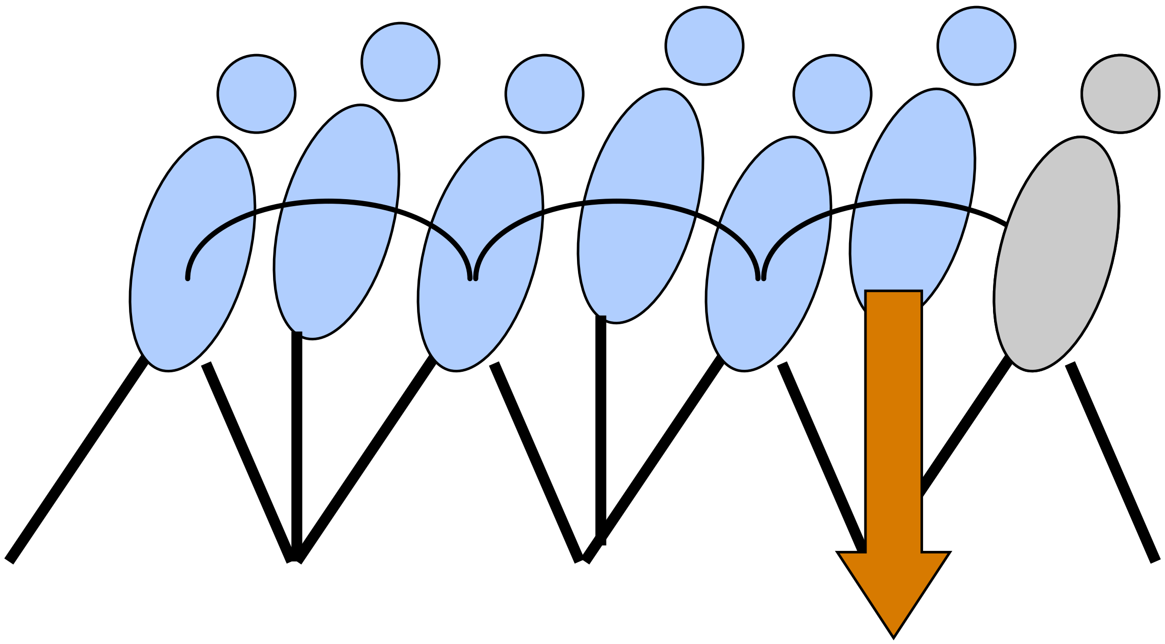
Energy



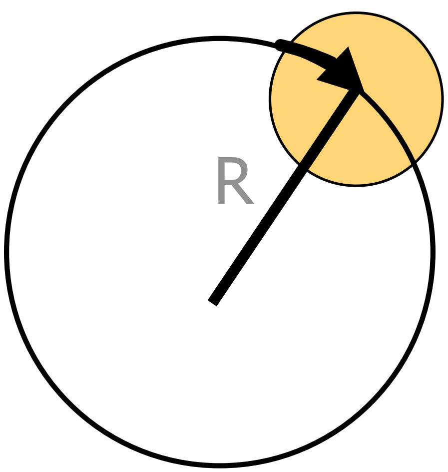
GAITS

- stride length: distance between footfalls of the same foot
- stride frequency: number of footfalls per time
- duty factor: fraction of stride time that a foot is on the ground (human walking = 0.5 - 0.6, running = 0.35). Gaits with duty factors less than 0.5 imply airborne phases.
- relative phase: time a foot is set down as a fraction of the stride time.

Simple Quantifiers of Movement on Land

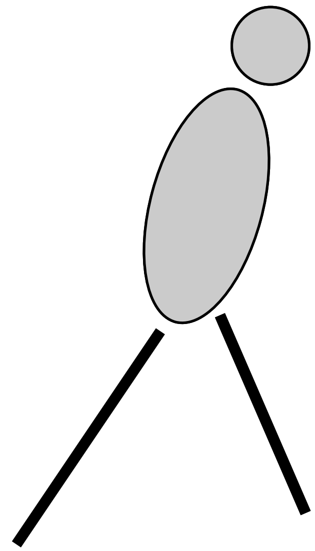


$$V^2 < g L$$

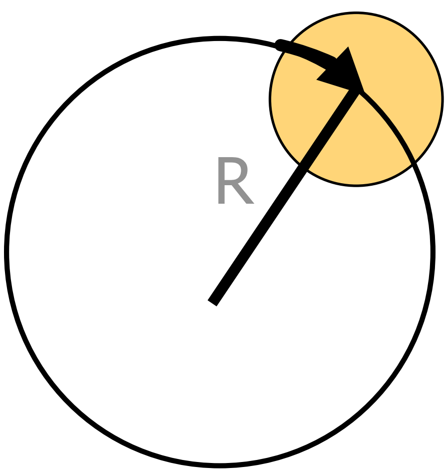


$$V^2 = a R$$

Simple Quantifiers of Movement on Land



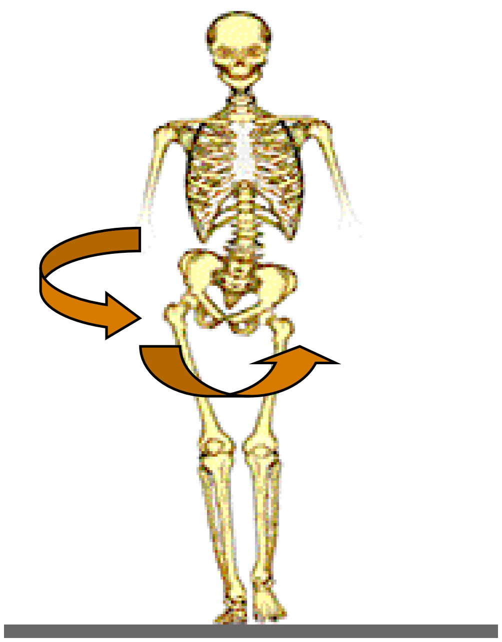
$$V^2 < g\,L$$

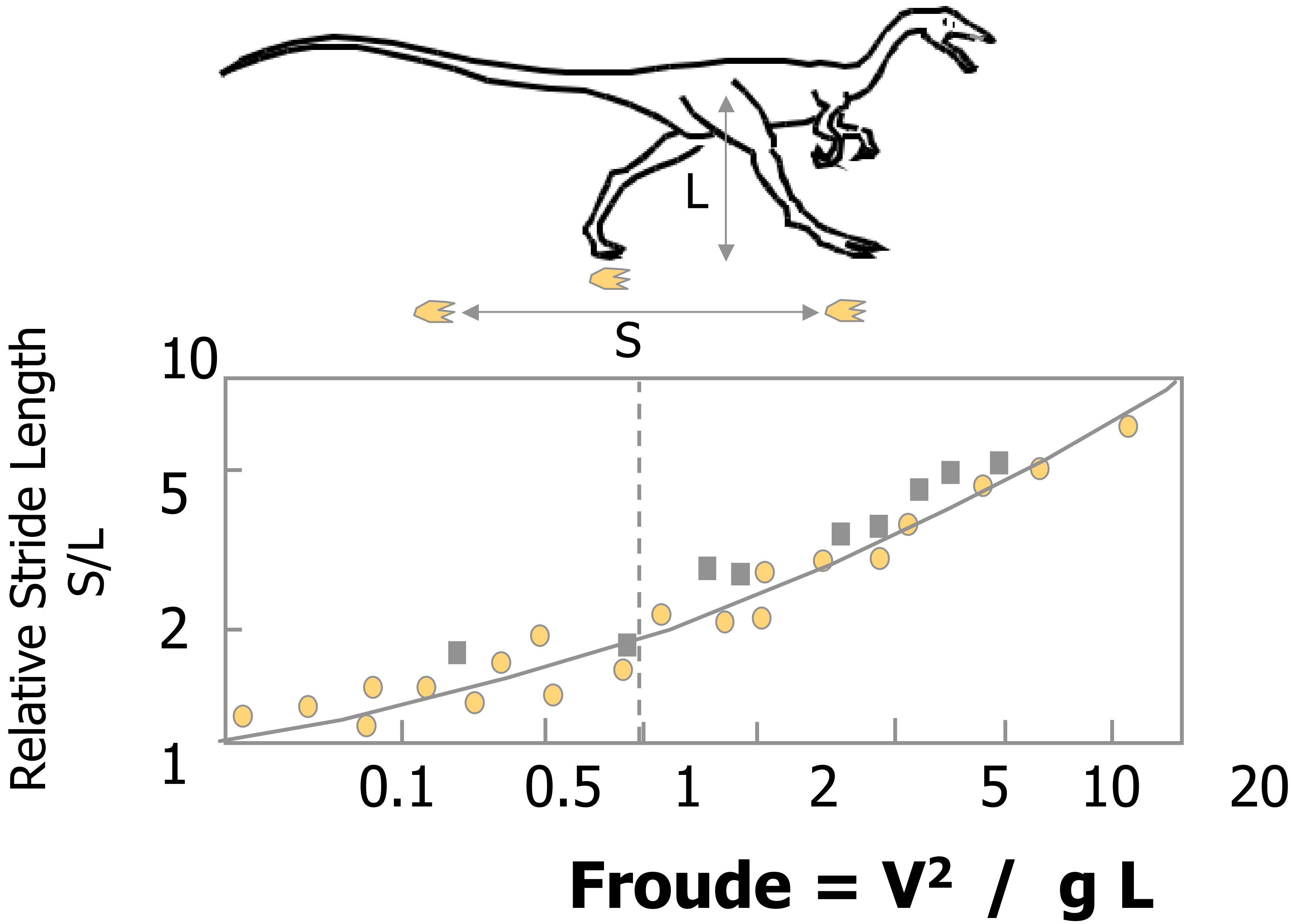


$$V^2 = a\,R$$

Modifiers of the radius of curvature

Lumbar flexion
Pelvic rotation
Pelvic tilt




L
S
Relative Stride Length
S/L
10
5
2
1
0.1
0.5
1
2
5
10
20
Froude = V^2 / g L

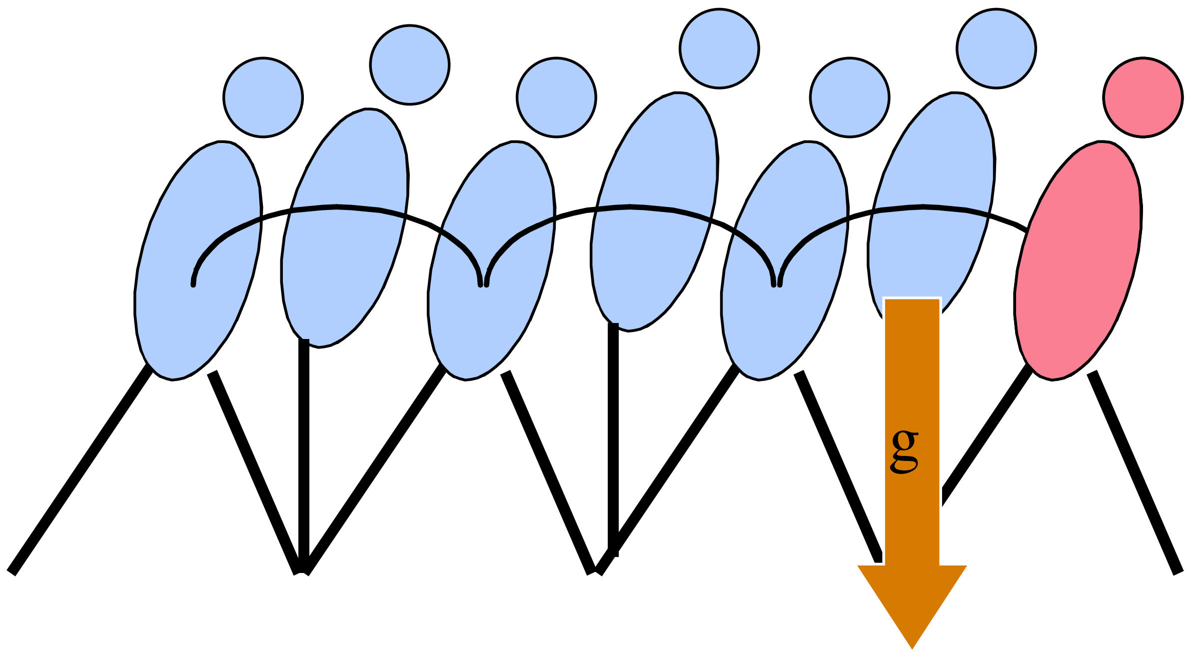
Biology 427 Biomechanics

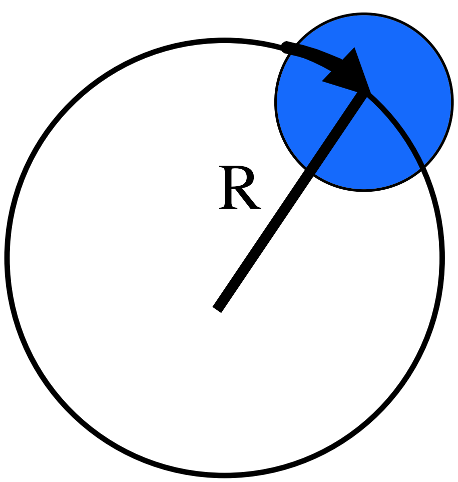
Lecture 5. Terrestrial locomotion II: mechanical analysis of gaits and jumpiness.

- Recap: gaits and ballistic walking
- When the Froude Number ($V^2/g\ L$) is greater than 1, simple ballistic walking is no longer possible.
- The jumper model accounts for an airborne phase of movement.
- Calculating optimal gaits for energy expenditure

Simple Quantifiers of Movement on Land

“Ballistic Waking”

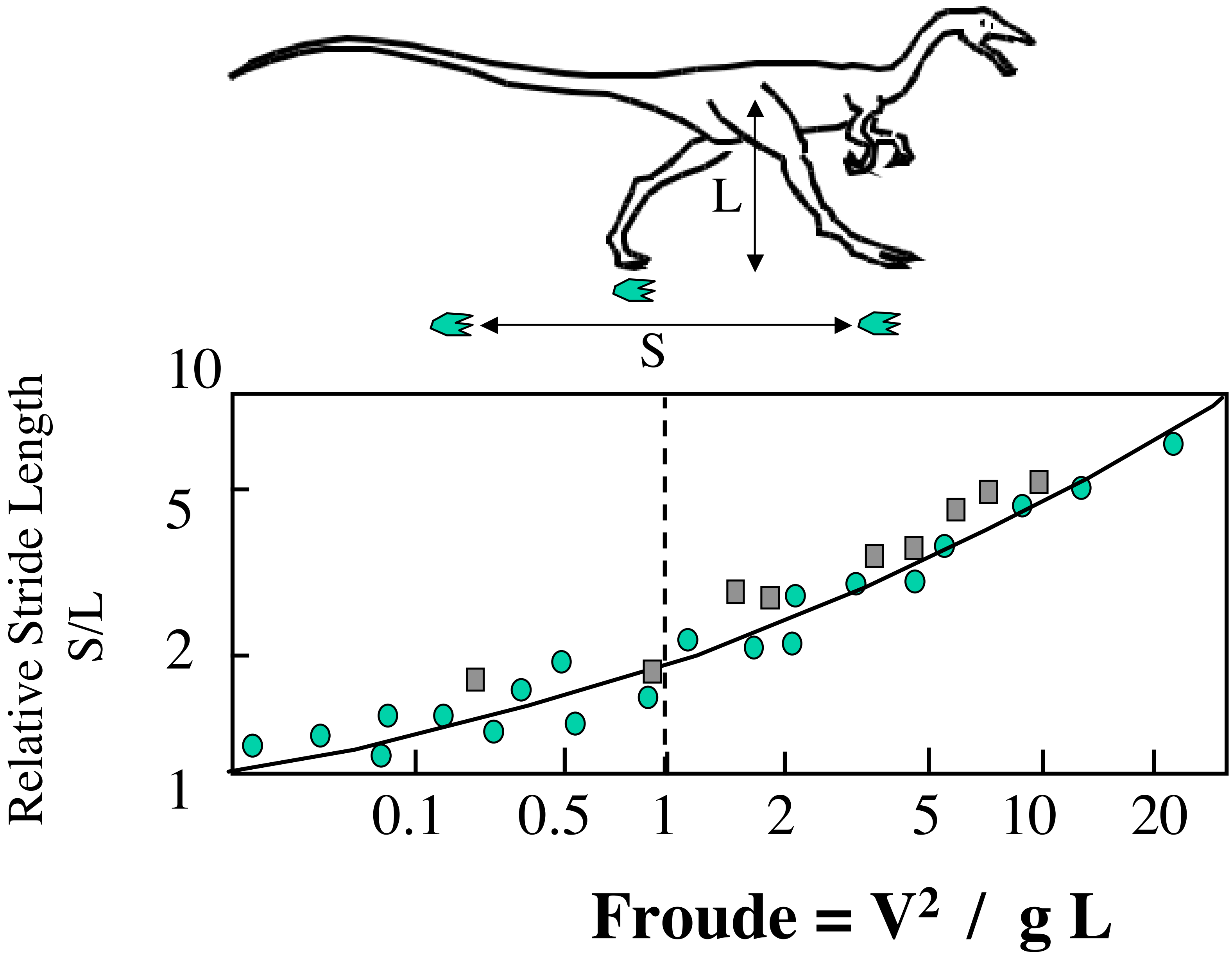




$V^2 = a\ R$

$$\mathbf{V^2 < g\ L}$$

$$\mathbf{Fr = V^2/gL}$$



L
S
Relative Stride Length
S/L
10
5
2
1
0.1
0.5
1
2
5
10
20
Froude = V² / g L

The jumper: for speeds greater than Fr = 1, gait must change with an airborne phase

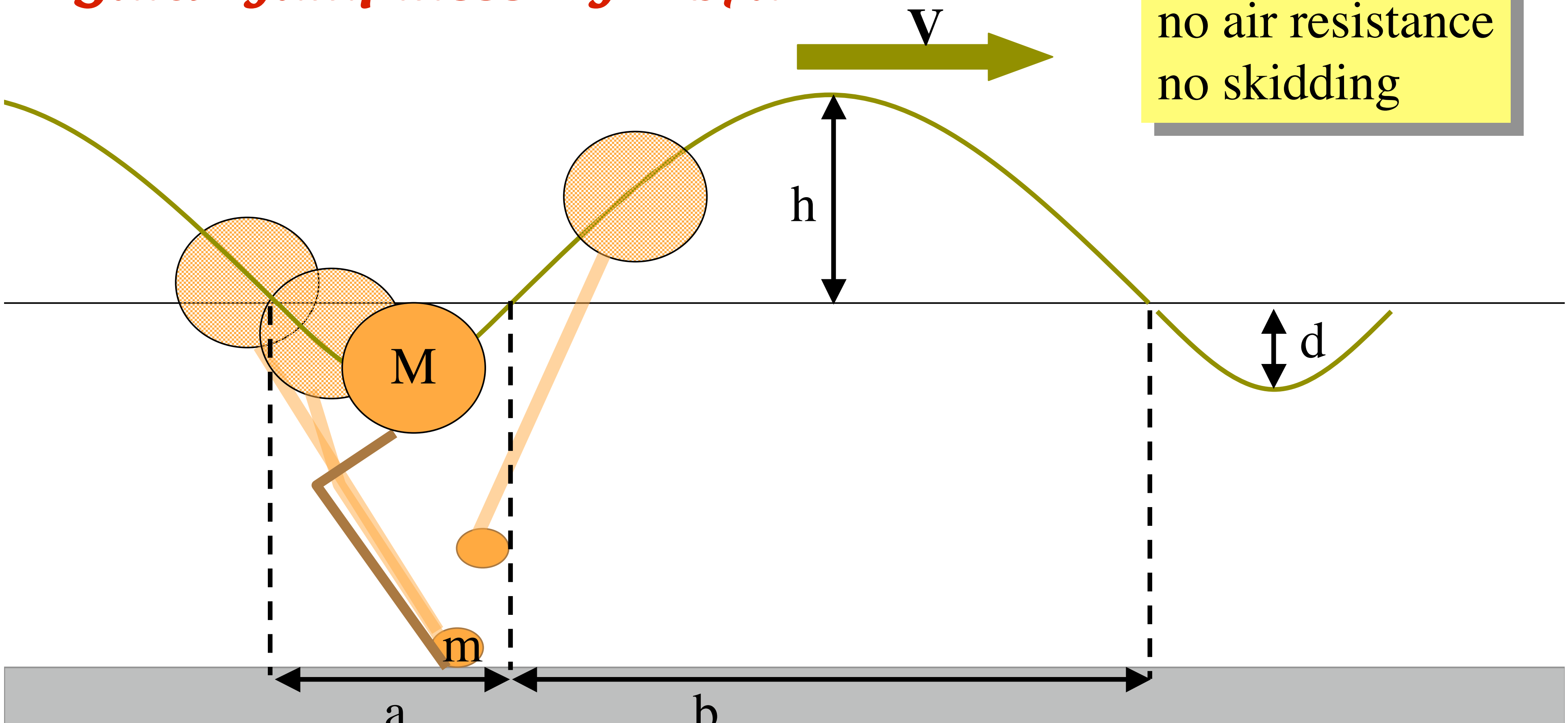
gait: jumpiness = j = b/a

Assume

constant V

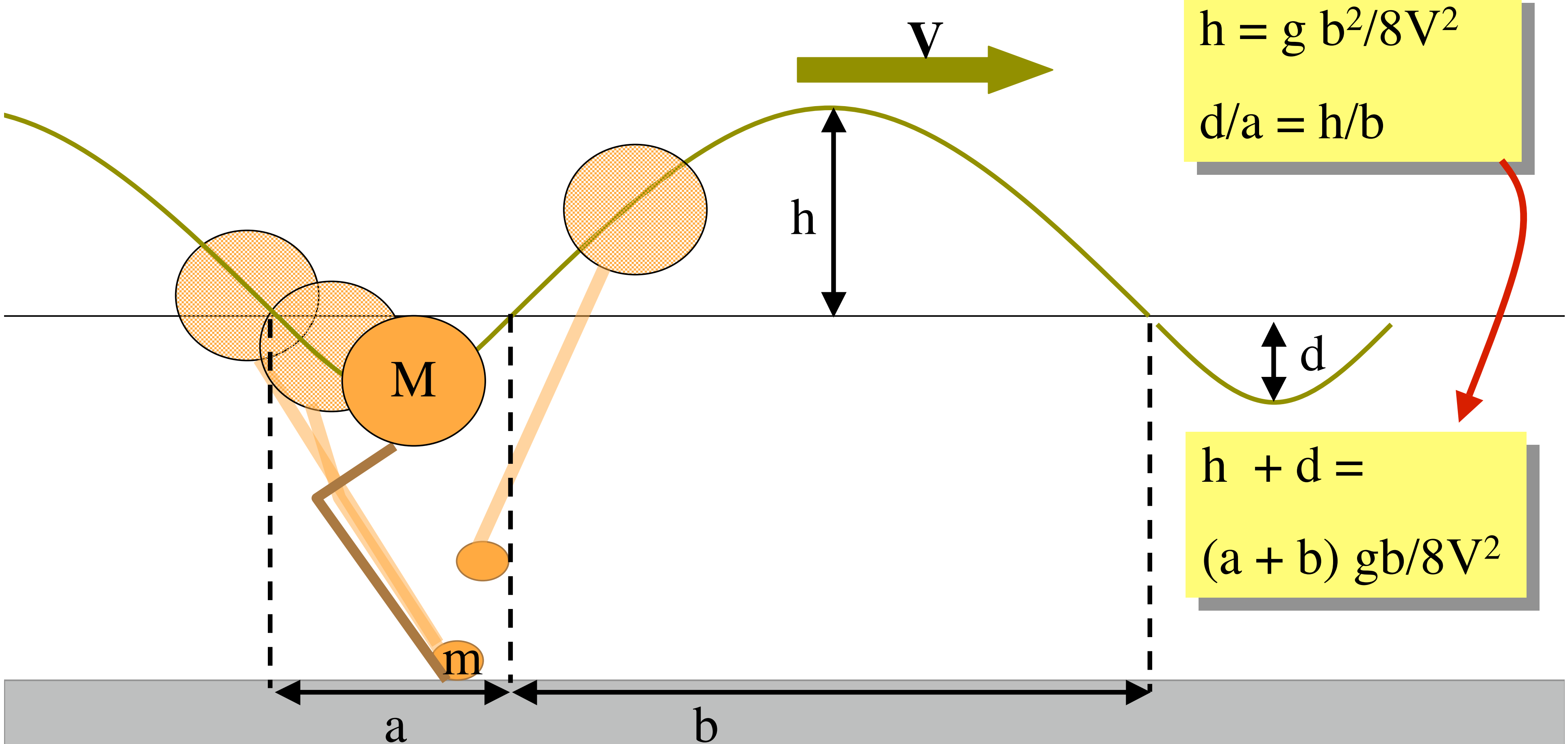
no air resistance

no skidding



Goal: compute the power (rate of energy expenditure) as a function of size (M,m) and gait (j = b/a)

Goal: compute the power (rate of energy expenditure) as a function of size (M,m) and gait (j = b/a)



$h = g\, b^2/8V^2$

$d/a = h/b$

$h + d =$

$(a + b)\, gb/8V^2$

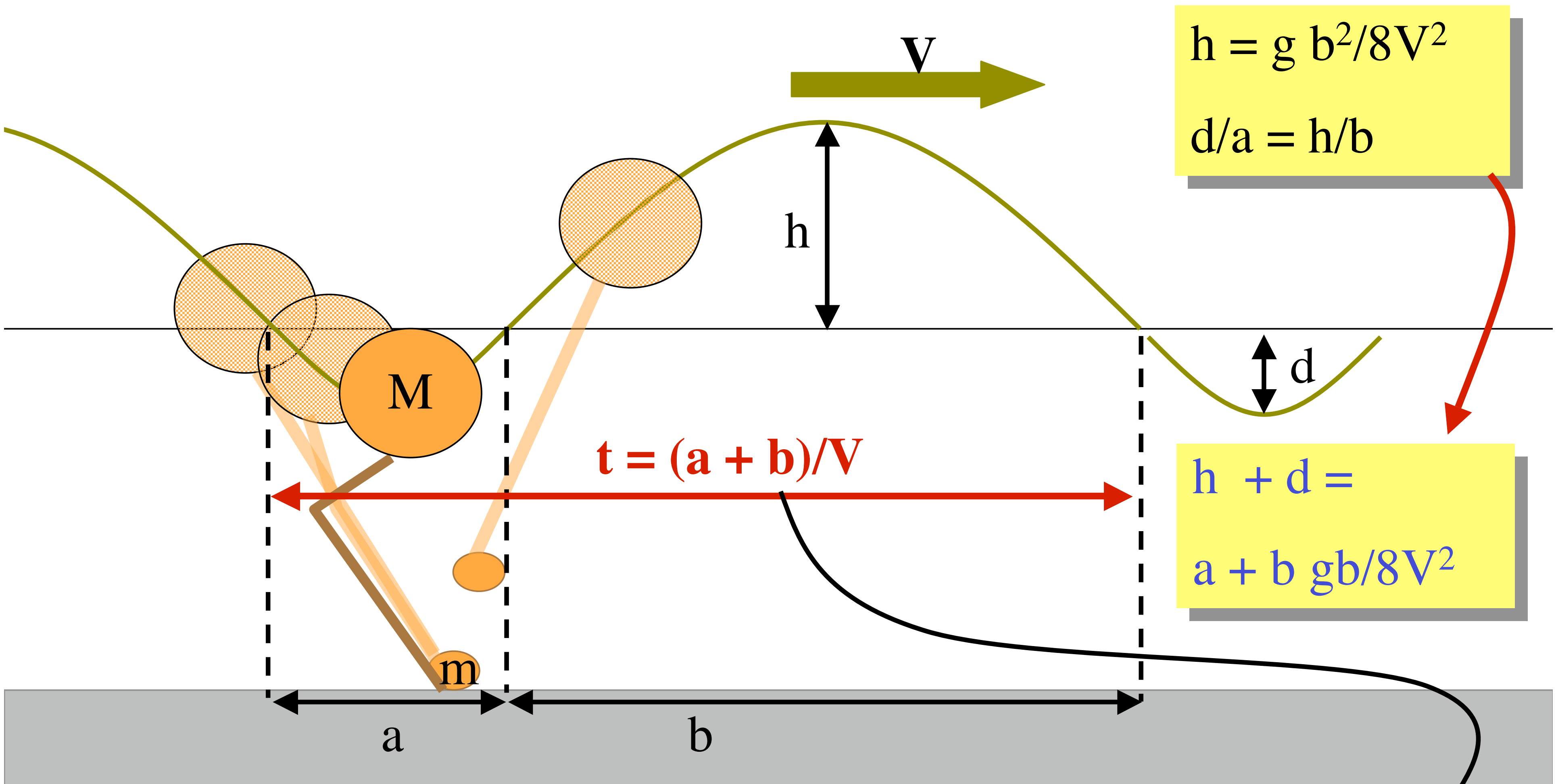
Energy =

work to launch the body $= M g (h + d)$

work to break the fall $= \gamma M g (h + d)$

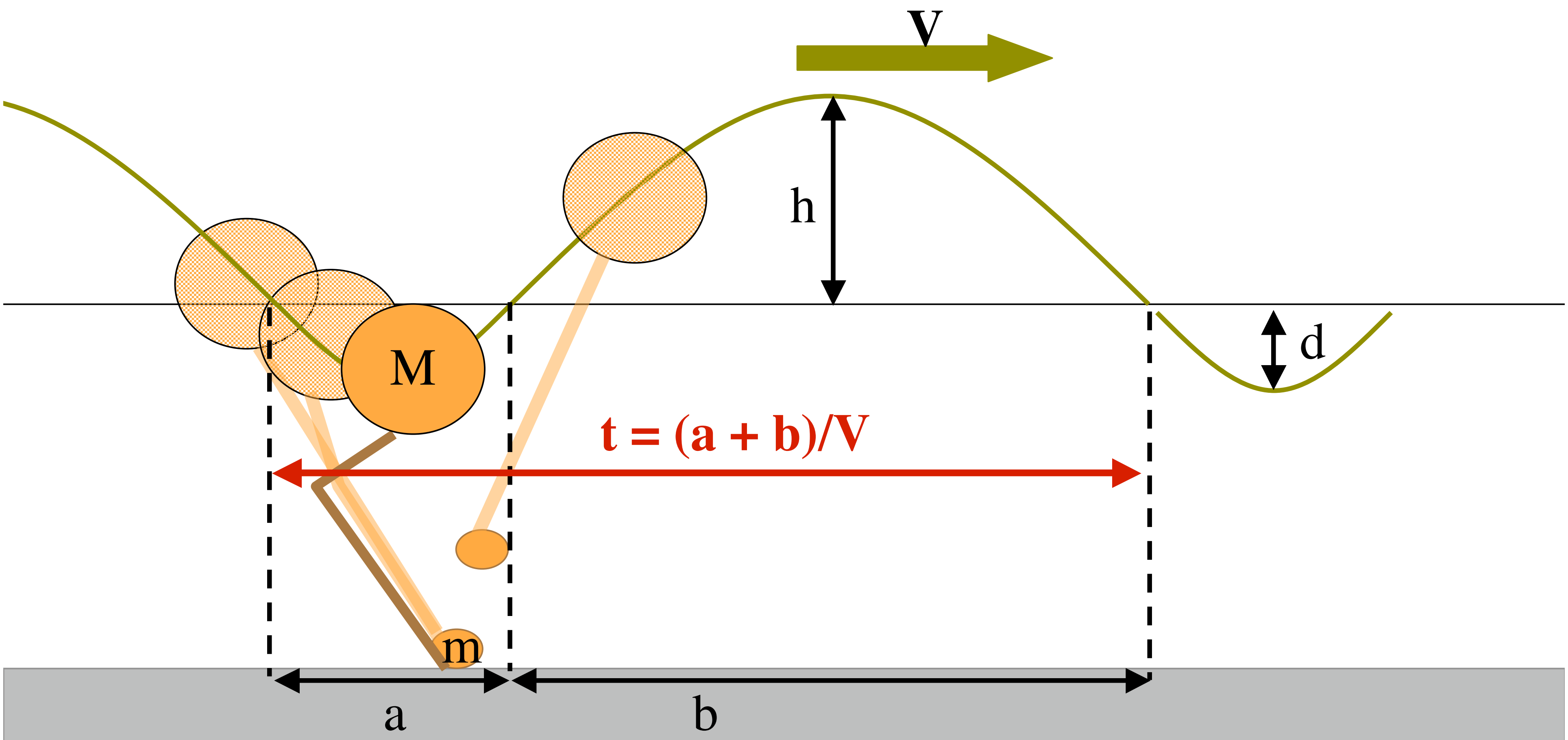
work to move the foot $= m V^2/2$

Goal: compute the power (rate of energy expenditure) as a function of size (M,m) and gait (j = b/a)



Energy = γ M g (h + d) **+** M g (h + d) + m $V^2/2$
= $(1 + \gamma)$ M g (h + d) **+** m $V^2/2$

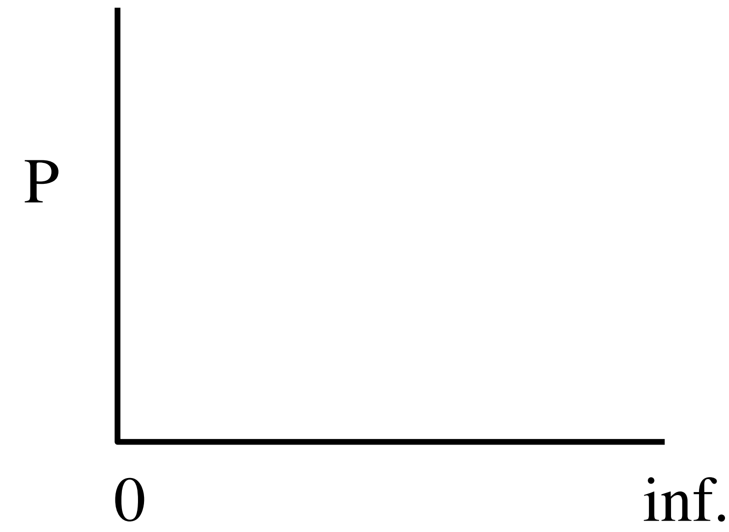
Goal: compute the power (rate of energy expenditure) as a function of size (M,m) and gait (j = b/a)

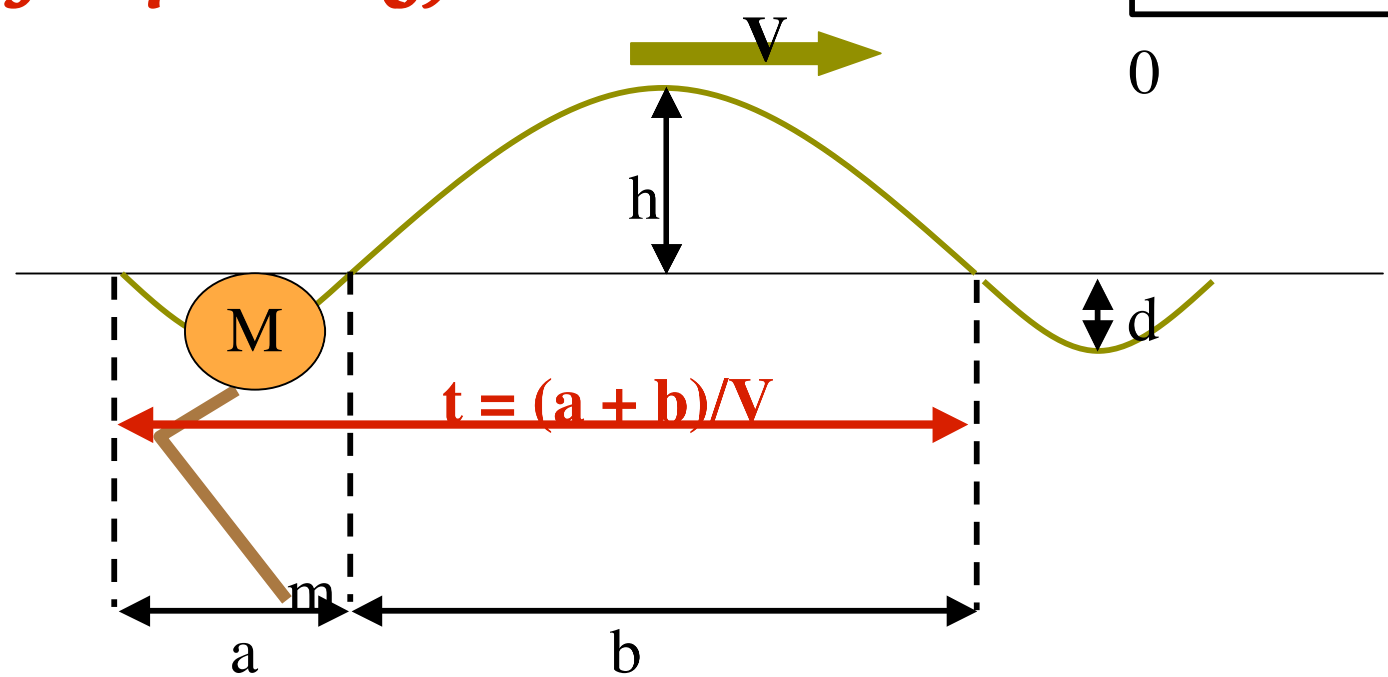


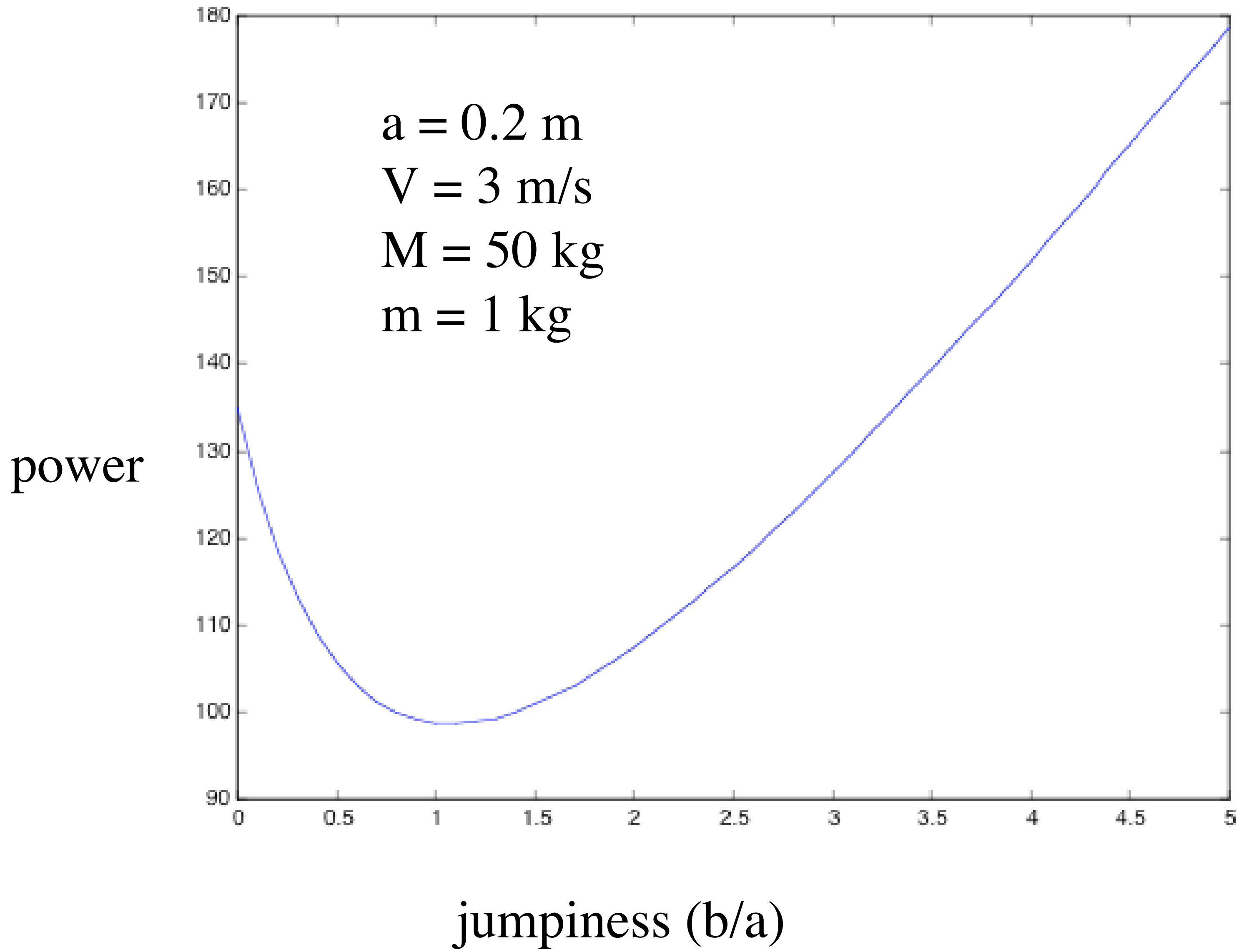
$$\textbf{Power} = (1+\gamma)\, M\, g^2\, b/8V + m\, V^3/2(a+b)$$
$$= (1+\gamma)\, M\, g^2\, a\, j\, /8V + m\, V^3/2a(1+j)$$

$$\text{Power} = (1+\gamma)\, M\, g^2\, b/8V + m\, V^3/2(a+b)$$
$$= (1+\gamma)\, M\, g^2\, a\, j\, /8V + m\, V^3/2a(1+j)$$

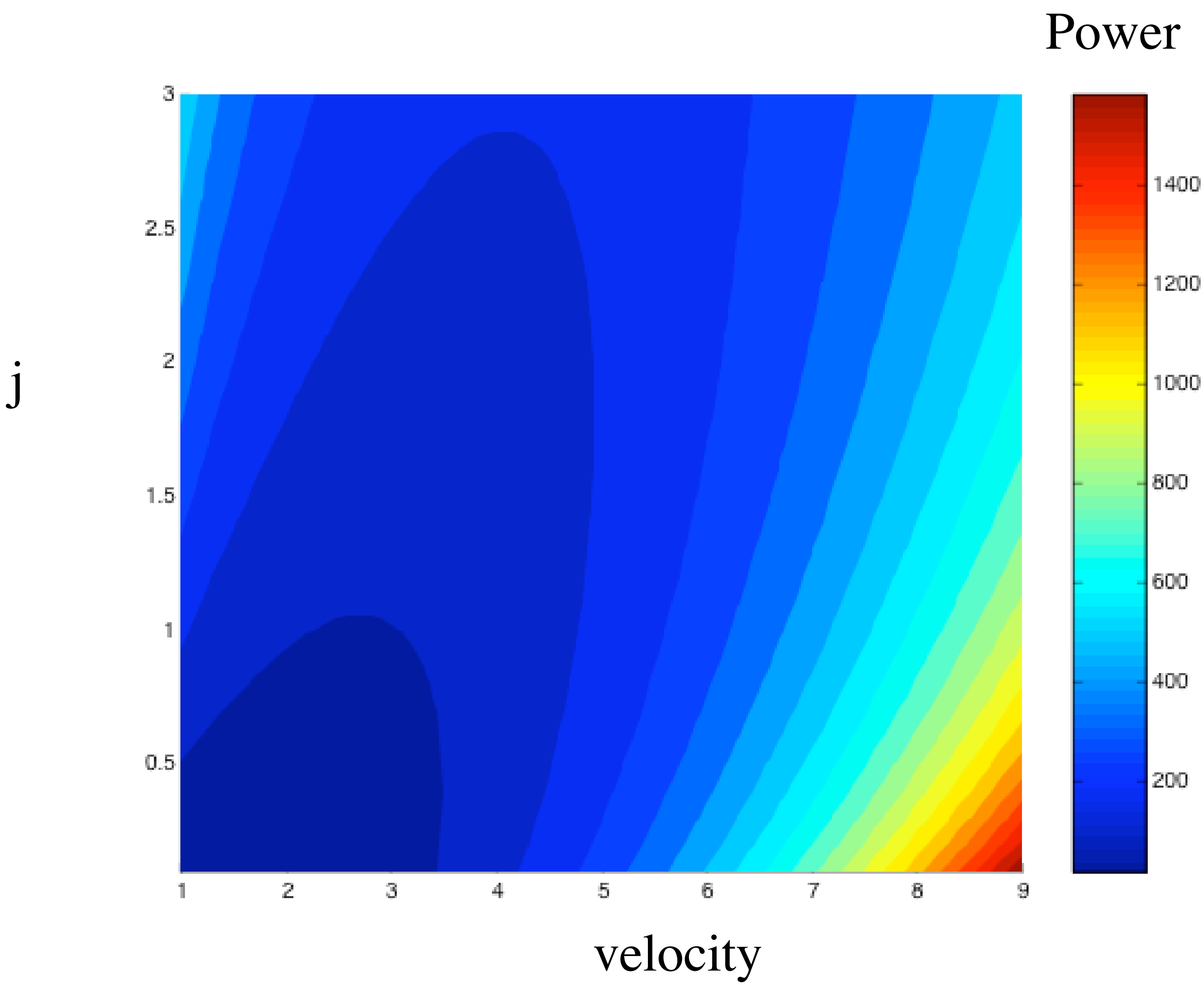
How does Power vary with
foot mass (m)?
body mass (M)?
body velocity (V)?
leg length (~a)?
jumpiness (j)?







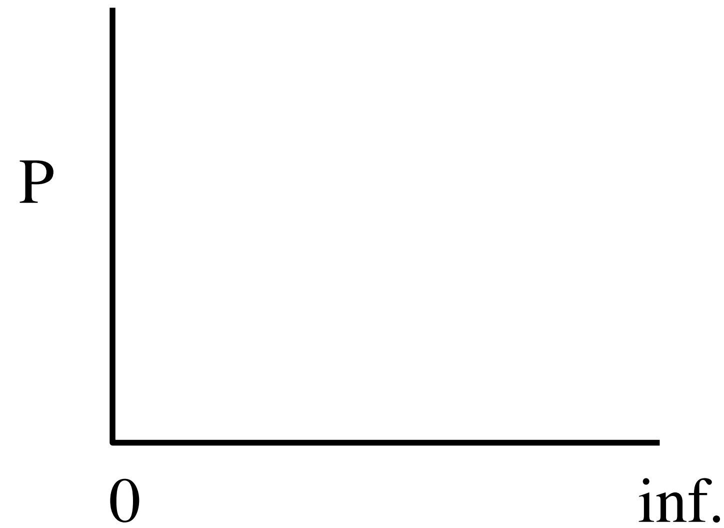
a = 0.2 m
V = 3 m/s
M = 50 kg
m = 1 kg
power
jumpiness (b/a)
180
170
160
150
140
130
120
110
100
90
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

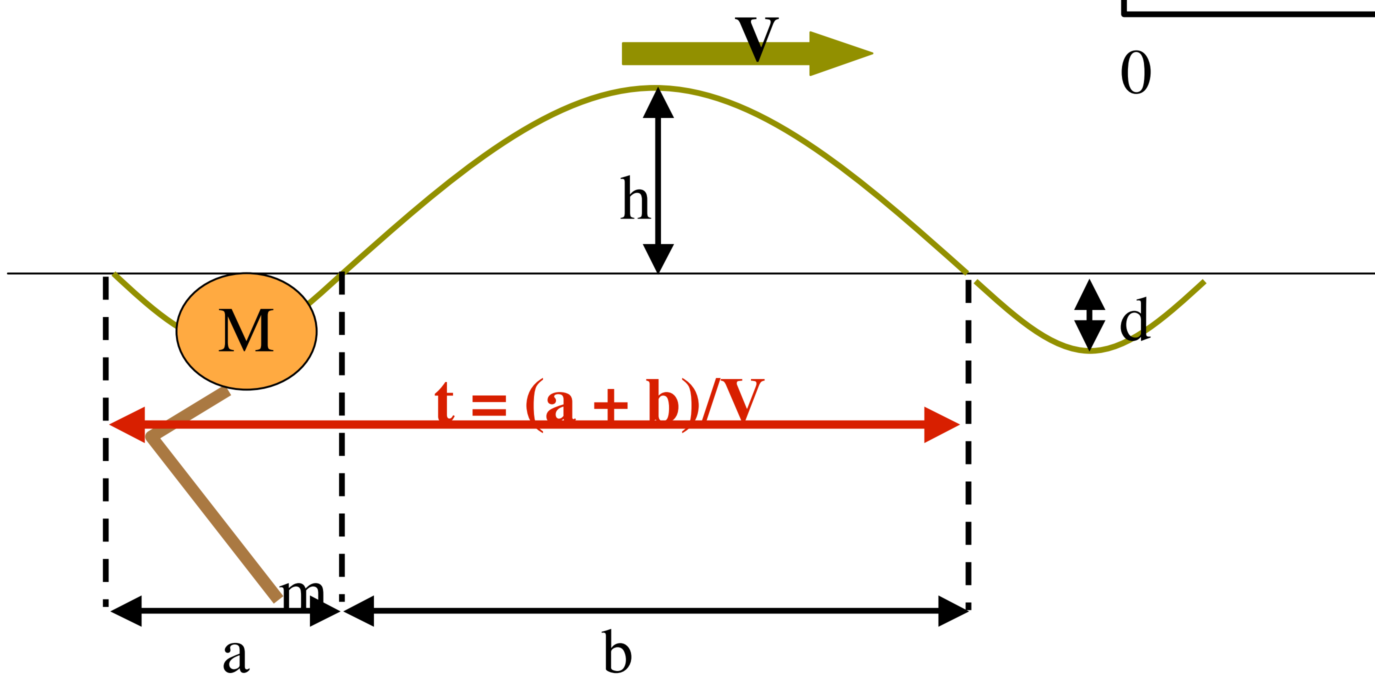


Power
1400
1200
1000
800
600
400
200
j
3
2.5
2
1.5
1
0.5
1
2
3
4
5
6
7
8
9
velocity

$$\text{Power} = (1+\gamma)\, M\, g^2\, b/8V + m\, V^3/2(a+b)$$
$$= (1+\gamma)\, M\, g^2\, a\, j\,/8V + m\, V^3/2a(1+j)$$

dP/dj = 0 defines a maximum





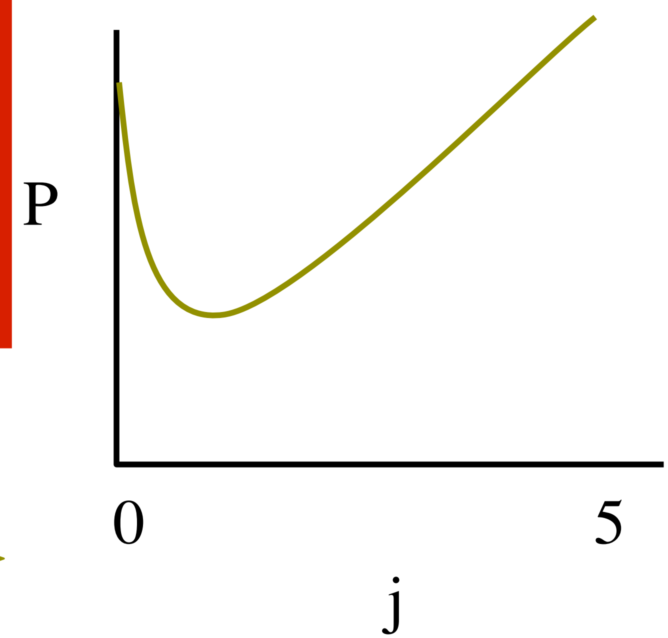
Biology 427 Biomechanics

Lecture 6. Everyday stress and strain and the stiffness of biological materials I: terms, definitions and other basics

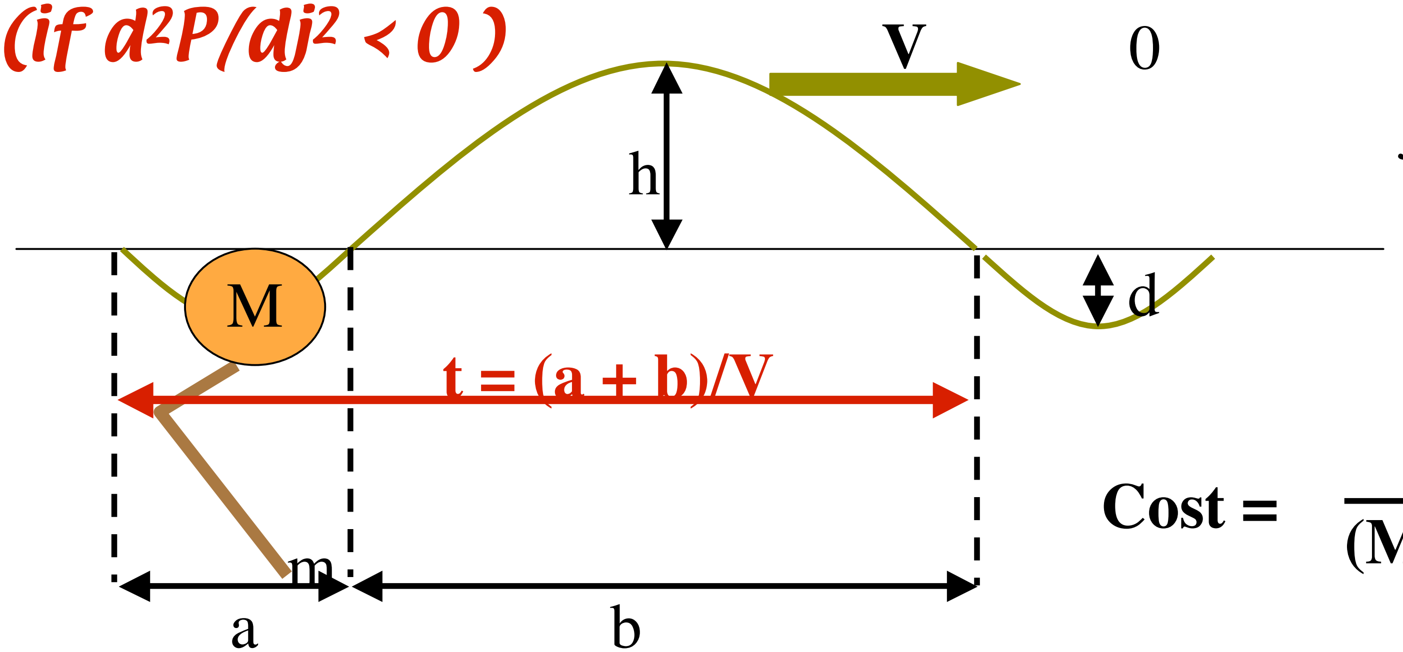
- Recap optimization of gaits for minimum power out put and cost of transport.
- Loads and deformations for basic stress and strain
- Stiffness: a measure of how materials respond to loads
- Strength

$$\textbf{Power} = (1+\gamma)\, M\, g^2\, b/8V + m\, V^3/2(a+b)$$
$$= (1+\gamma)\, M\, g^2\, a\, j\, /8V + m\, V^3/2a(1+j)$$

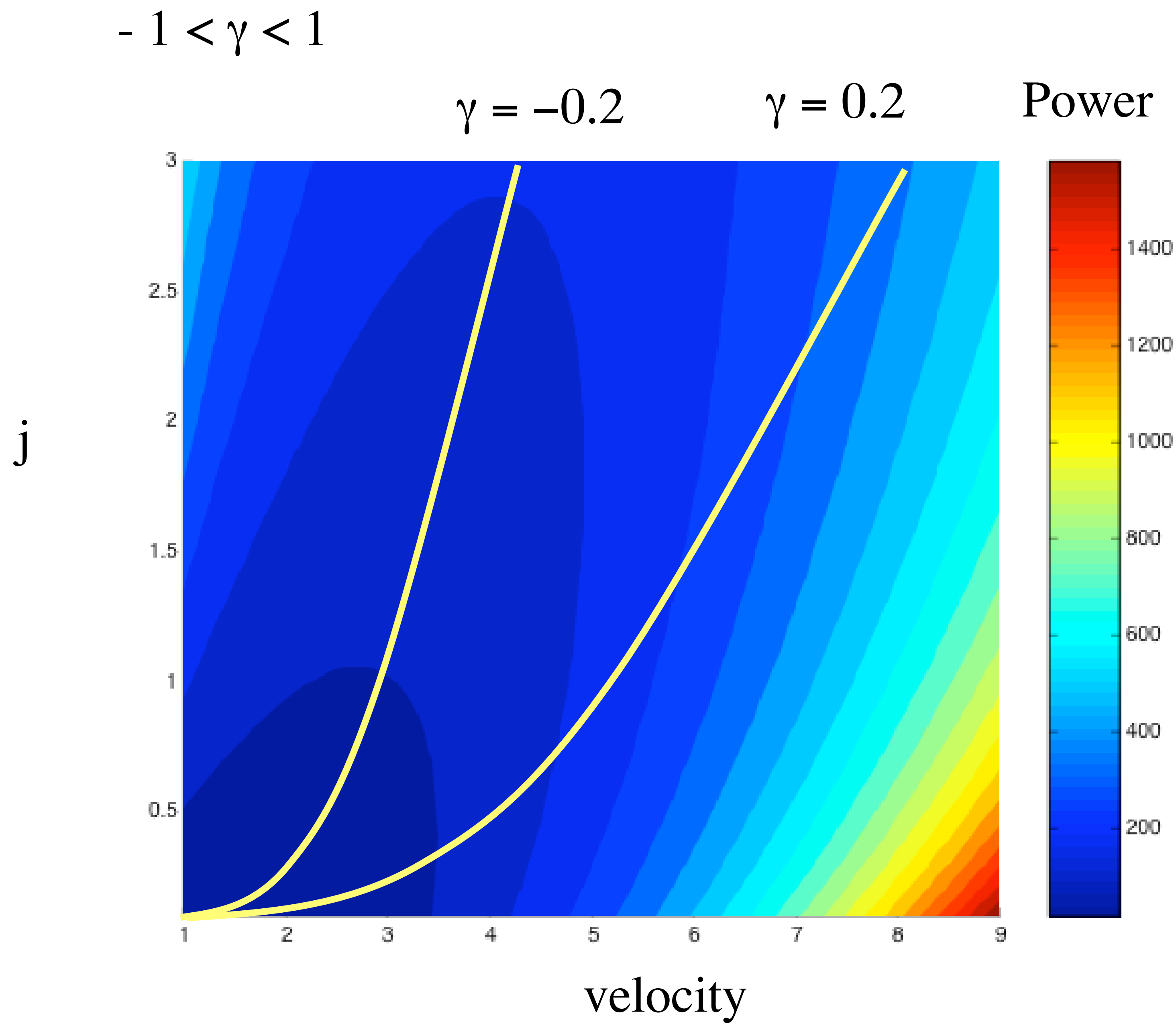
Comes from a calculation for the work to launch and land and the work to move the feet



dP/dj = 0 defines a maximum (if $d^2P/dj^2 < 0$)

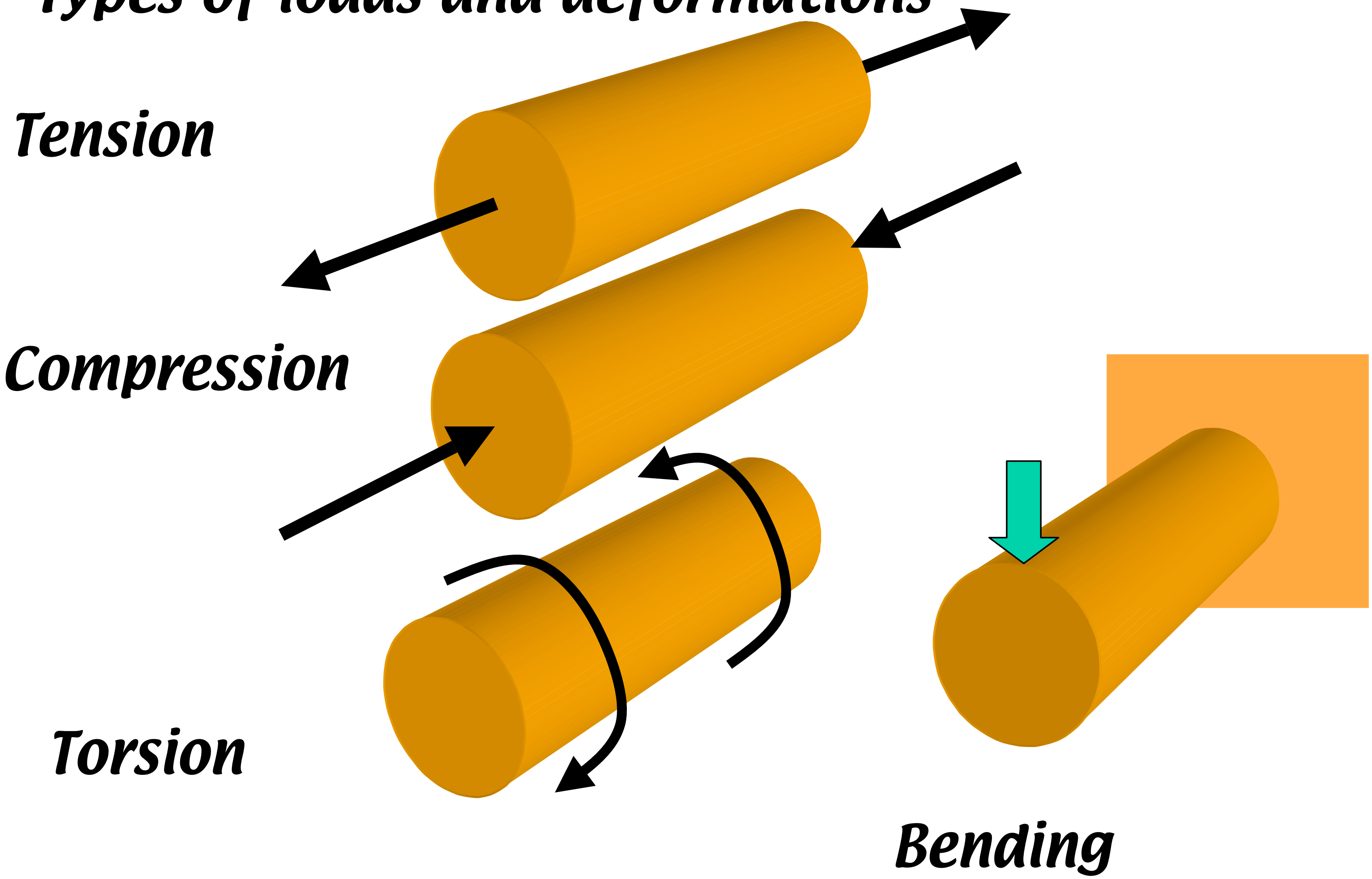


$$\textbf{Cost} = \frac{P}{(M+m)\, g\, V}$$

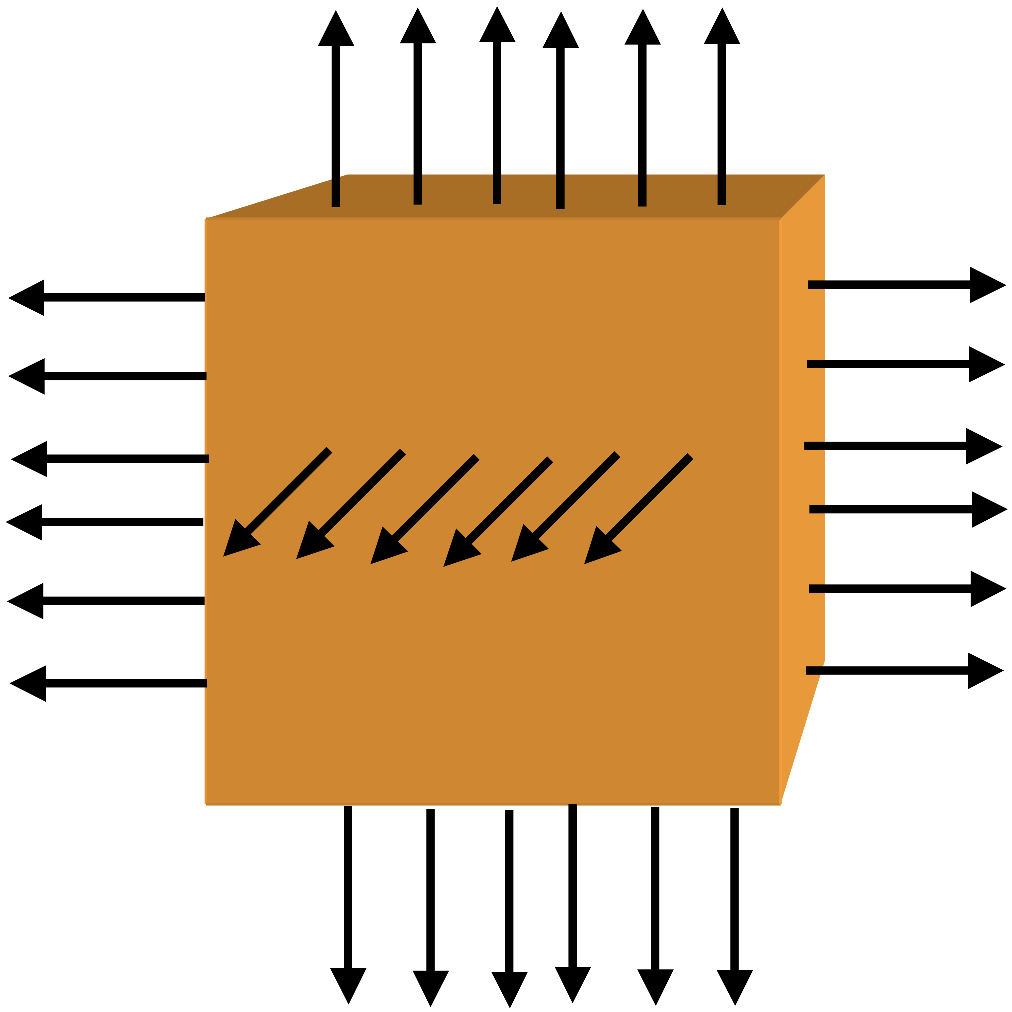


- 1 < γ < 1
γ = −0.2
γ = 0.2
Power
j
velocity
3
2.5
2
1.5
1
0.5
1
2
3
4
5
6
7
8
9
1400
1200
1000
800
600
400
200

Types of loads and deformations



Types of loads and deformations

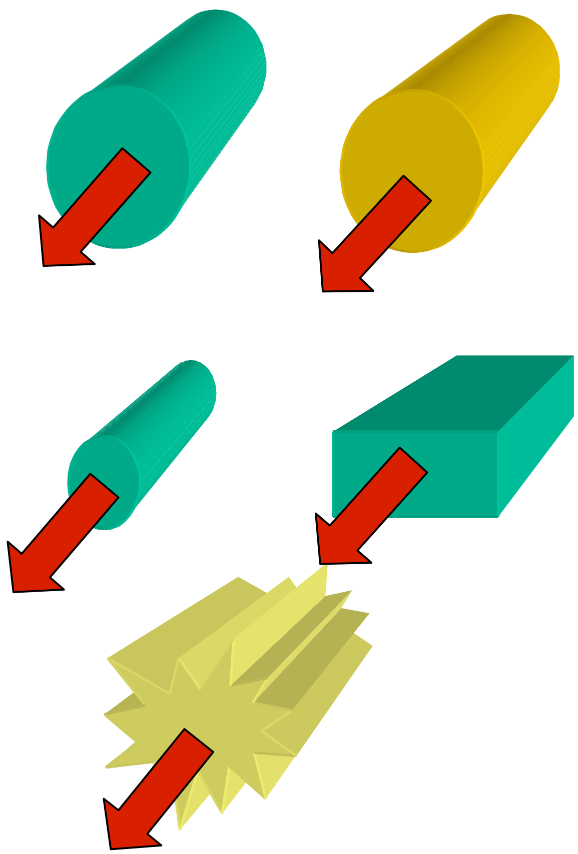


uniaxial loads

biaxial loads

triaxial loads

Material vs. structural properties

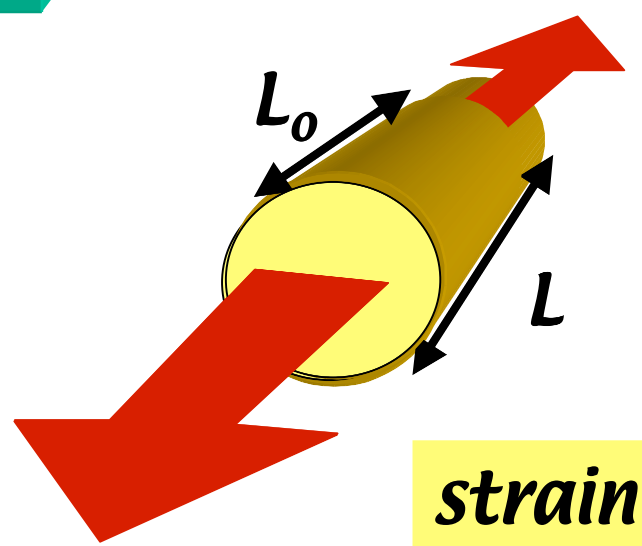


Need to eliminate the effect of size and shape to define material properties.

stress (σ) = Force/Area

$\sigma = F/A$

$Pa = N/m^2$



strain (ε) = $(L - L_o)/L_o$

Material vs. structural properties

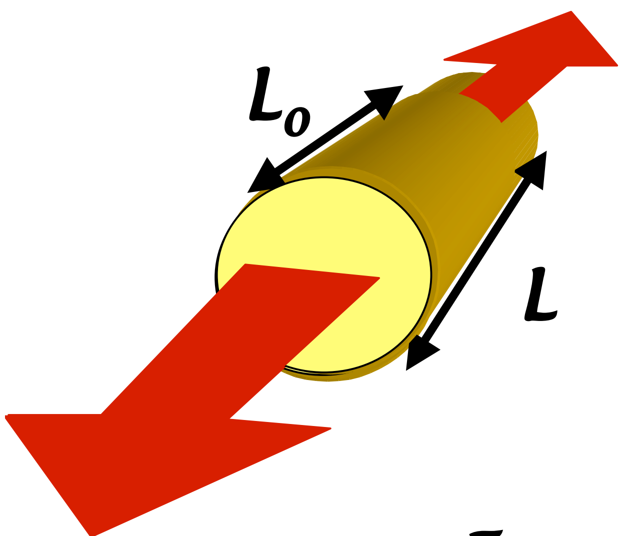
stress (σ) = Force/Area

σ = F/A

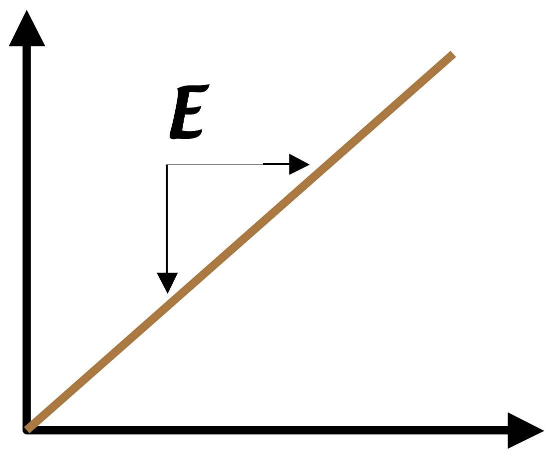
strain (ε)= $(L - L_o)/L_o$

$\varepsilon_T = \int dL/L = \ln(L)$

$\lambda = L/L_o$



$\sigma = \dfrac{\text{Force}}{\text{Area}}$



$\varepsilon = \Delta L / L_o$

Hooke's Law

$F = k\,\Delta L$

Young's modulus (E) measures the stiffness of a *material*

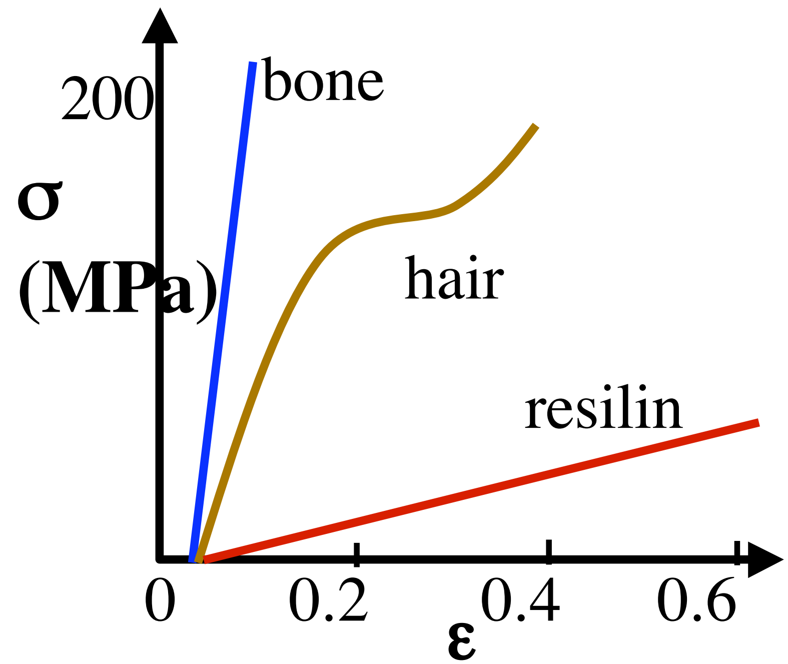
$E = \sigma/\varepsilon$

stress* (σ) : *the distribution of force over an area

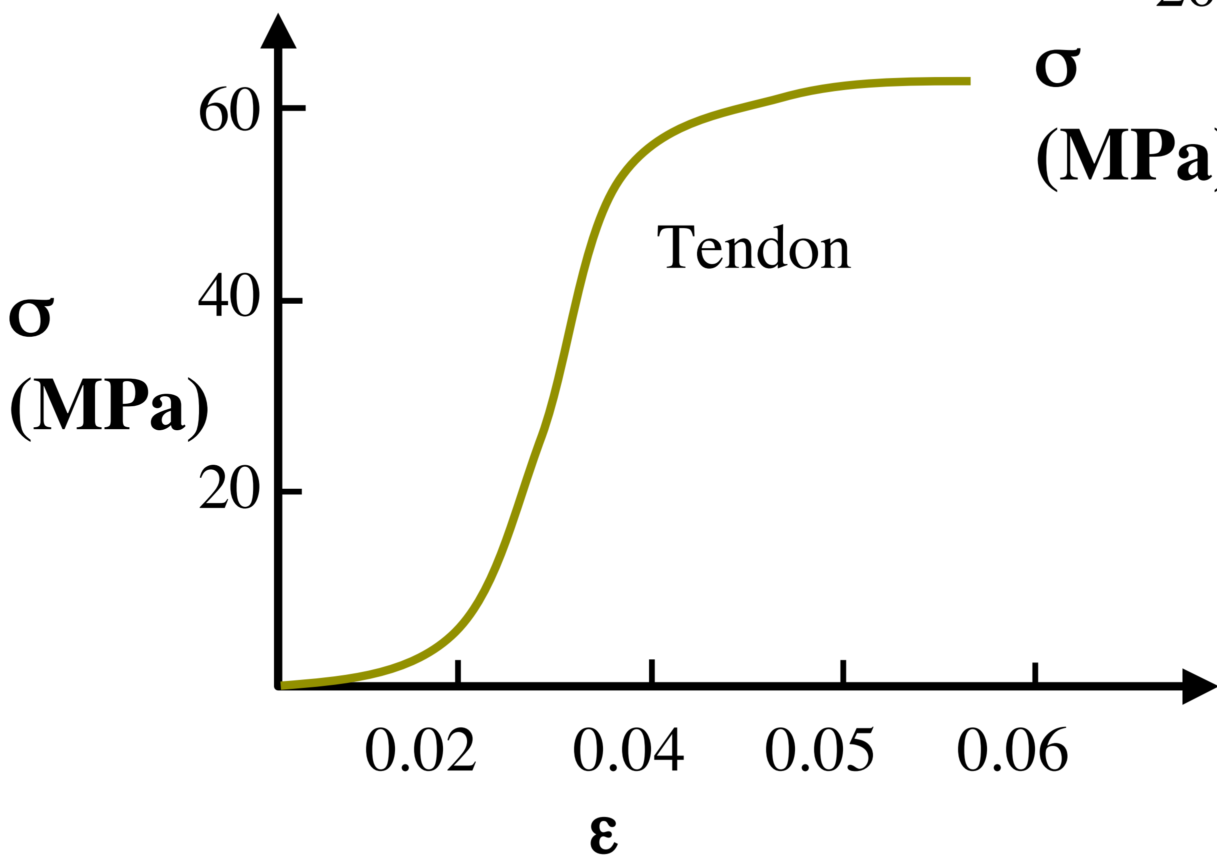
strain* (ε): *a dimensionless measure of length change

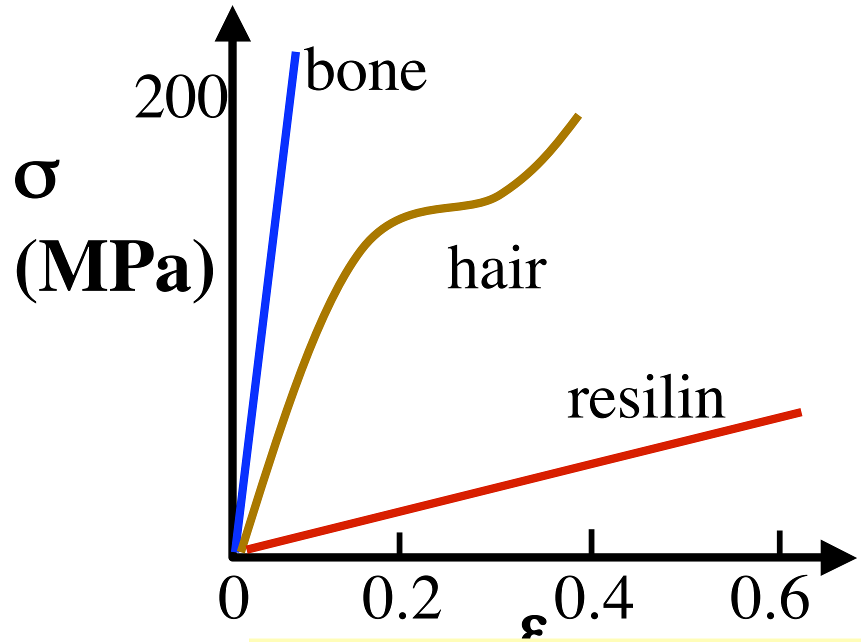
stiffness (E): the change in stress required for a change in strain (the slope of a stress-strain curve)

Material	*Youngs Modulus (Mpa)*
Locust cuticle	**0.2**
Rubber	**7**
Human cartilage	**24**
Human tendon	**600**
Cheap plastic	**1,400**
Plywood	**14,000**
Human bone	**21,000**
Glass	**70,000**
Brass	**120,000**
Iron	**210,000**
Diamond	**1,200,000**

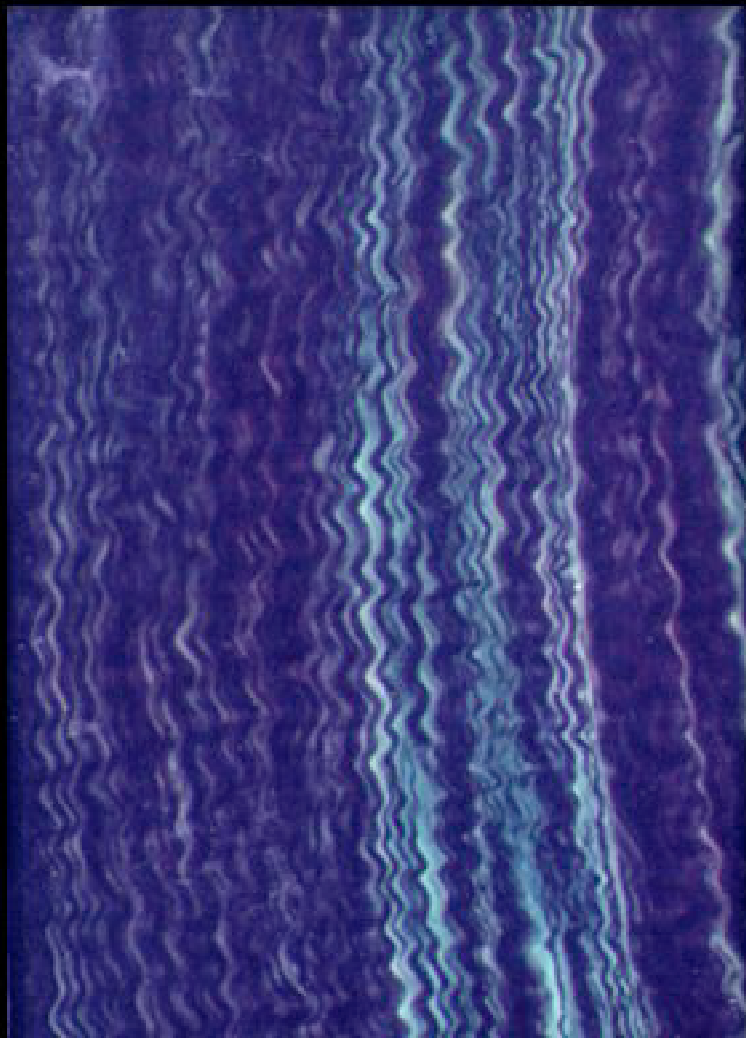


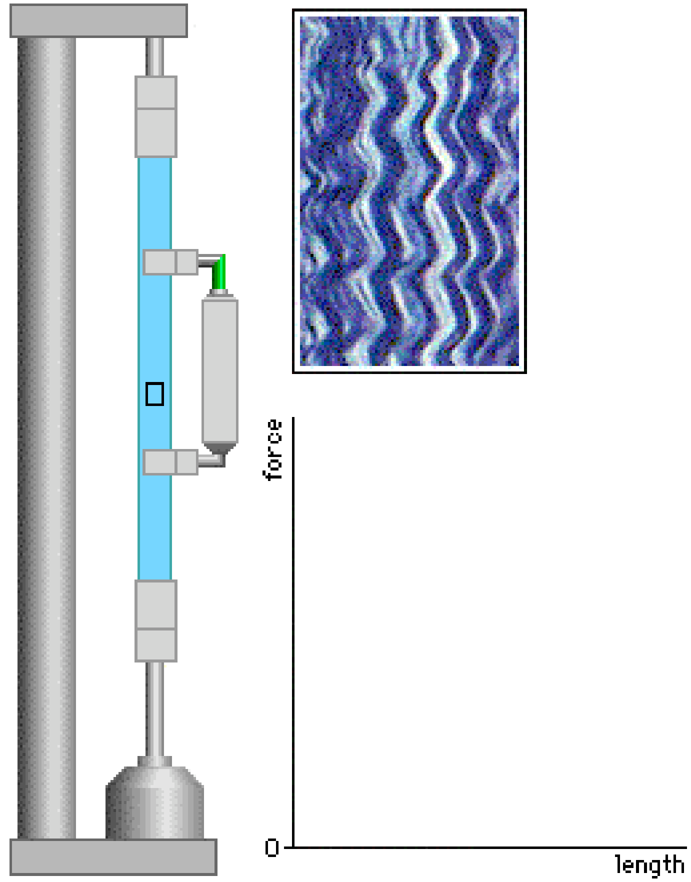
stress (σ) : the distribution of force over an area
strain (ε): a dimensionless measure of length change
stiffness (E): the change in stress required for a change in strain (the slope of a stress-strain curve)





Many (most) biological materials are non-linearly elastic



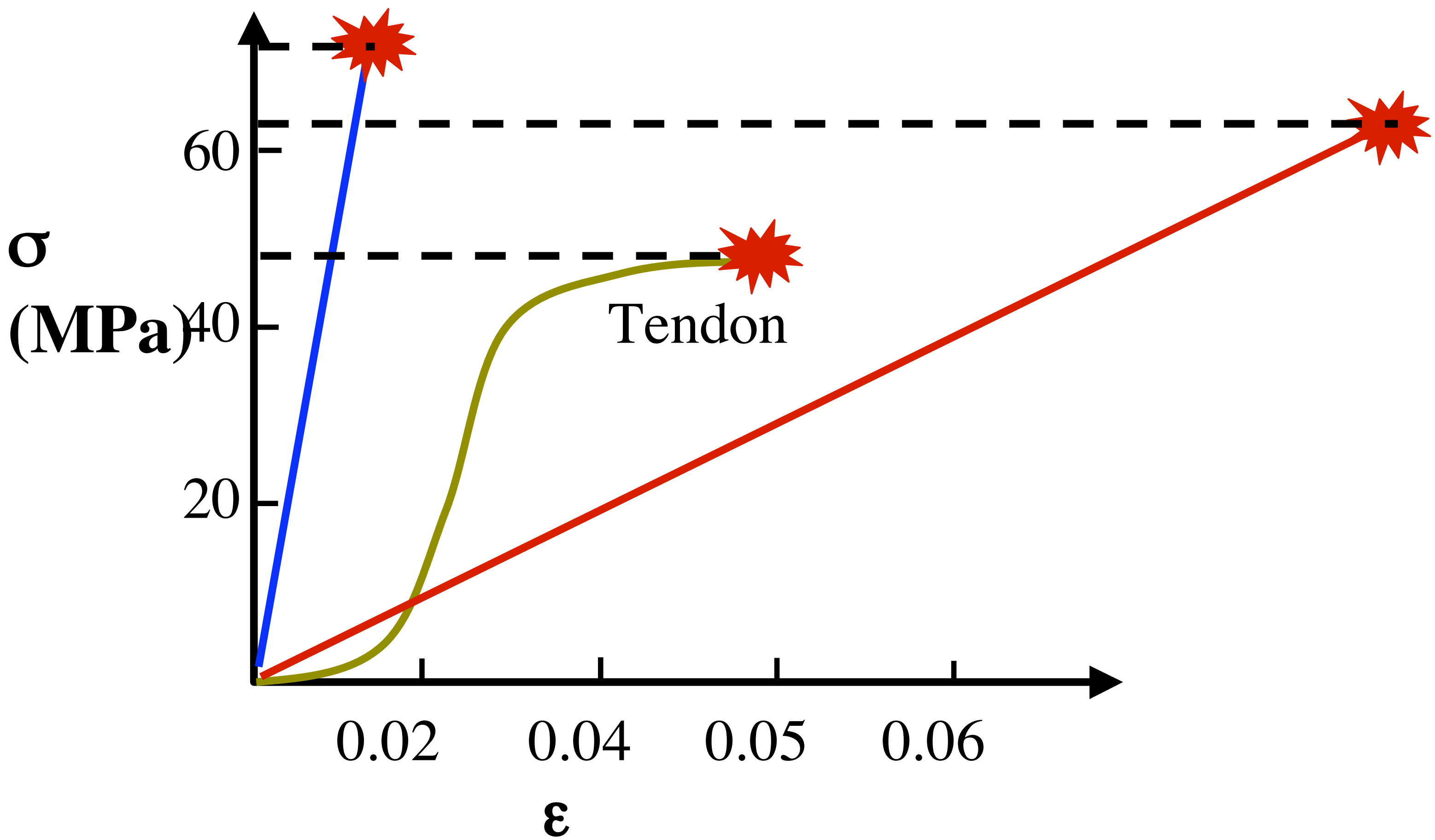

force
0
length

stress **(σ) :** ***the distribution of force over an area***

strain **(ε):** ***a dimensionless measure of length change***

stiffness (E): ***the change in stress required for a change in strain (the slope of a stress-strain curve)***

strength **(σ_{max}):** ***the stress at failure***

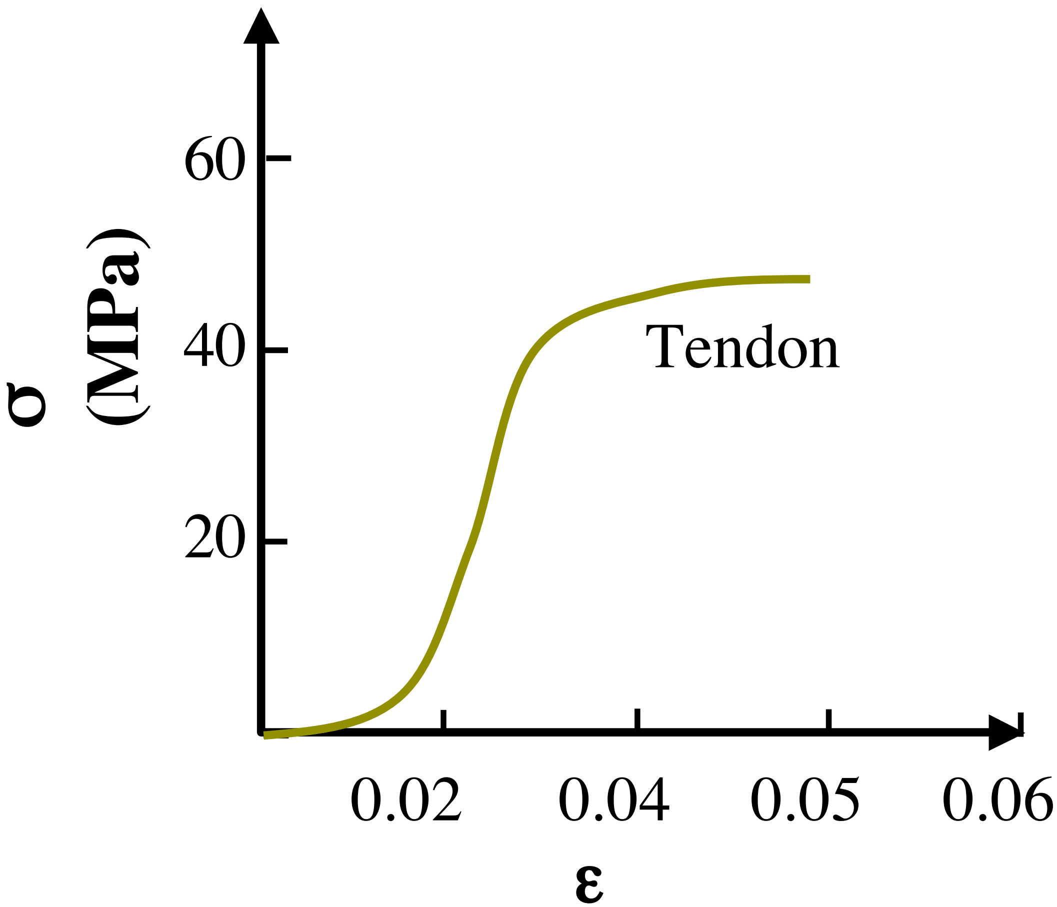


stress (σ) : the distribution of force over an area

strain (ε): a dimensionless measure of length change

stiffness (E): the change in stress required for a change in strain (the slope of a stress-strain curve)

strength (σ_{max}): the stress at failure



material	strength (MPa)
arterial wall	2
human cartilage	3
cement	4
cheap aluminum	70
glass	100
human tendon	100
human bone	110
human hair	200
spider silk	350
titanium	1000
steel wire	3000

Biology 427

Lecture 7. Strength and toughness of biological materials

Recap stress, strain, stiffness and strength of biomaterials: measures of material properties

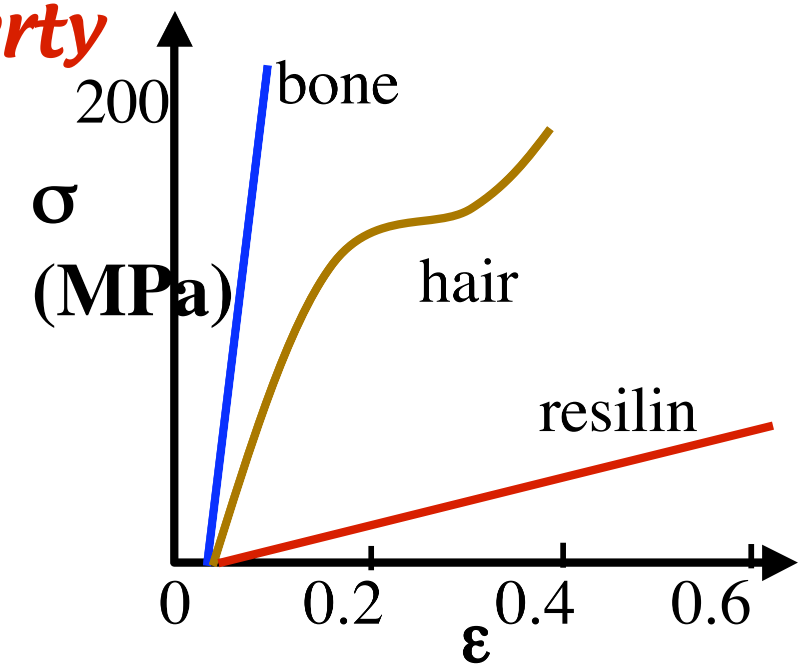
Strength revisited and the limits to the size of terrestrial vertebrates

Energy relations in biological materials: toughness and resilience

Plastic deformations: an introduction to time-dependent material properties.

stress (σ) : the distribution of force over an area
strain (ε): a dimensionless measure of length change
stiffness (E): the change in stress required for a change in strain (the slope of a stress-strain curve): a material property

Material	*Youngs Modulus (Mpa)*
Locust cuticle	**0.2**
Rubber	**7**
Human cartilage	**24**
Human tendon	**600**
Cheap plastic	**1,400**
Plywood	**14,000**
Human bone	**21,000**
Glass	**70,000**
Brass	**120,000**
Iron	**210,000**
Diamond	**1,200,000**

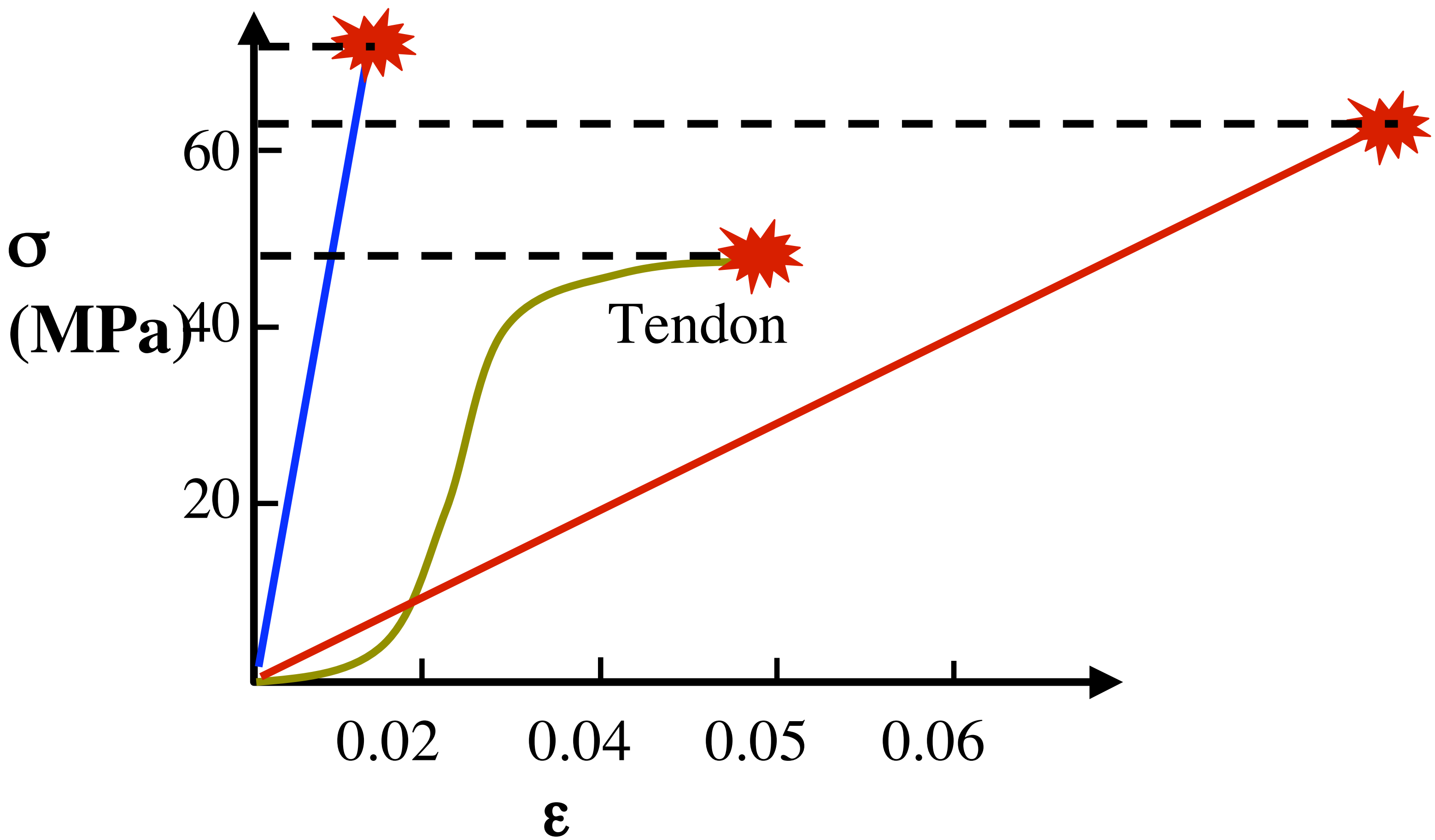


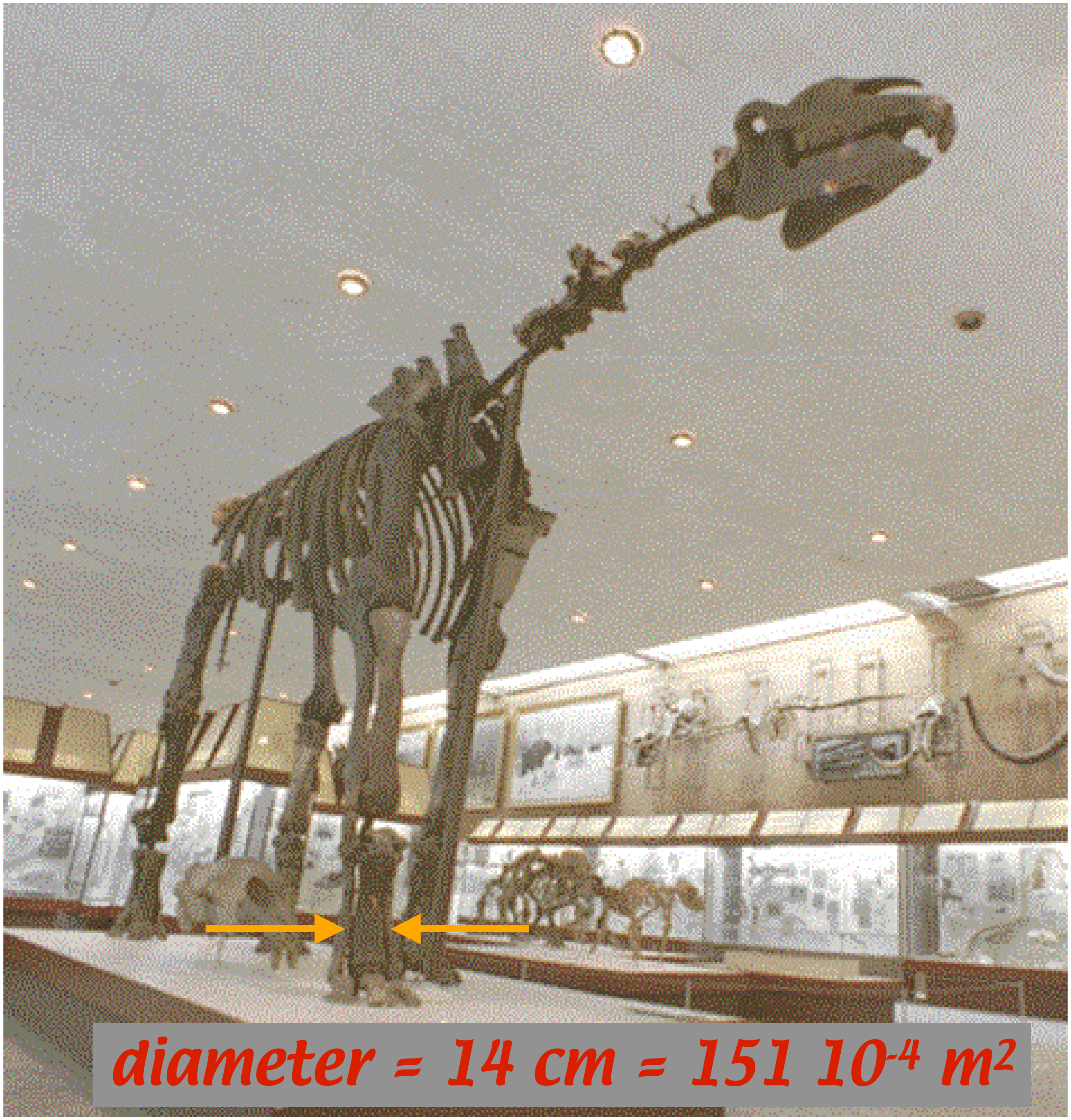
stress (σ) : the distribution of force over an area

strain (ε): a dimensionless measure of length change

stiffness (E): the change in stress required for a change in strain (the slope of a stress-strain curve) a material property

strength (σ_{max}): the stress at failure



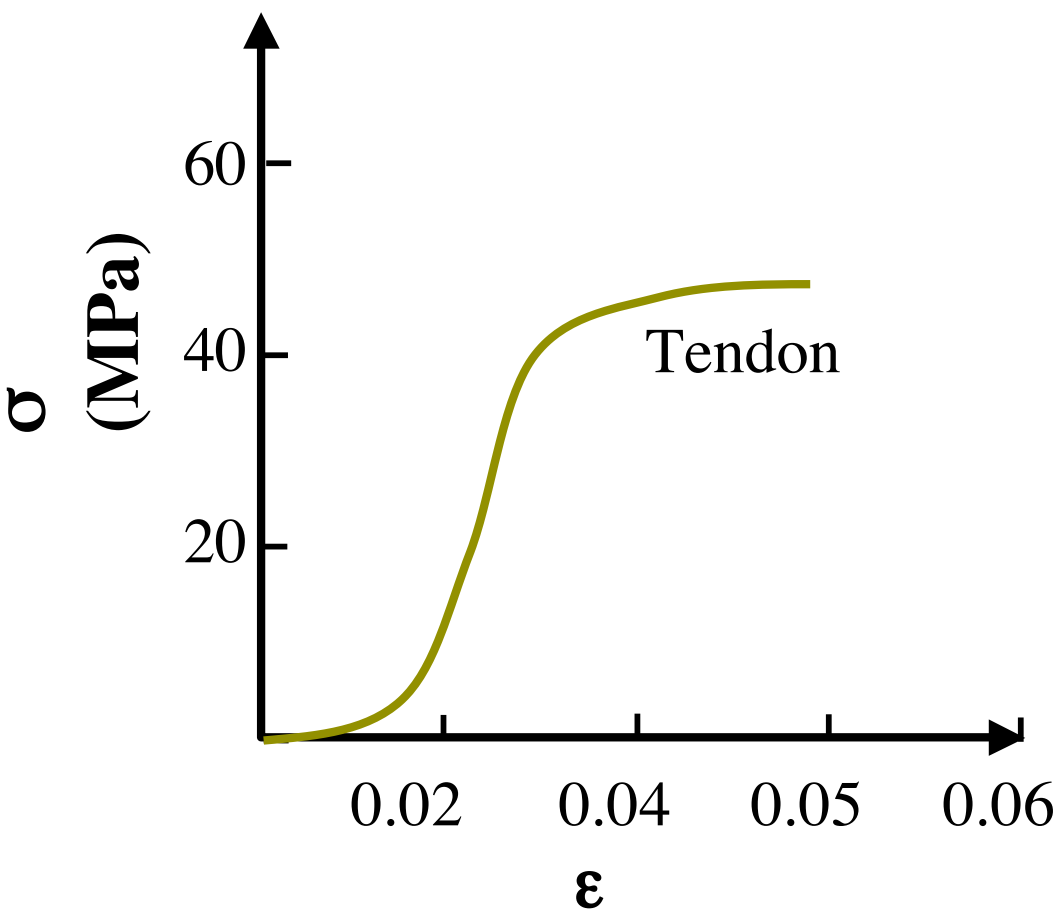


Baluchitherium: about 30 Tons Could the foot bones support its weight?

σ_{max} = 100 MPa
= F_{max}/Area

F_{max} = 10^8*Area
= 151 10^4 N
m = 151 10^3 Kg
= 151 T

stress (σ) : the distribution of force over an area
strain (ε): a dimensionless measure of length change
stiffness (E): the change in stress required for a change in strain (the slope of a stress-strain curve)
strength (σ_{max}): the stress at failure

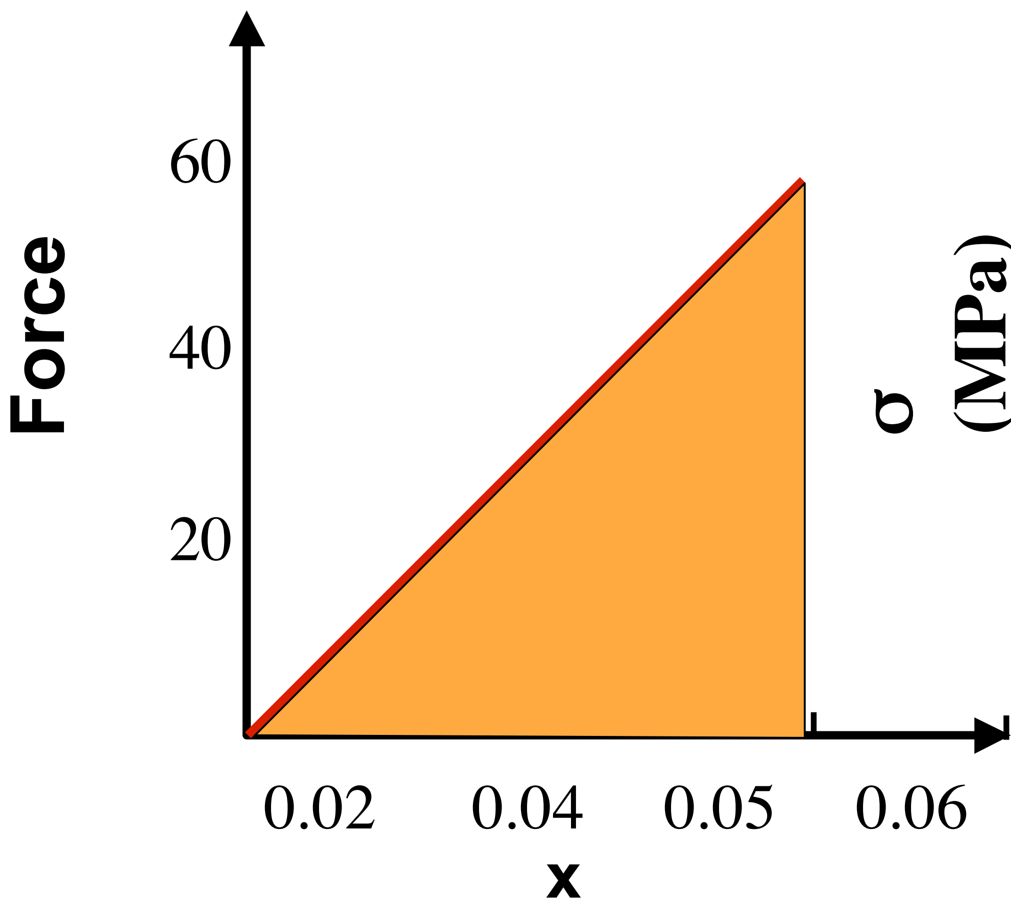


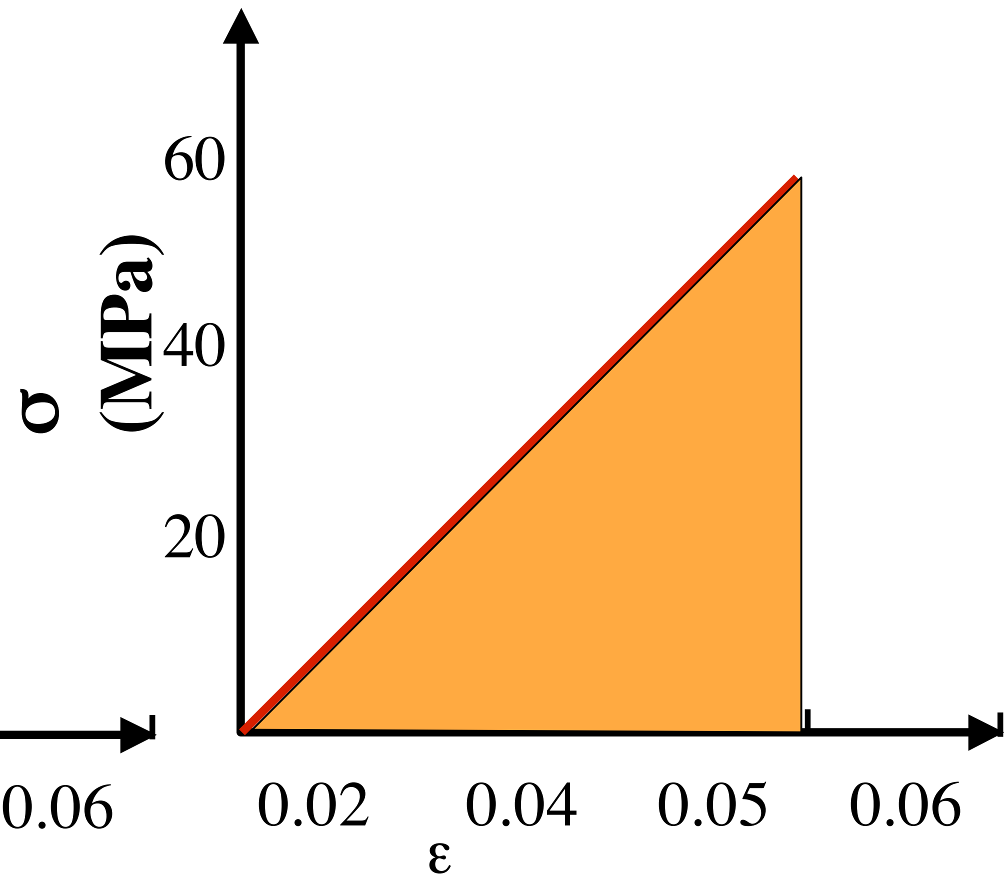
material	*strength (MPa)*
arterial wall	2
human cartilage	3
cement	4
cheap aluminum	70
glass	100
human tendon	100
human bone	110
human hair	200
spider silk	350
titanium	1000
steel wire	3000

Energy Basics for Materials

$$W = \int F\,dx$$

$$\frac{W}{vol} = \int \frac{F\,dx}{A\,L}$$





The energy imparted is the ***mechanical strain energy***

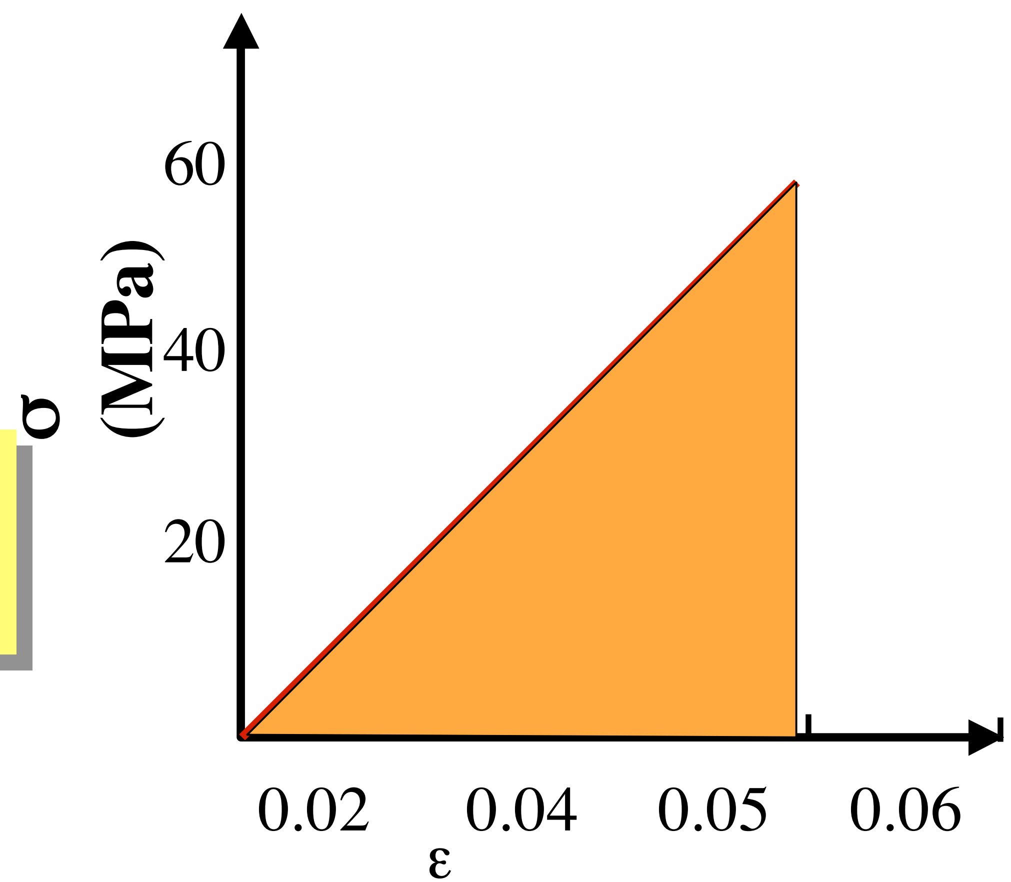
$$U = \int \sigma \, d\varepsilon \qquad \frac{W}{\text{vol}} = \int \frac{F\,dx}{A\,L}$$

For Hookean materials

$$\sigma = E\,\varepsilon$$

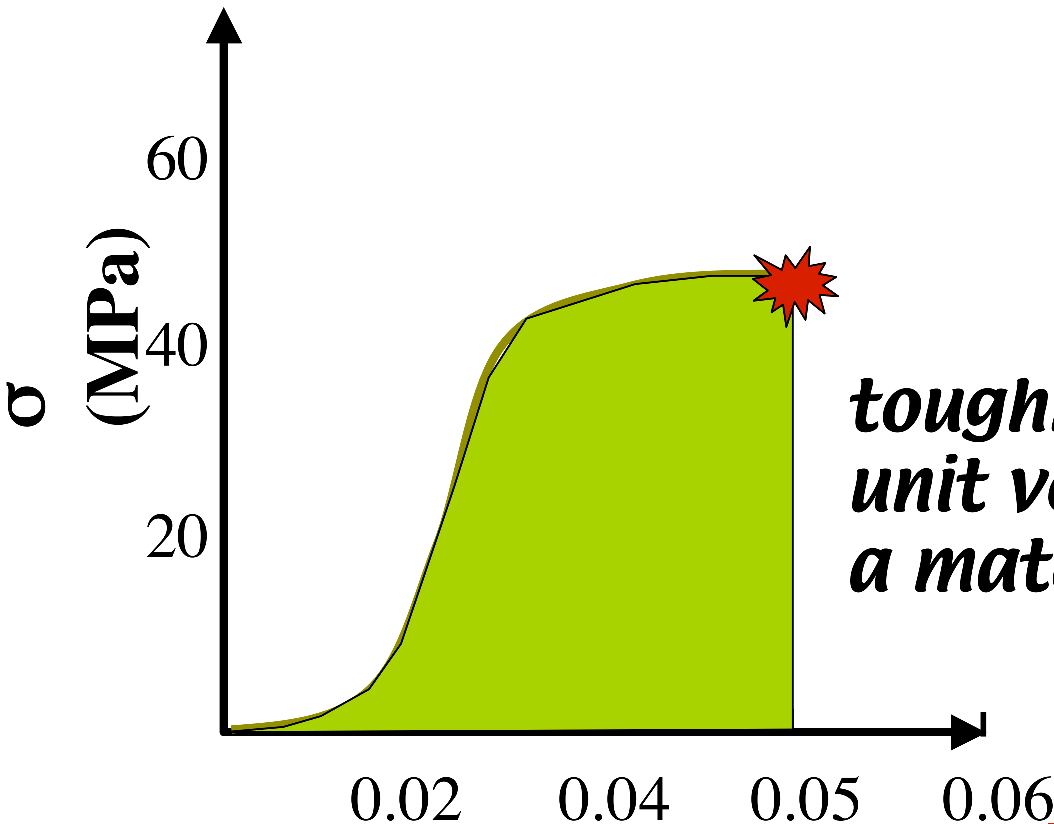
$$U = \int E\,\varepsilon\,d\varepsilon = E\,\varepsilon^2/2$$

$$U = \int \sigma\,d\sigma/E = \sigma^2/2E$$



The energy imparted is the mechanical strain energy that can be returned or be so great as to break the material

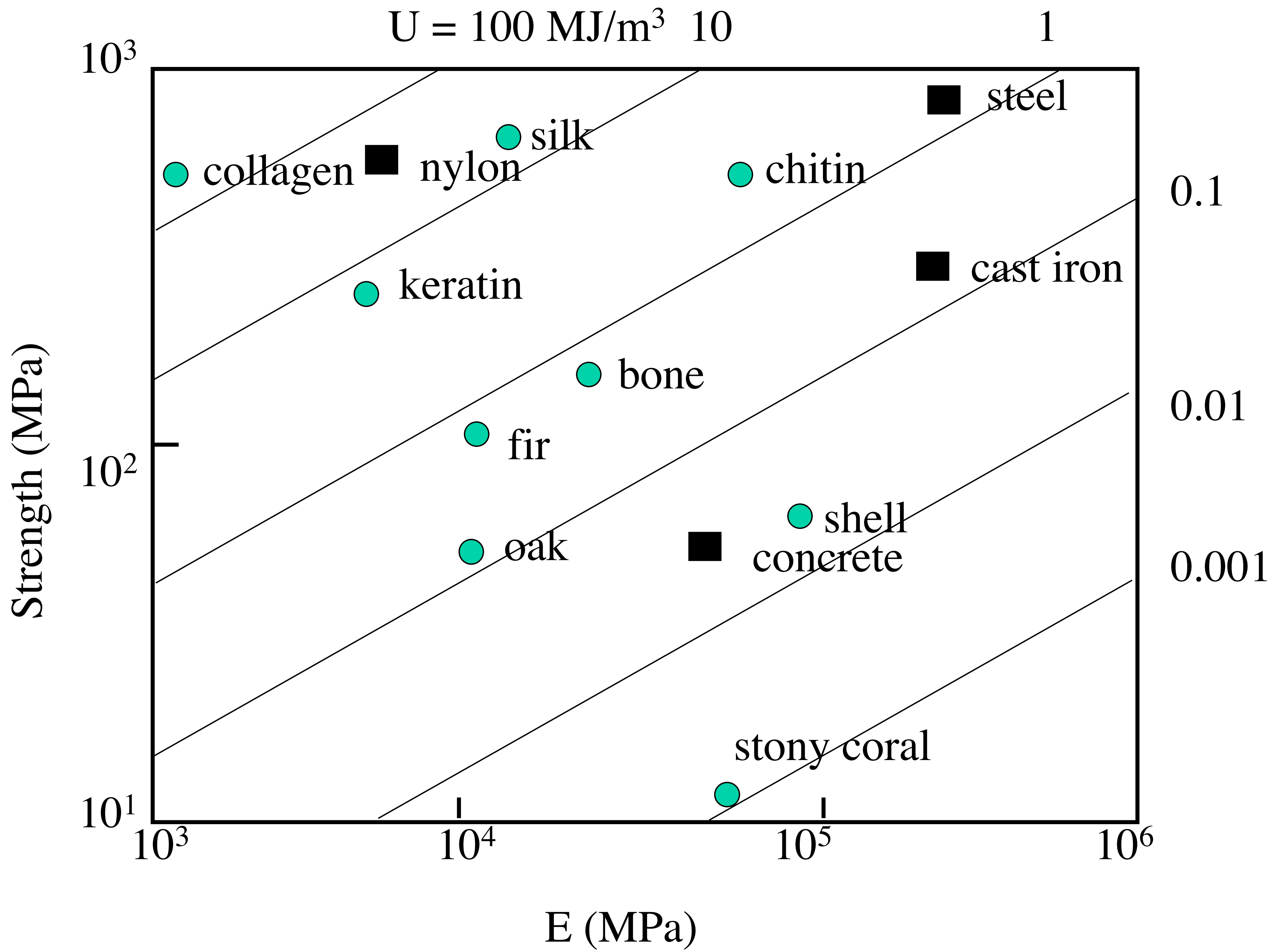
$$U = \int \sigma \, d\varepsilon$$



toughness: the energy per unit volume required to break a material

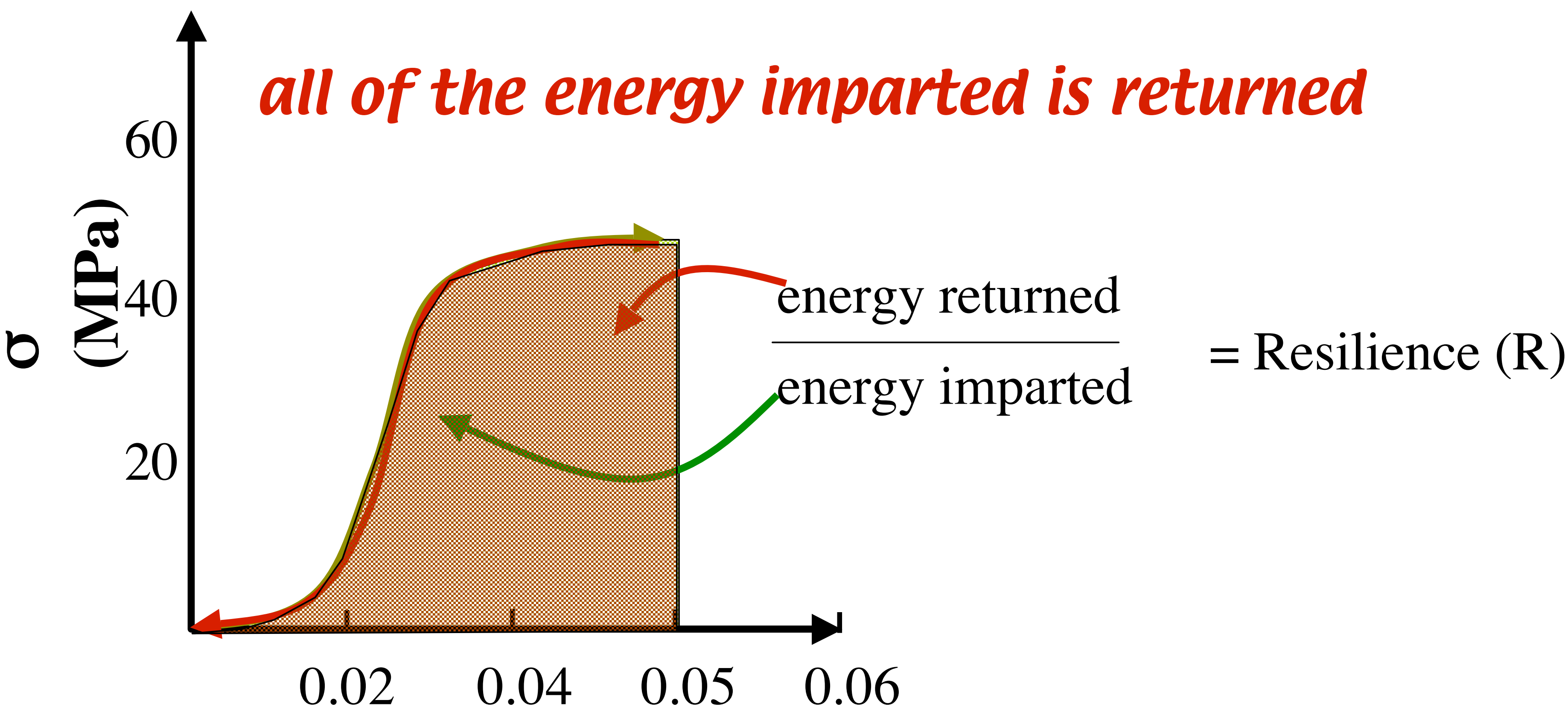
$$T = \int \sigma \, d\varepsilon$$

$$T = \sigma^2_{max} / 2E \text{ (if Hookean)}$$


U = 100 MJ/m3
10
1
0.1
0.01
0.001
steel
silk
collagen
nylon
chitin
cast iron
keratin
bone
fir
shell
oak
concrete
stony coral
Strength (MPa)
E (MPa)
10^3
10^2
10^1
10^4
10^5
10^6

The energy imparted is the mechanical strain energy that can be returned or be so great as to break the material

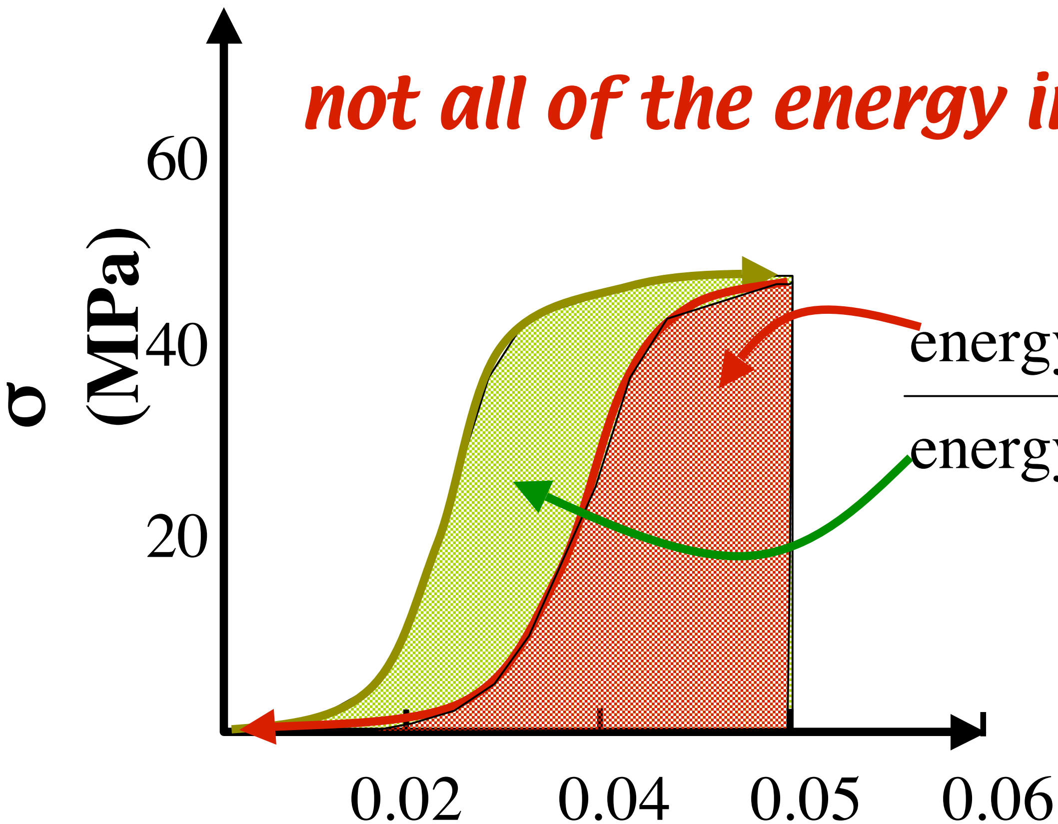
$$U = \int \sigma \, d\varepsilon$$



The energy imparted is the mechanical strain energy that can be returned or be so great as to break the material

$$U = \int \sigma \, d\varepsilon$$

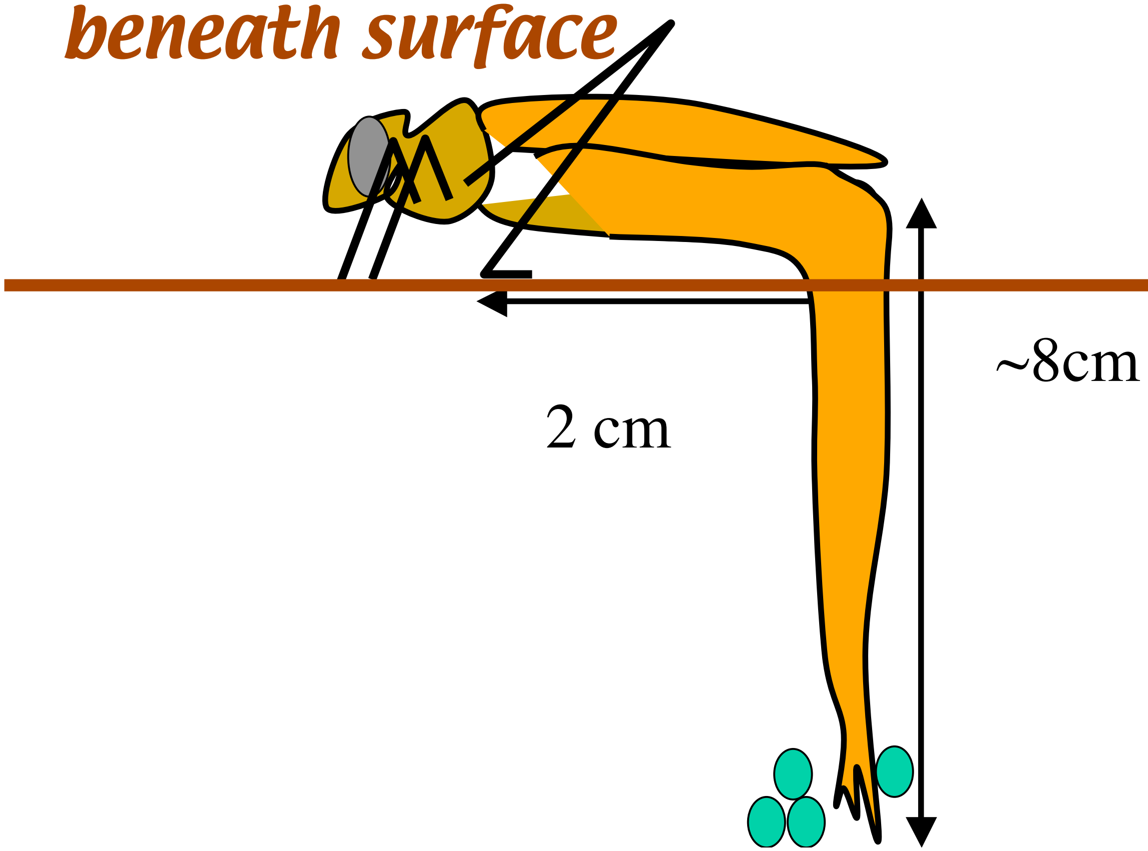
not all of the energy imparted is returned



$$\frac{\text{energy returned}}{\text{energy imparted}} = \text{Resilience (R)}$$

The story of the pregnant (gravid) locust

- *locusts are migratory*
- *light weight important in flight*
- *fertilized eggs in dehydrated state*
- *live in very arid climates*
- *trick: bury the eggs ~ 8 cm beneath surface*



$E = 10^5$ MPa

$U = \int E\,\varepsilon\, d\varepsilon$

$= E\,\varepsilon^2/2$

$= 10^{11}\ 9/2$

$\sim 5\ 10^{11}\ \mathrm{J/m^3}$

$= 5\ 10^8\ \mathrm{J/kg}$

with 1 g of abdomen

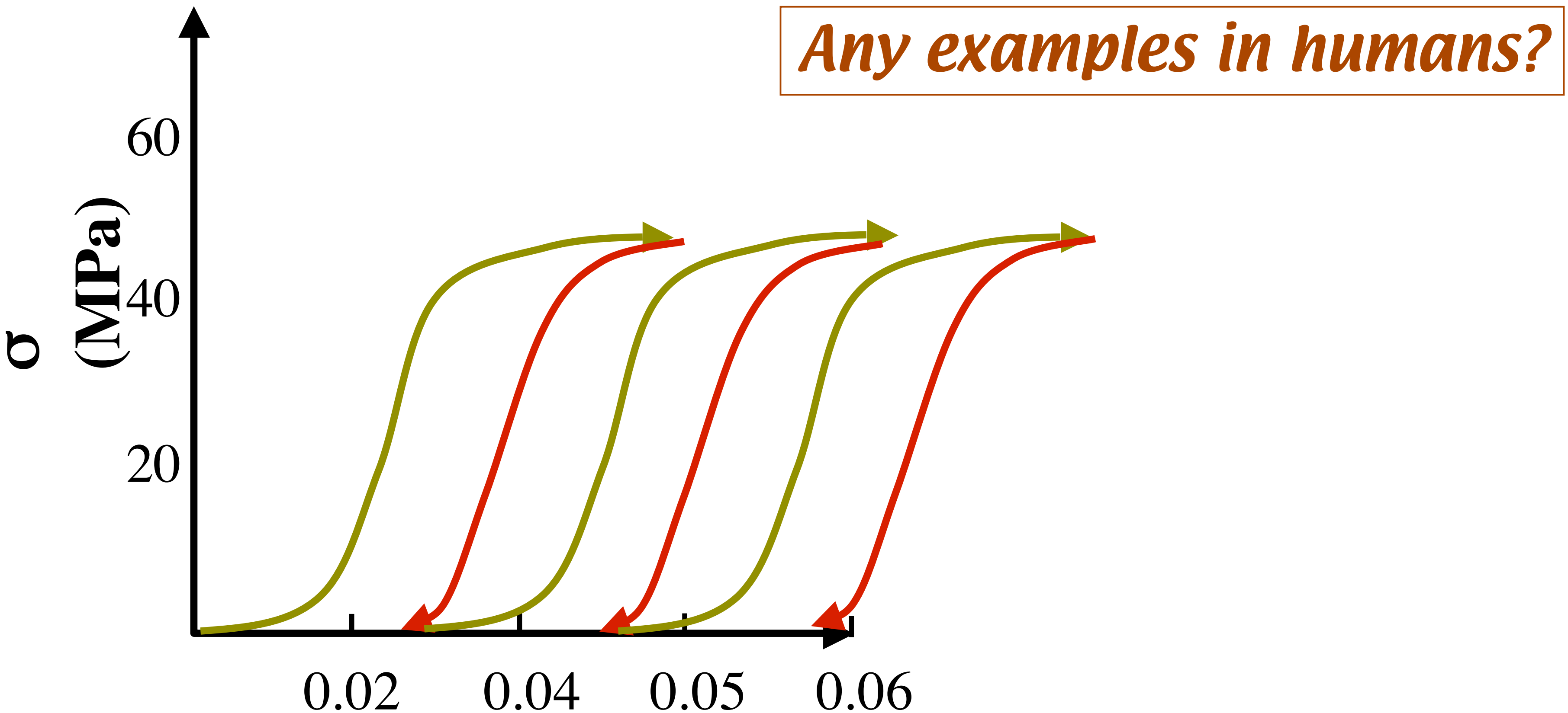
$\to 5\ 10^5$ J

$M = 0.005$ kg

$PE = Mg\,h$

$h = 10^7$m

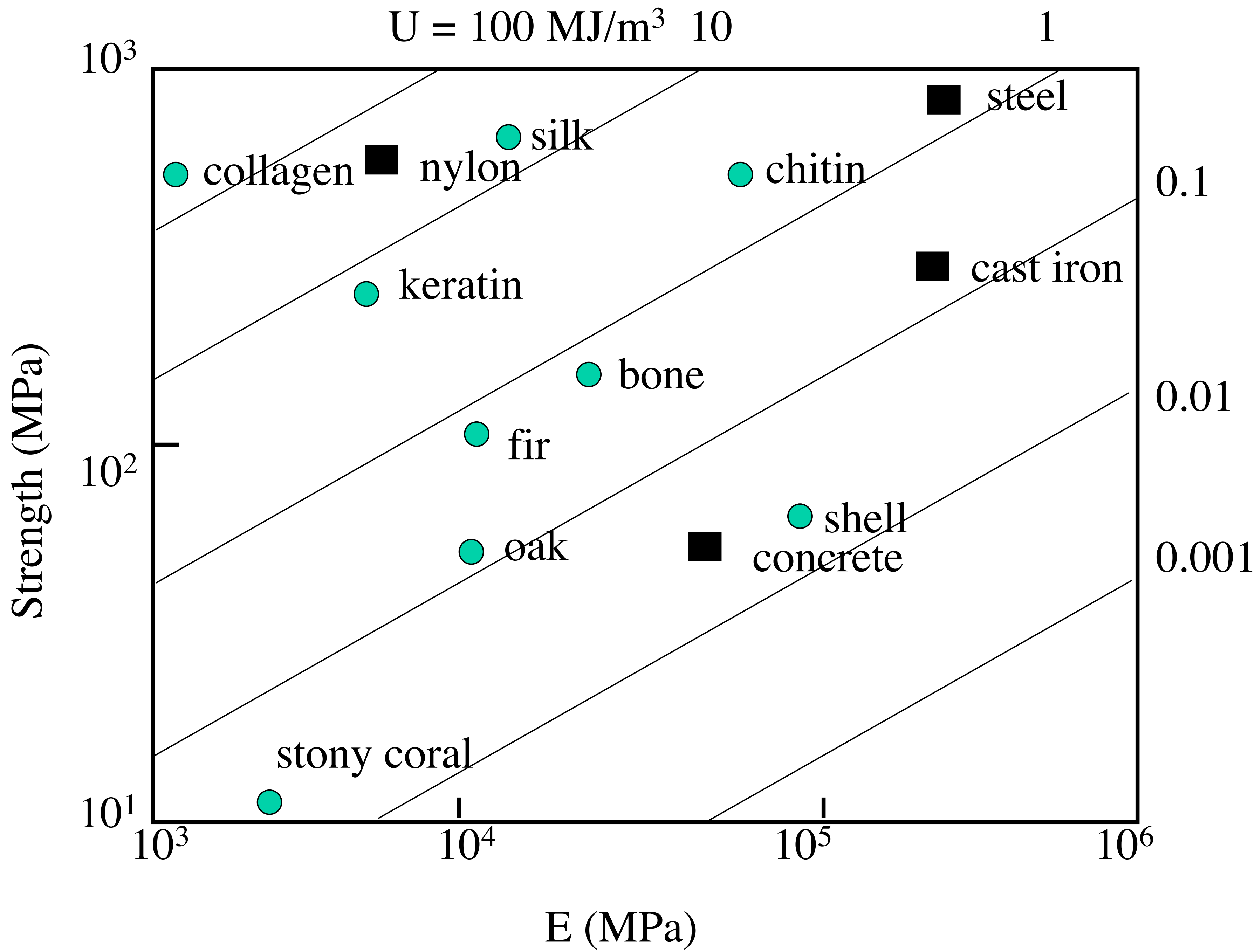
The energy imparted is the mechanical strain energy that can be returned or be so great as to break the material or be lost as a permanent deformation (plastic deformation)



Biology 427 Biomechanics

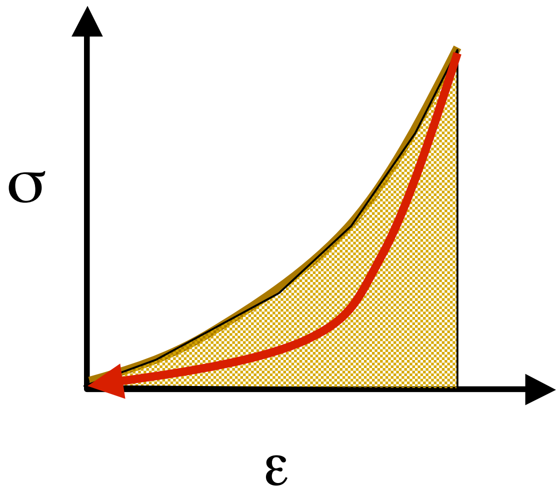
Lecture 8. Visco-elasticity: time-dependent properites of biological materials

- Recap of basic elasticity
- Differentiating fluids (viscous) from solid (elastic) behaviors
- Experimental results for some biomaterials
- Elementary descriptions of visco-elastic material properties.
- Comments about term project 1



U = 100 MJ/m^3
10
1
0.1
0.01
0.001
steel
silk
collagen
nylon
chitin
cast iron
keratin
bone
fir
shell
oak
concrete
stony coral
Strength (MPa)
10^3
10^2
10^1
10^3
10^4
10^5
10^6
E (MPa)

Recap Material Properties

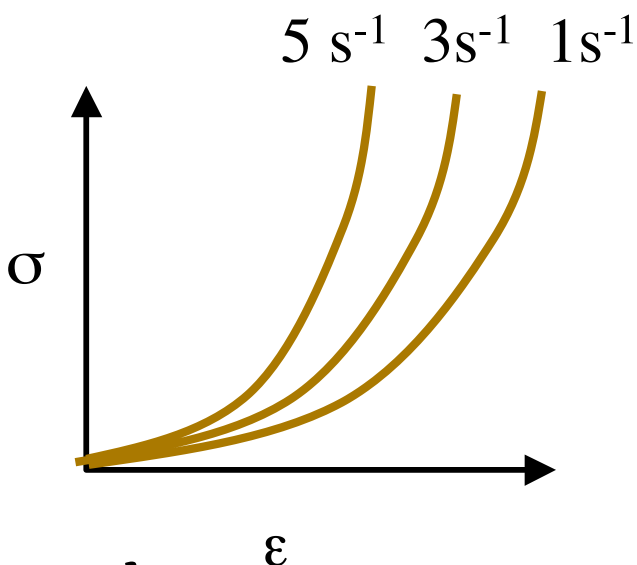


stiffness $d\sigma/d\varepsilon$

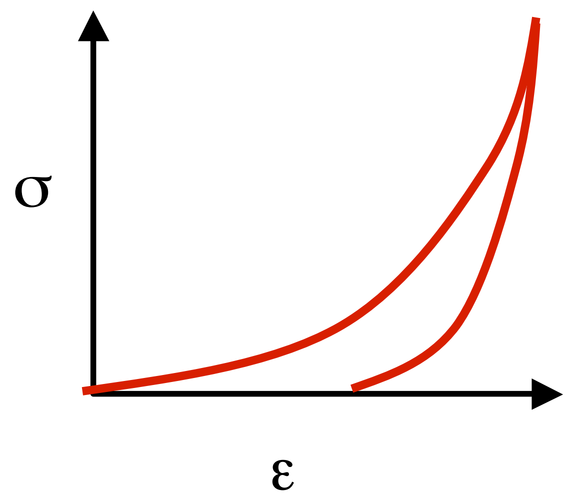
strength (σ_{max})

toughness $\int \sigma \, d\varepsilon$

resilience $\int \sigma \, d\varepsilon \, / \int \sigma \, d\varepsilon$



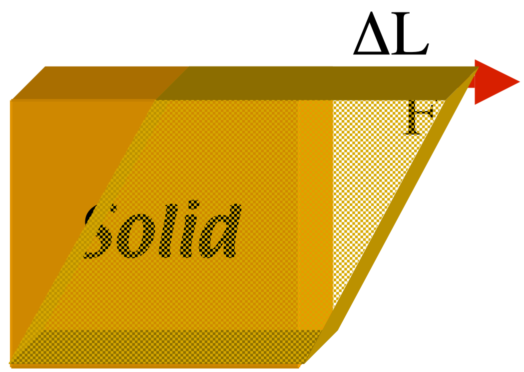
time dependent

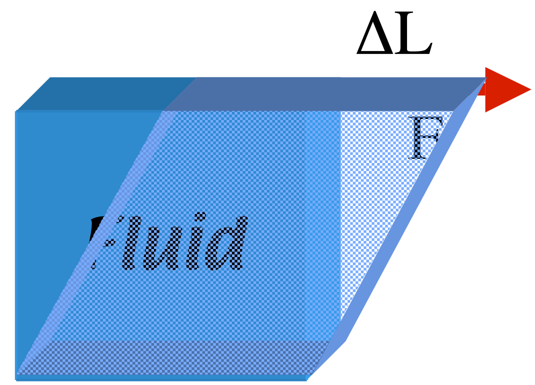


plastic

Tendon, muscle, cuticle, cartilage, mucus, hair, mesoglea, skin, all show time-dependent properties. They are, therefore, vicso-elastic.

Key: how do solids and fluids respond to a shear force?





$F \sim \Delta L$

$\sigma \sim \varepsilon$

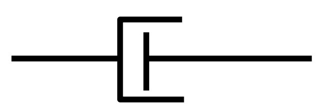
$\sigma = E\,\varepsilon$

$F \sim \Delta L/\Delta t \sim dL/dt$

$\sigma \sim d\varepsilon/dt$

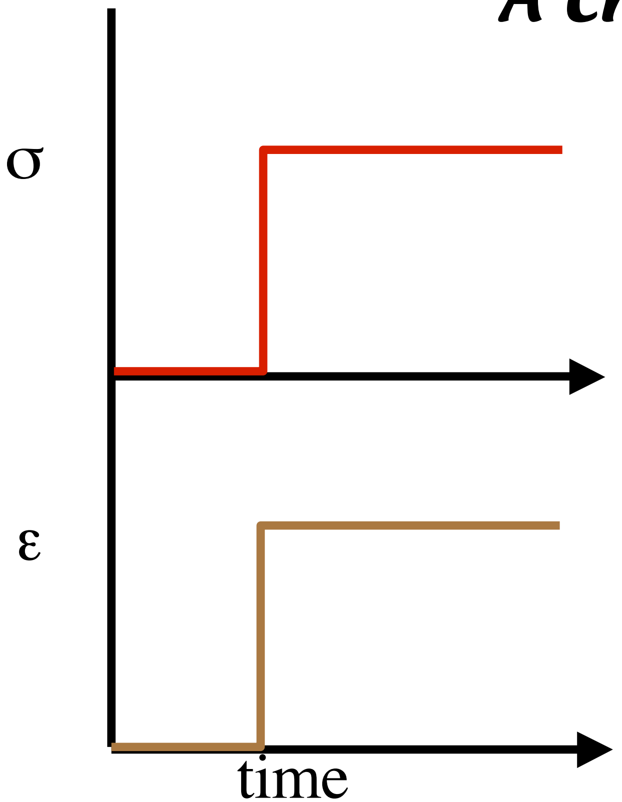
$\sigma = \mu\, d\varepsilon/dt$

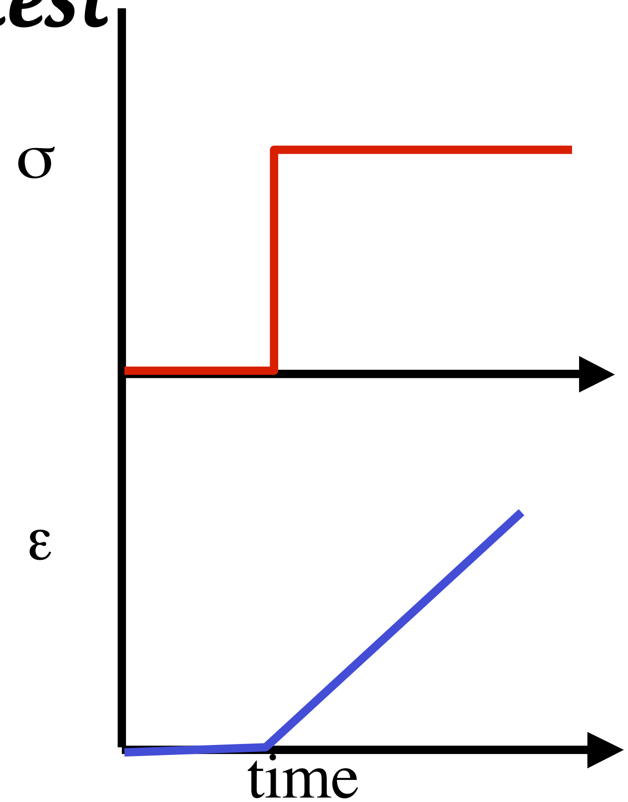
spring



dashpot

A creep test





$F \sim \Delta L$

$\sigma \sim \varepsilon$

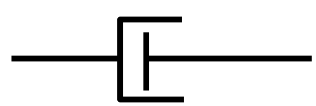
$\sigma = E\,\varepsilon$

spring

$F \sim \Delta L/\Delta t \sim dL/dt$

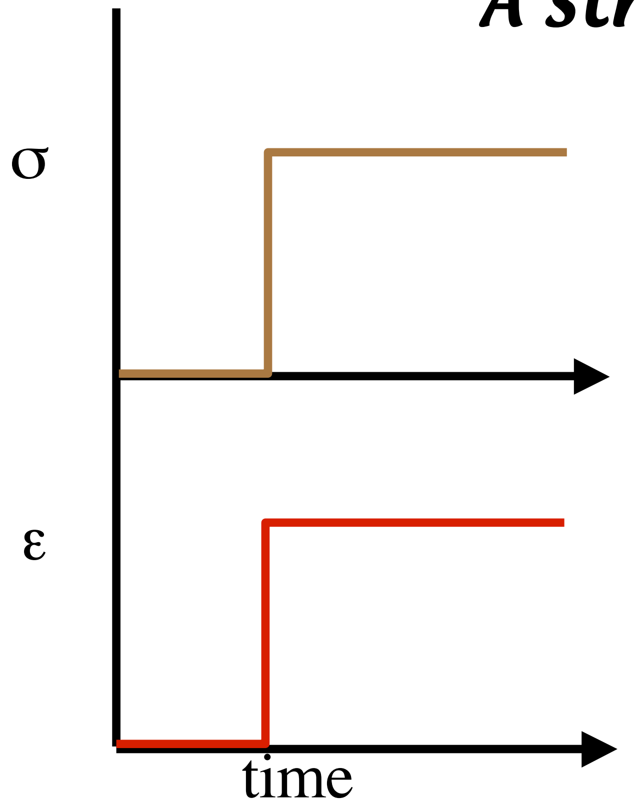
$\sigma \sim d\varepsilon/dt$

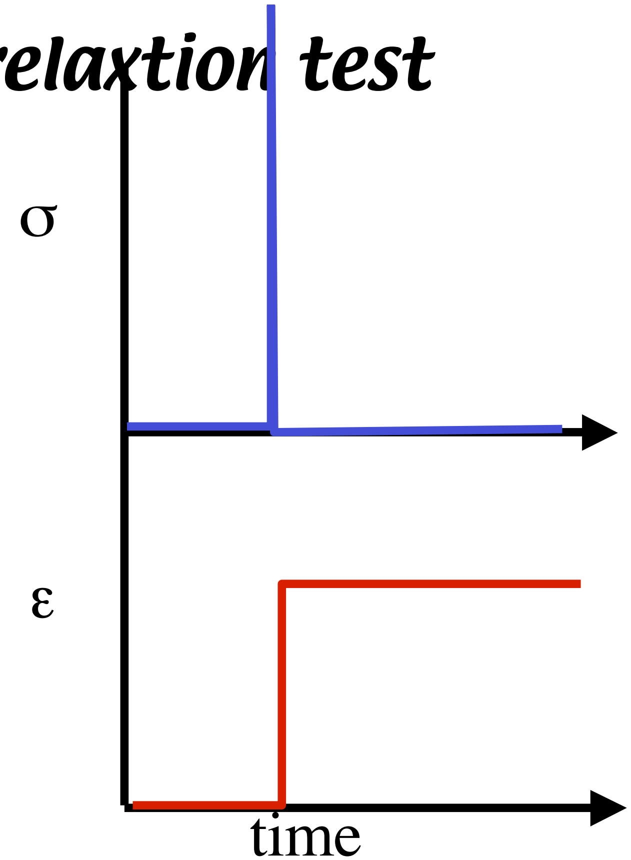
$\sigma = \mu\, d\varepsilon/dt$



dashpot

A stress relaxtion test





$F \sim \Delta L$

$\sigma \sim \varepsilon$

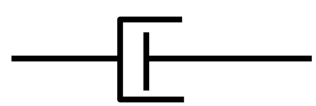
$\sigma = E\,\varepsilon$

spring

$F \sim \Delta L/\Delta t \sim dL/dt$

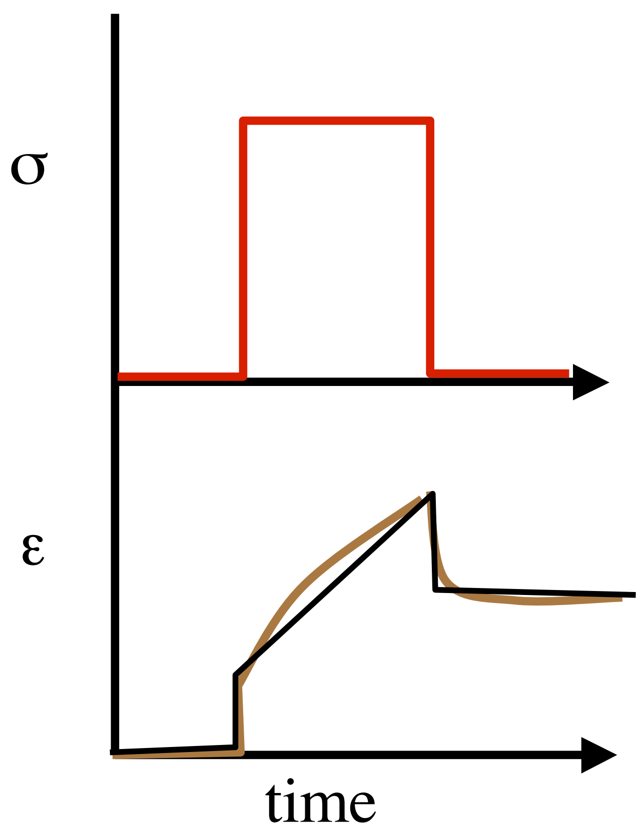
$\sigma \sim d\varepsilon/dt$

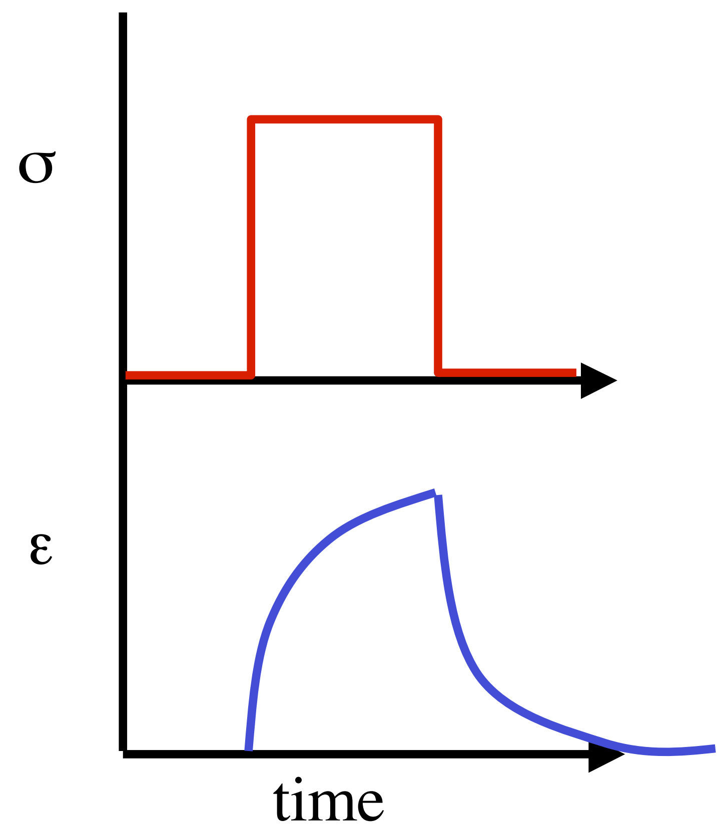
$\sigma = \mu\, d\varepsilon/dt$



dashpot

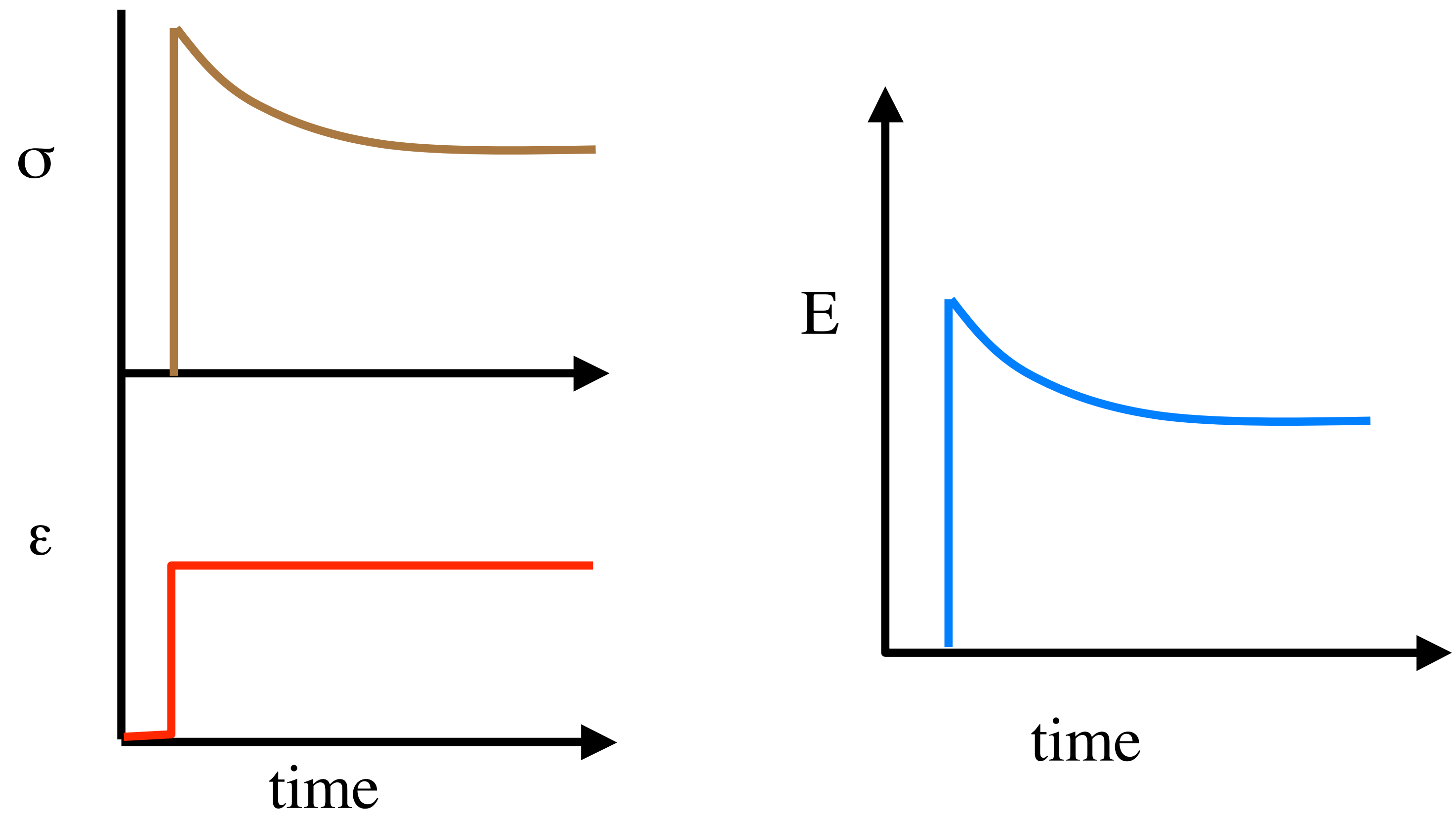
Creep characteristics of
mucus and cartilage





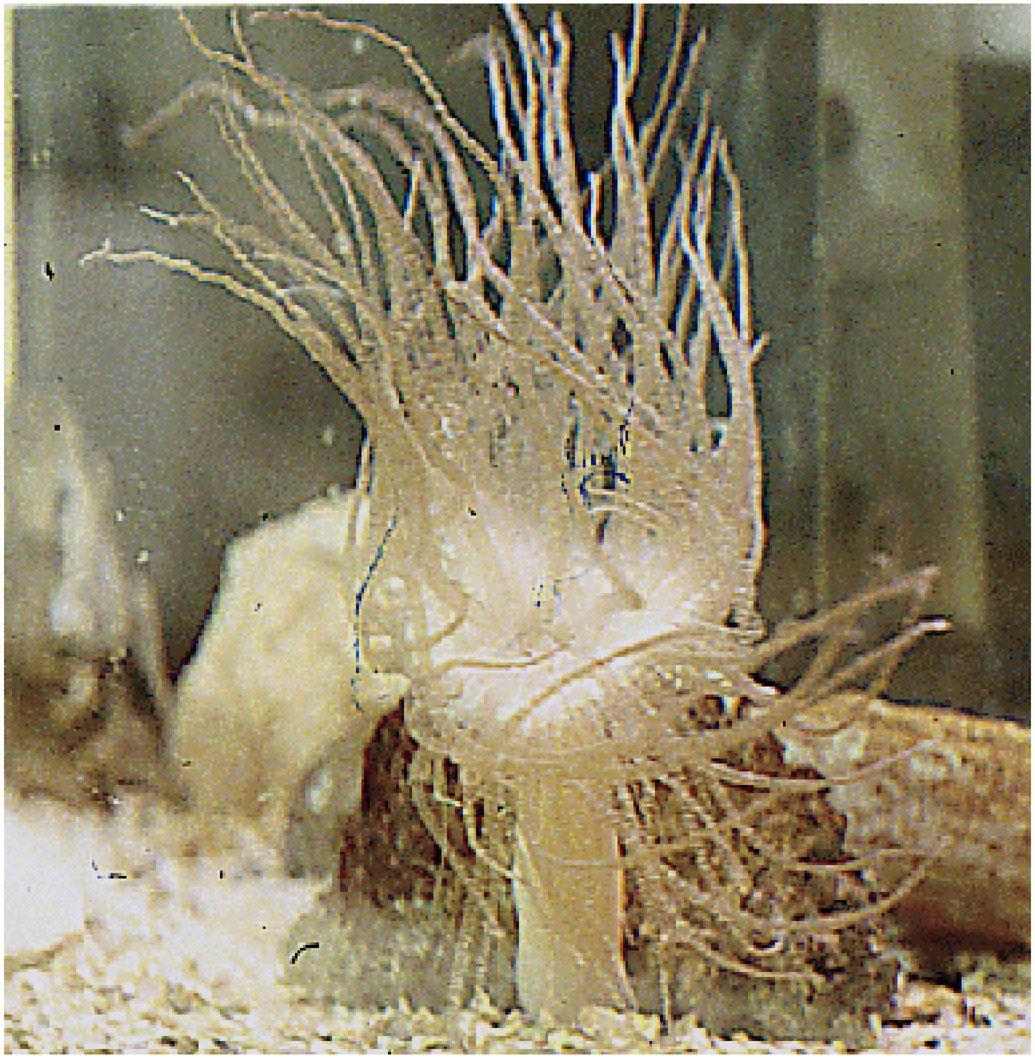
How does the stiffness vary in time?

A stress relaxtion test on the ACL



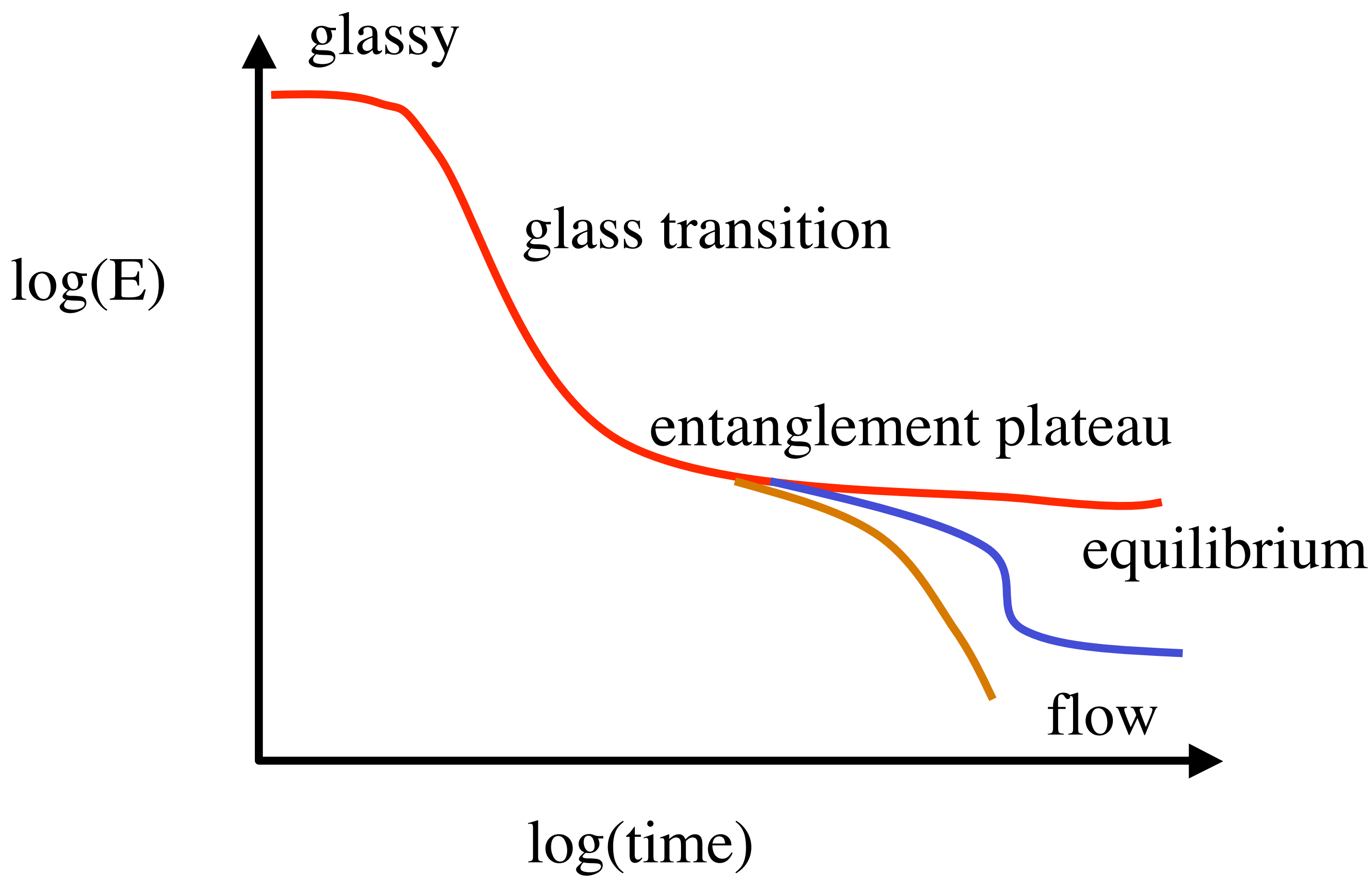
How does the stiffness vary in time?

Mesoglea (a protein polysaccharide)



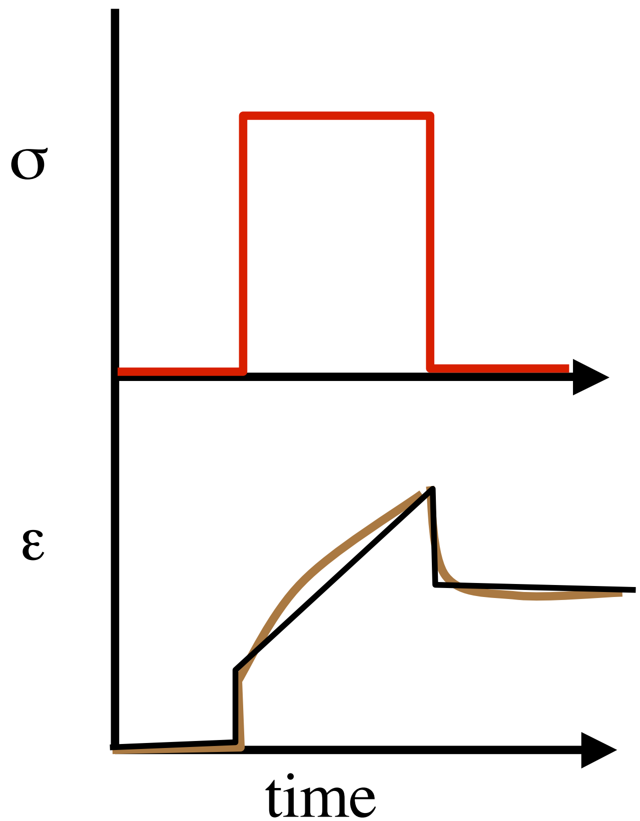
wave forces : seconds
postural forces: minutes
tidal changes : hours

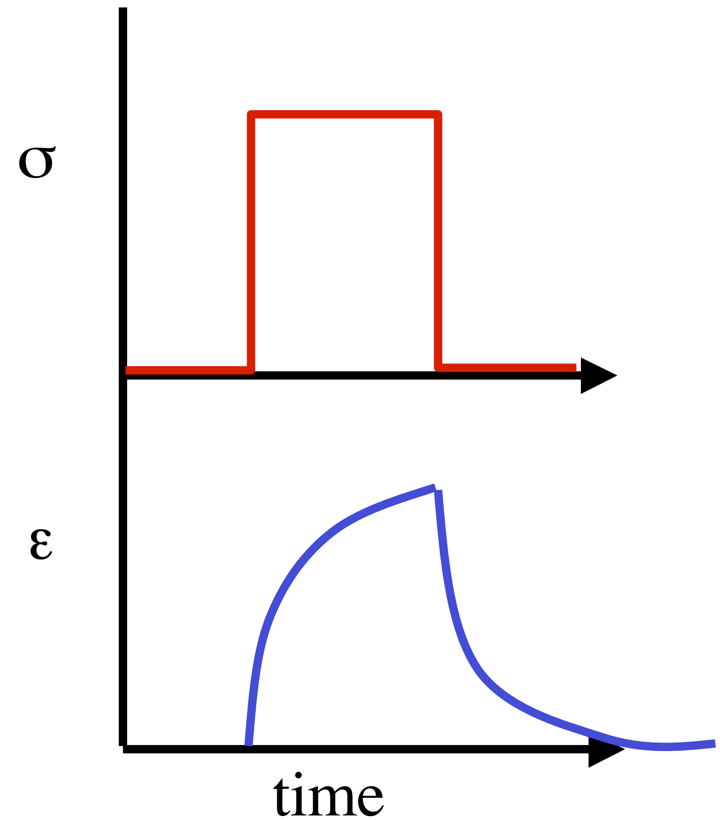
The Master Curve

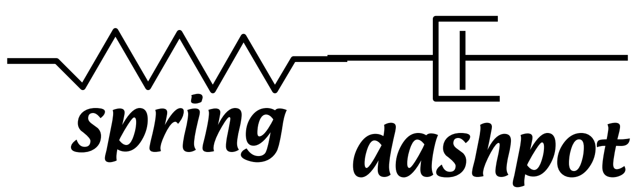


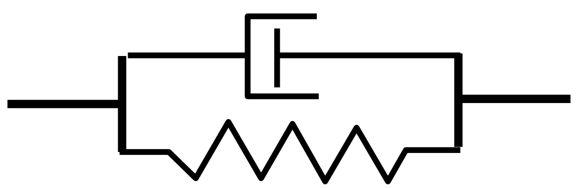
We often want to develop predictive models to how any material (structure) responds to a load.

Creep characteristics of *mucus* and *cartilage*

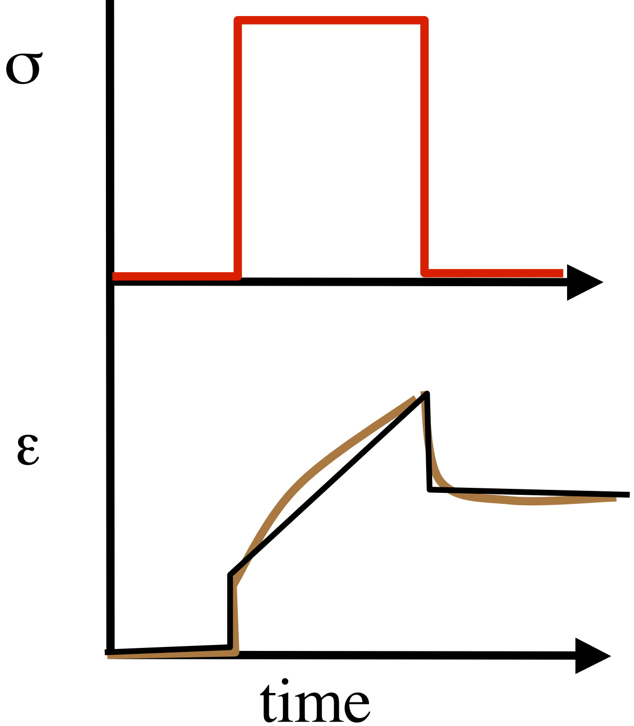


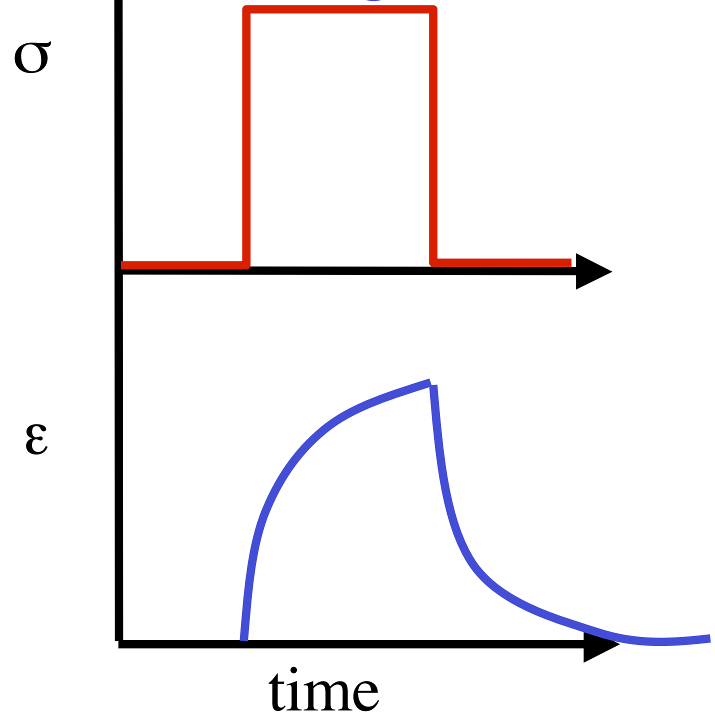






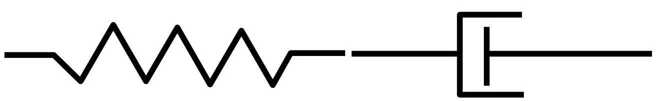
Creep characteristics of *mucus* *and* *cartilage*

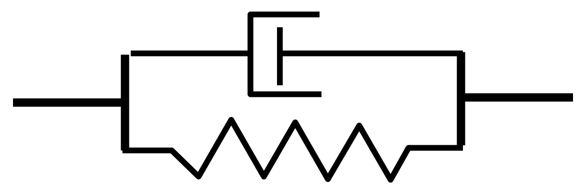




$$\sigma_s = \sigma_d = \sigma_{total}$$
$$\varepsilon_s + \varepsilon_d = \varepsilon_{total}$$
$$\mu\, d\varepsilon_d/dt = \sigma$$
$$\varepsilon_d = \sigma\, t/\mu$$
$$\sigma/E + \sigma\, t/\mu = \varepsilon_{total}$$

$$\sigma_s + \sigma_d = \sigma_{total}$$
$$\varepsilon_s = \varepsilon_d = \varepsilon_{total}$$
$$\mu\, d\varepsilon/dt + E\varepsilon = \sigma$$
$$\varepsilon = (1 - e^{-(E/\mu)\, t})$$



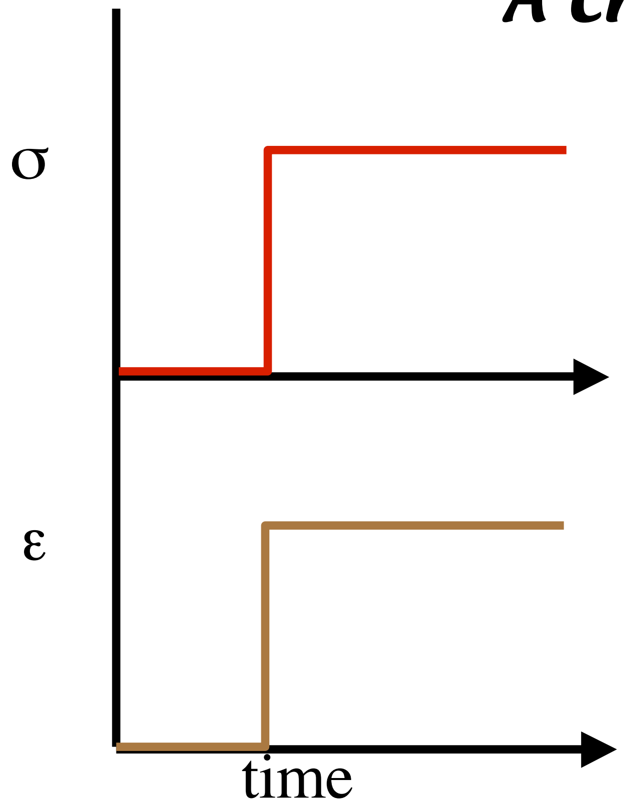


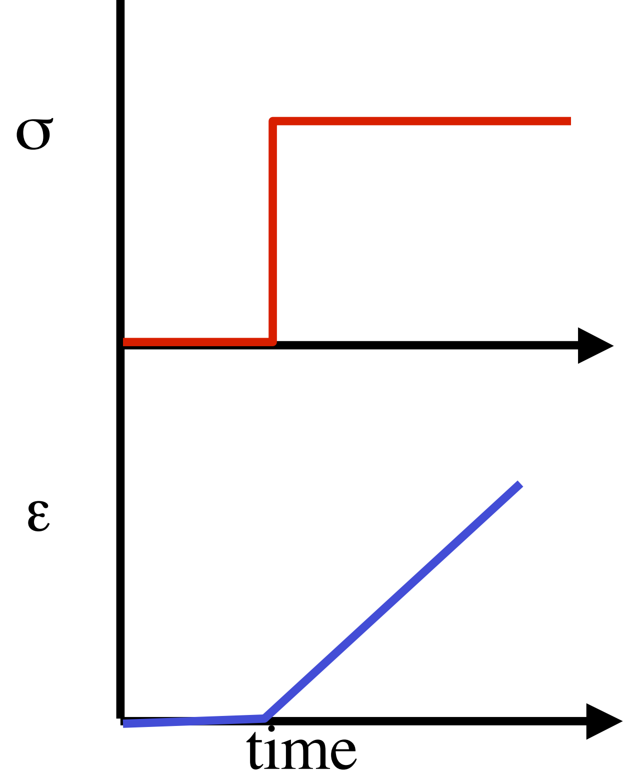
Biology 427 Biomechanics

Lecture 9. Models of the visco-elastic behavior of biological materials.

- *Recap visco-elasticity and time-dependent properties*
- *Simple theoretical models of visco-elastic materials*
- *Complex models applied to the cellular mechanism of sensory transduction. A tale of two sensors.*

A creep test





$F \sim \Delta L$

$\sigma \sim \varepsilon$

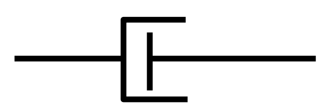
$\sigma = E\,\varepsilon$

spring

$F \sim \Delta L/\Delta t \sim dL/dt$

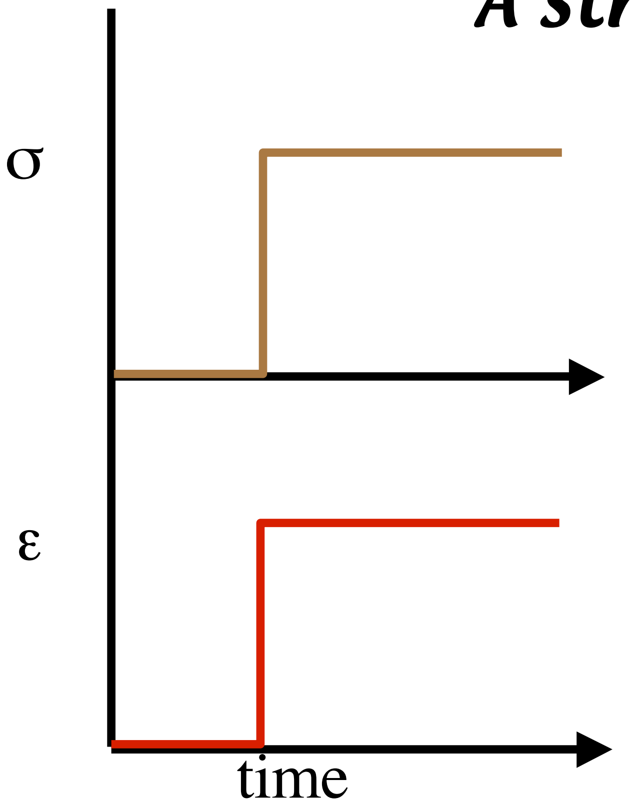
$\sigma \sim d\varepsilon/dt$

$\sigma = \mu\, d\varepsilon/dt$



dashpot

A stress relaxtion test

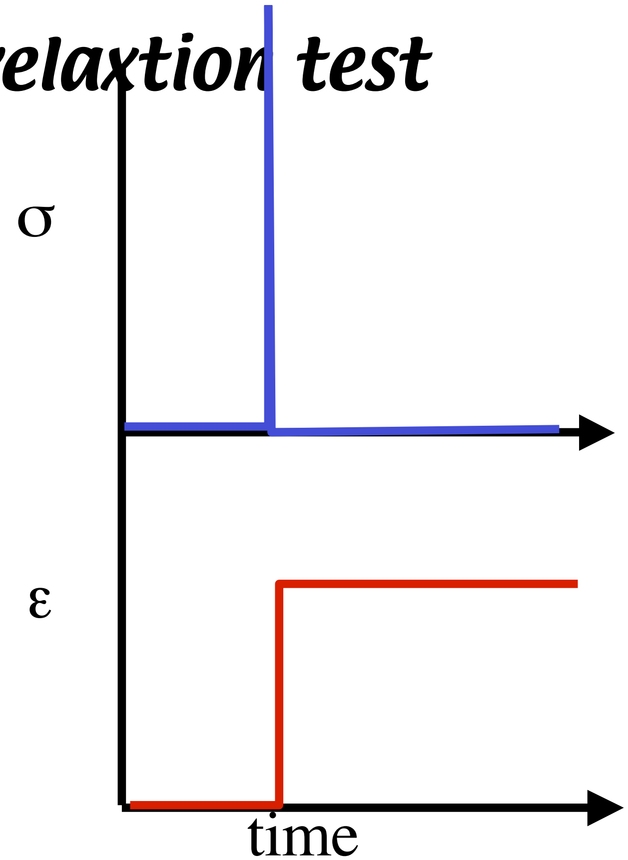


$F \sim \Delta L$

$\sigma \sim \varepsilon$

$\sigma = E\, \varepsilon$

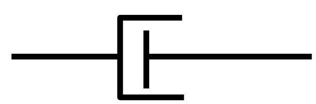
spring



$F \sim \Delta L/\Delta t \sim dL/dt$

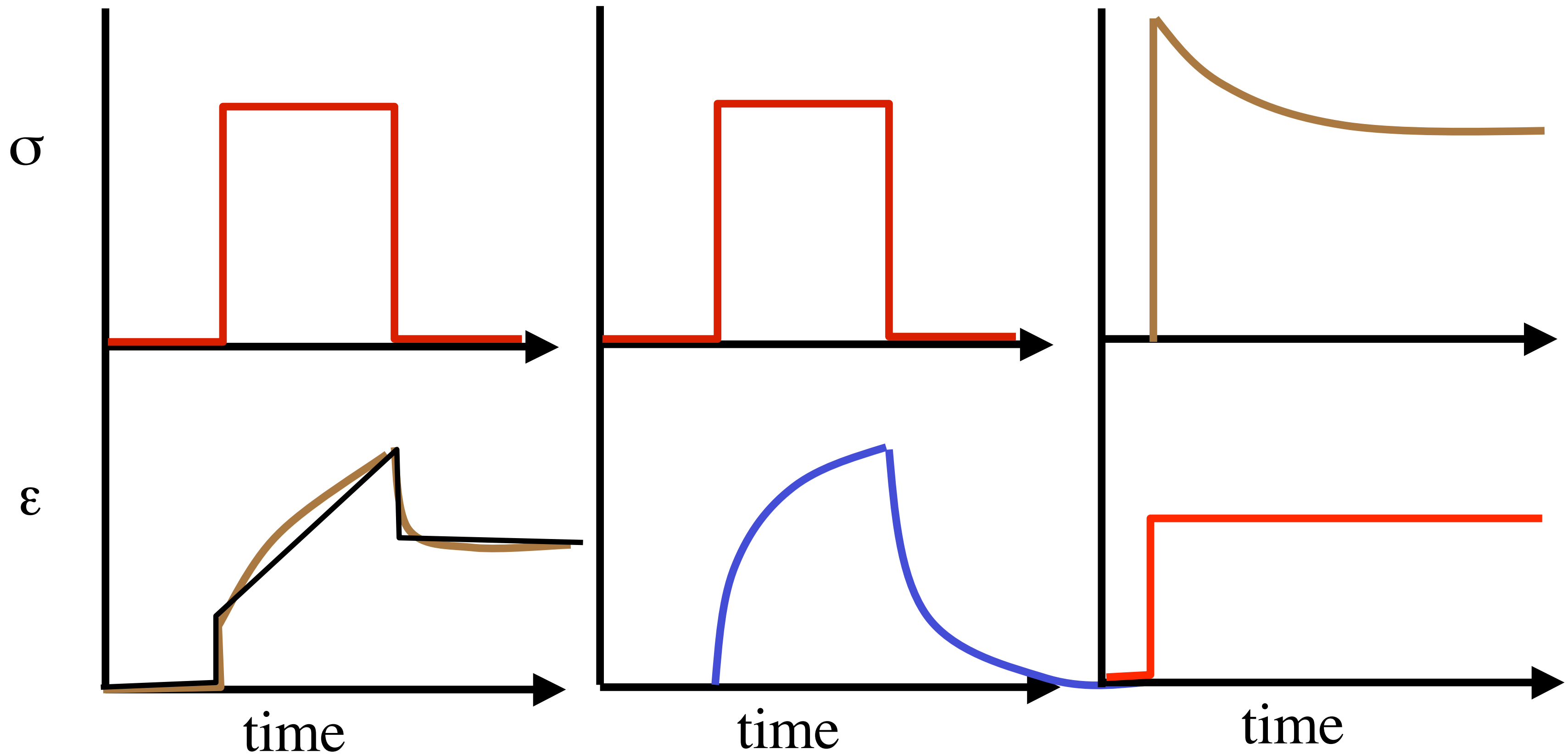
$\sigma \sim d\varepsilon/dt$

$\sigma = \mu\, d\varepsilon/dt$



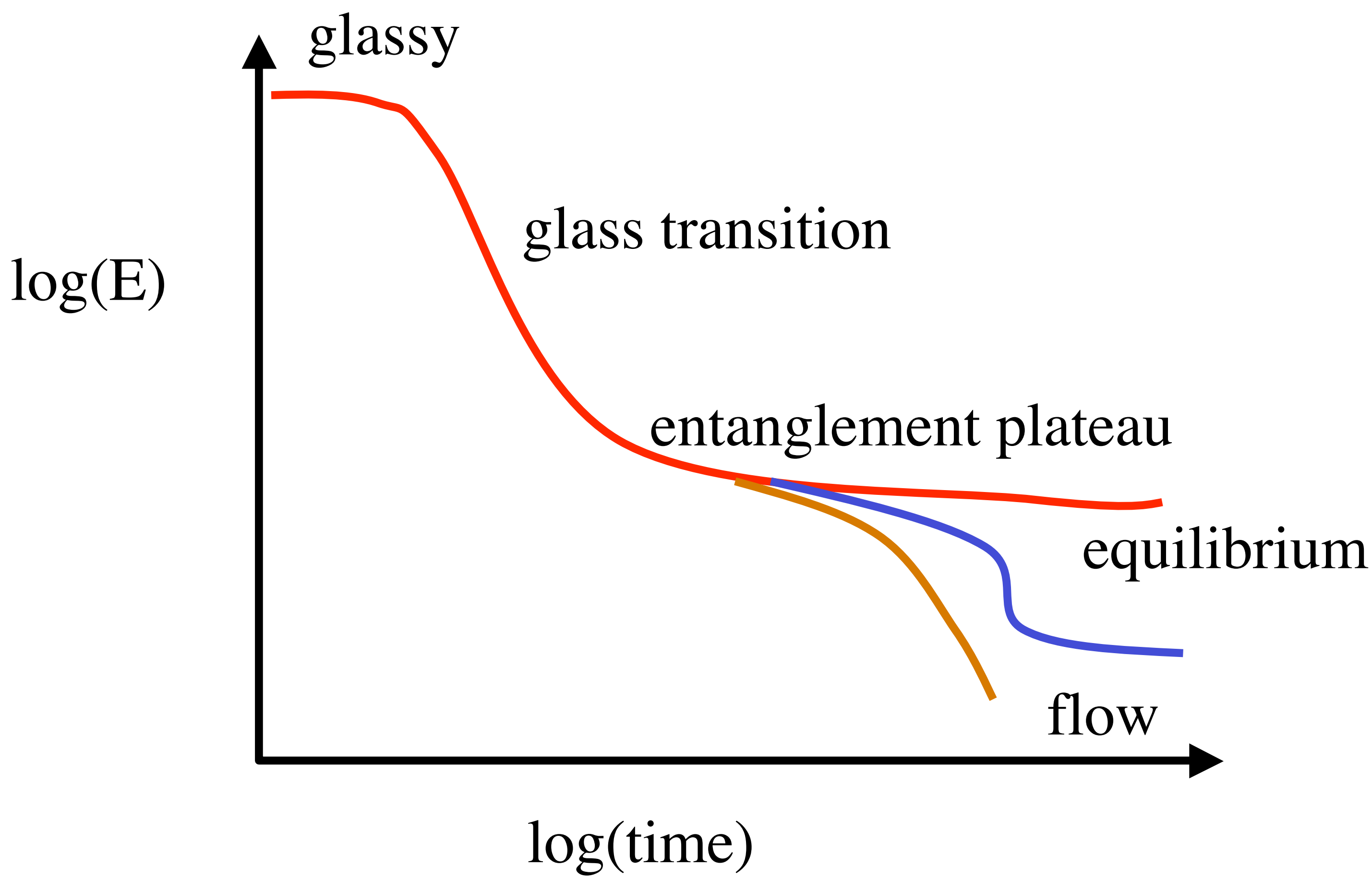
dashpot

Time dependent properties of
mucus cartilage and tendon



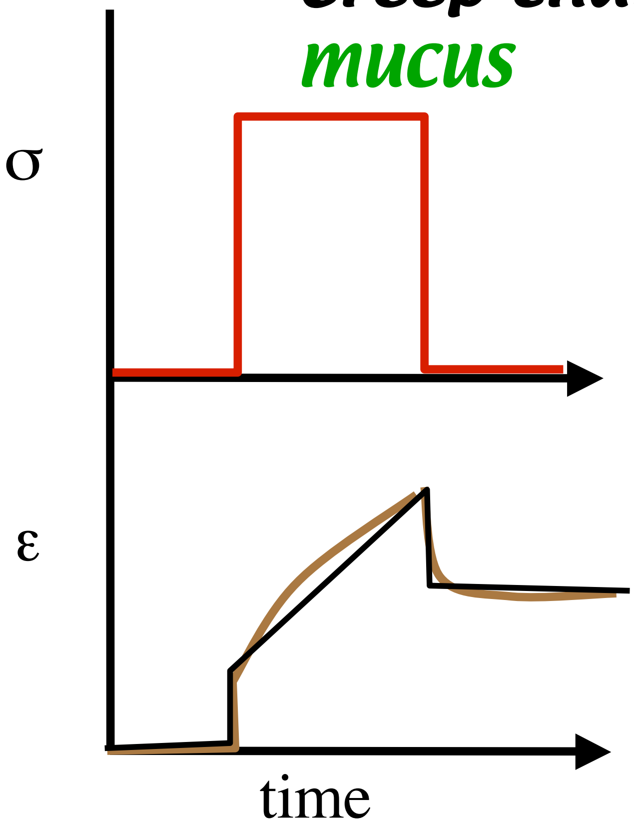
How does the stiffness vary in time?

The Master Curve



We often want to develop predictive models to how any material (structure) responds to a load.

Creep characteristics of mucus and cartilage



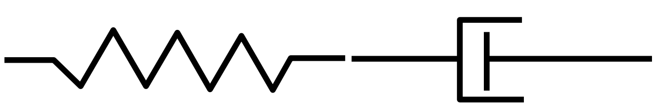
$\sigma_s = \sigma_d = \sigma_{total}$

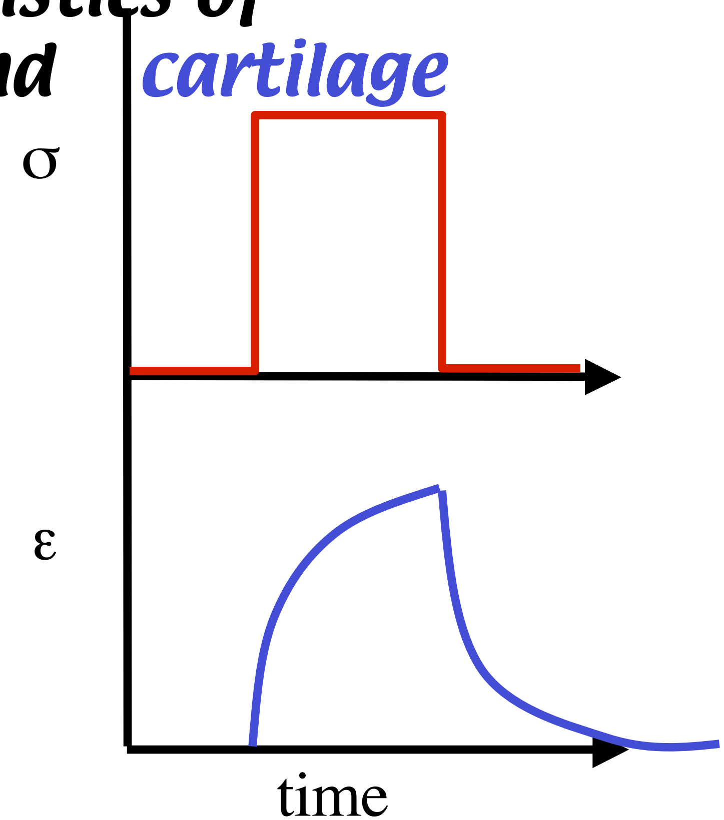
$\varepsilon_s + \varepsilon_d = \varepsilon_{total}$

$\mu \, d\varepsilon_d/dt = \sigma$

$\varepsilon_d = \sigma \, t/\mu$

$\sigma/E + \sigma \, t/\mu = \varepsilon_{total}$



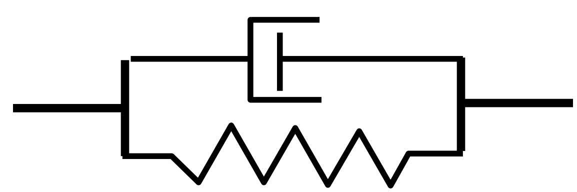


$\sigma_s + \sigma_d = \sigma_{total}$

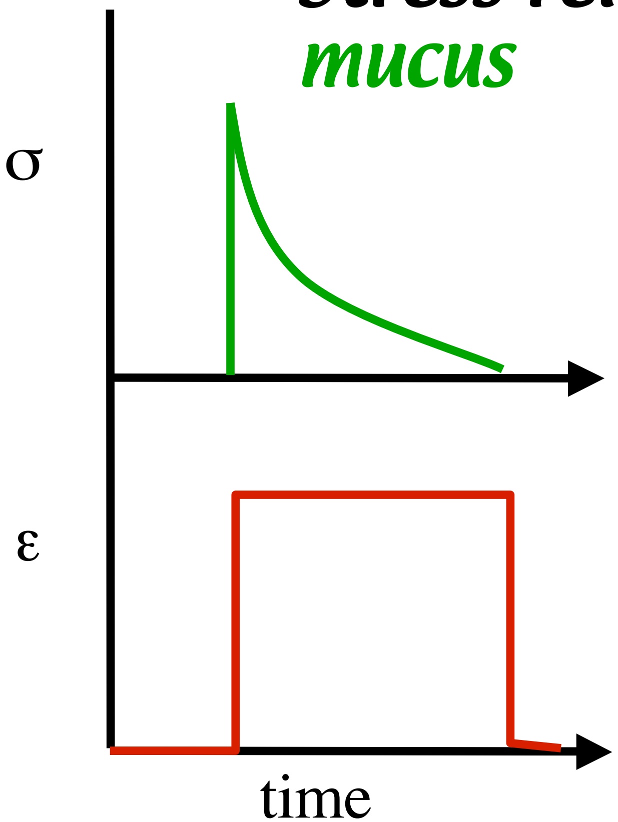
$\varepsilon_s = \varepsilon_d = \varepsilon_{total}$

$\mu \, d\varepsilon/dt + E\varepsilon = \sigma$

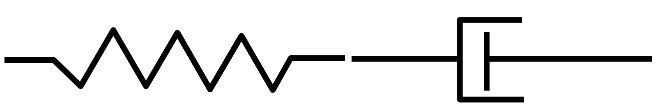
$\varepsilon = (1 - e^{-(E/\mu)\,t})$

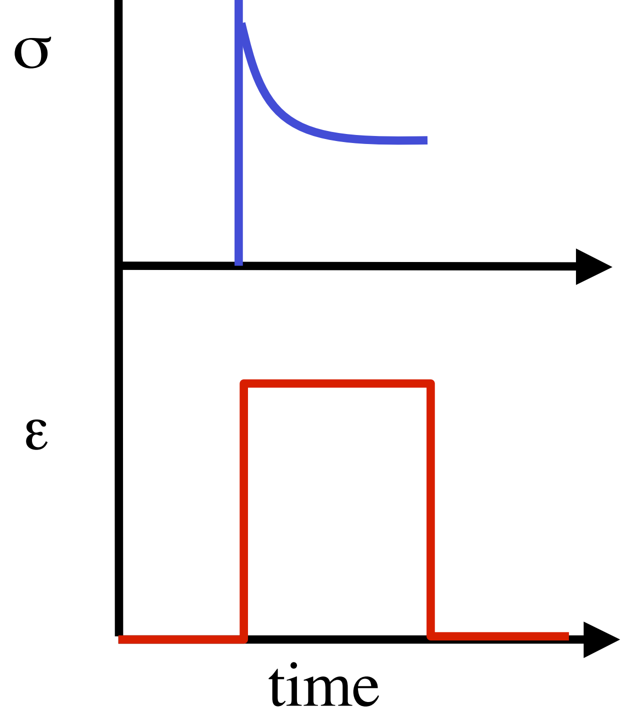


Stress relaxtion characteristics
mucus and cartilage

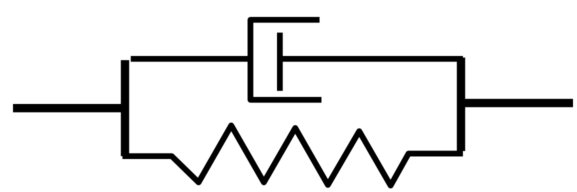


$$\sigma_s = \sigma_d = \sigma_{total}$$
$$\varepsilon_s + \varepsilon_d = \varepsilon_{total}$$
$$\sigma_{total} = E\,\varepsilon\,(1 - e^{\,Et/\mu})$$

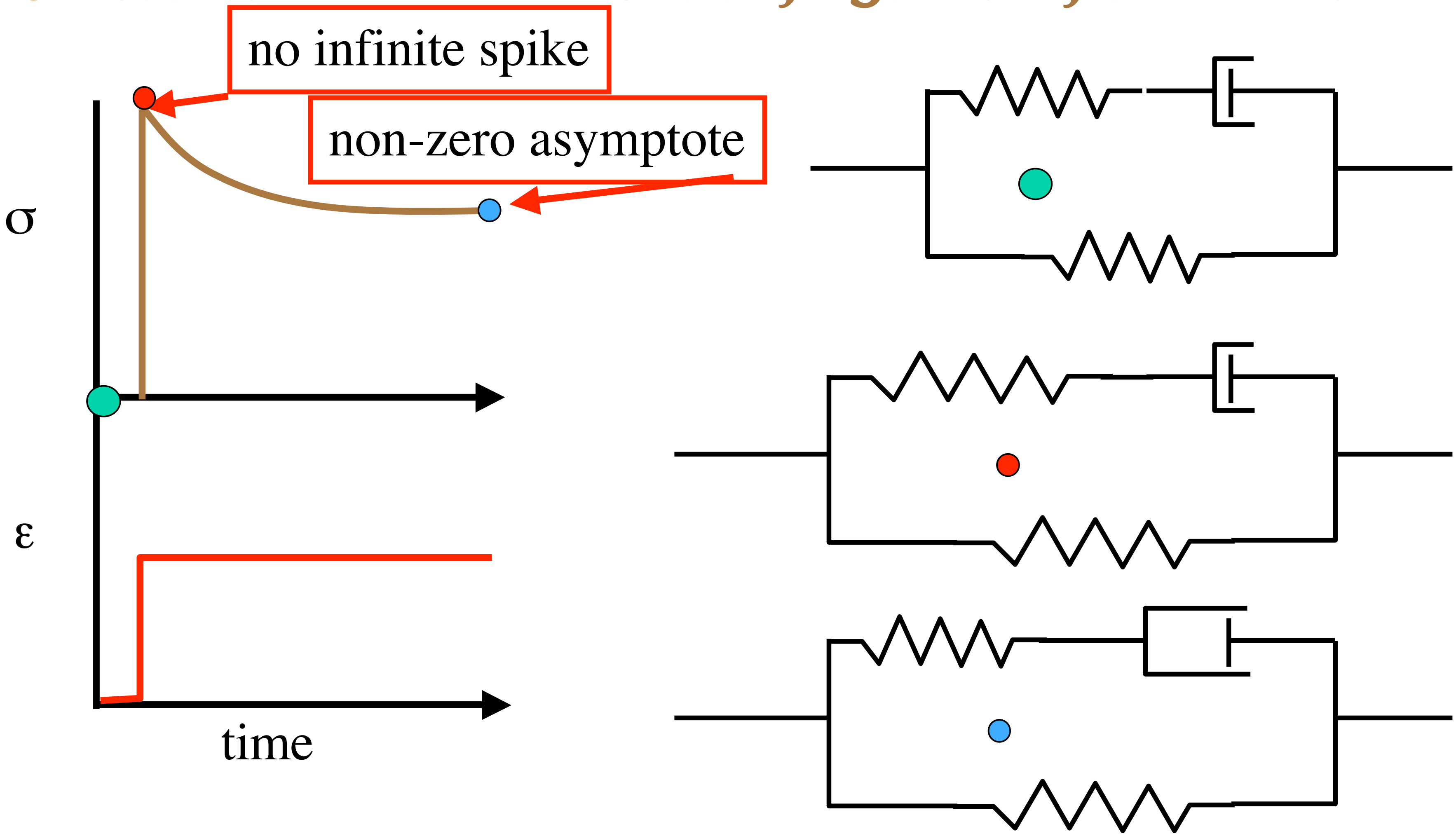




$$\sigma_s + \sigma_d = \sigma_{total}$$
$$\varepsilon_s = \varepsilon_d = \varepsilon_{total}$$

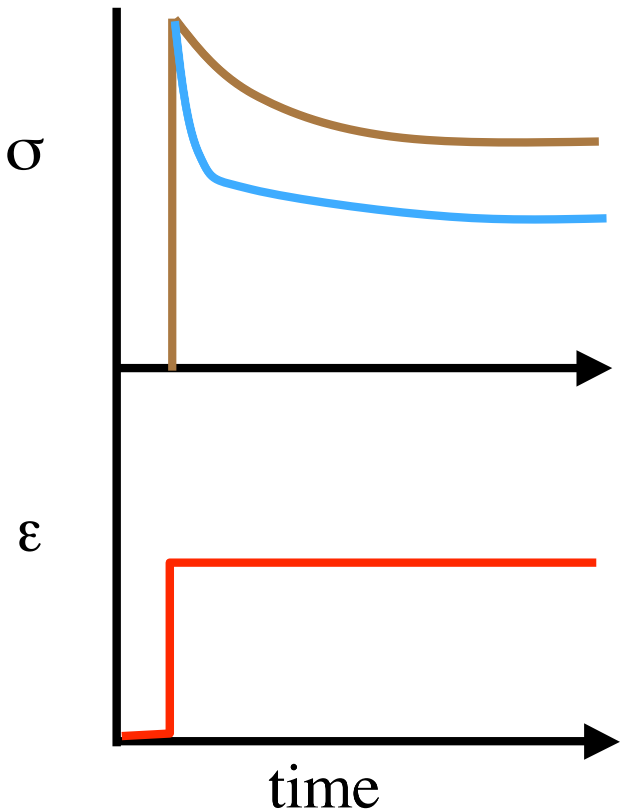


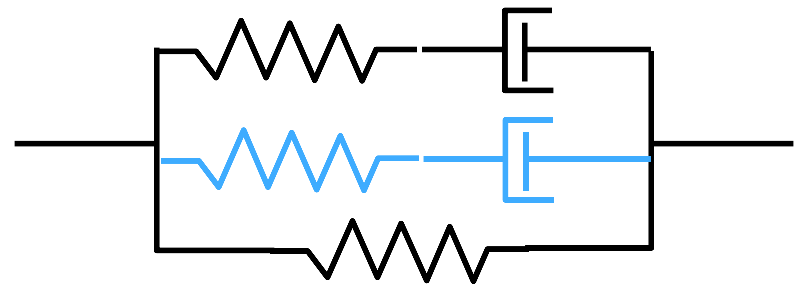
Stress relaxation in tendon, ligament, chitin ....

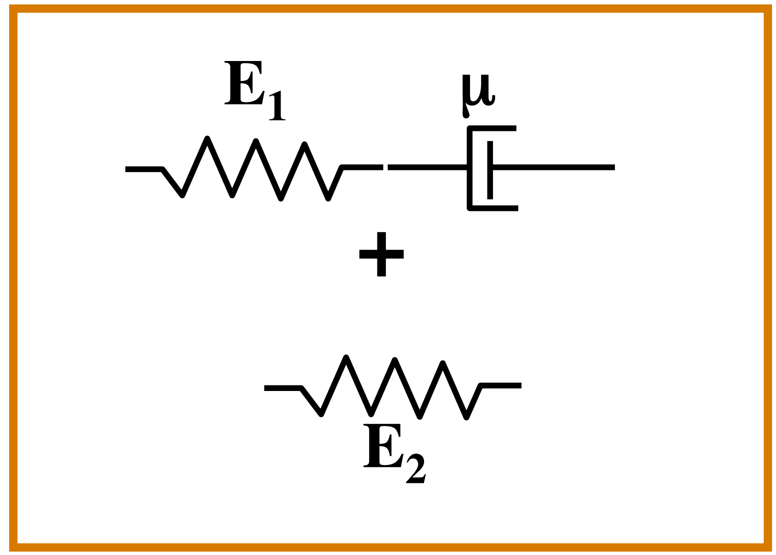


We need a more general model

Stress relaxation in tendon, ligament, chitin ….







Standard linear solid

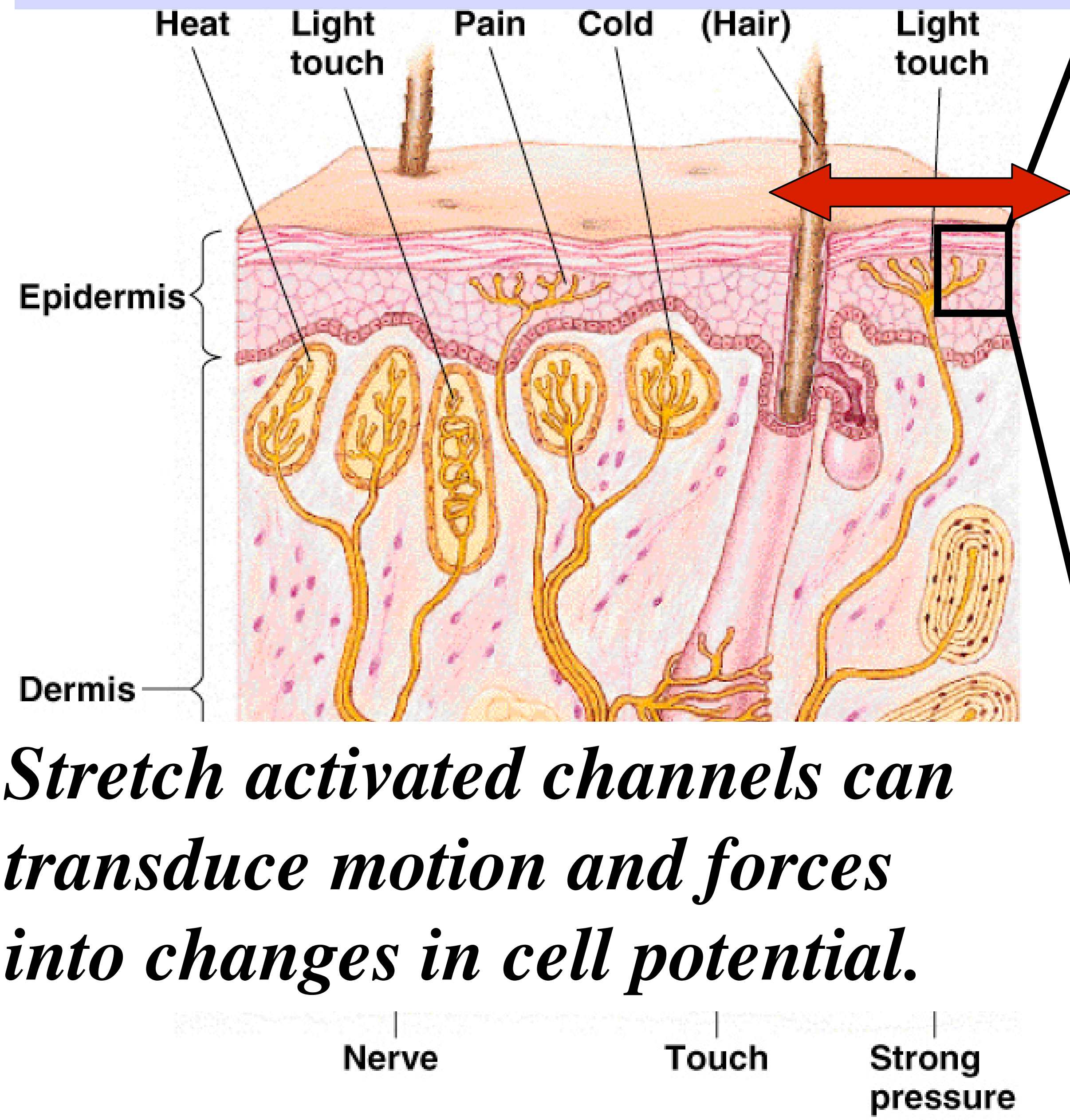
$\sigma_1 = E_1\,\varepsilon\,(1 - e^{-E_1 t/\mu})$

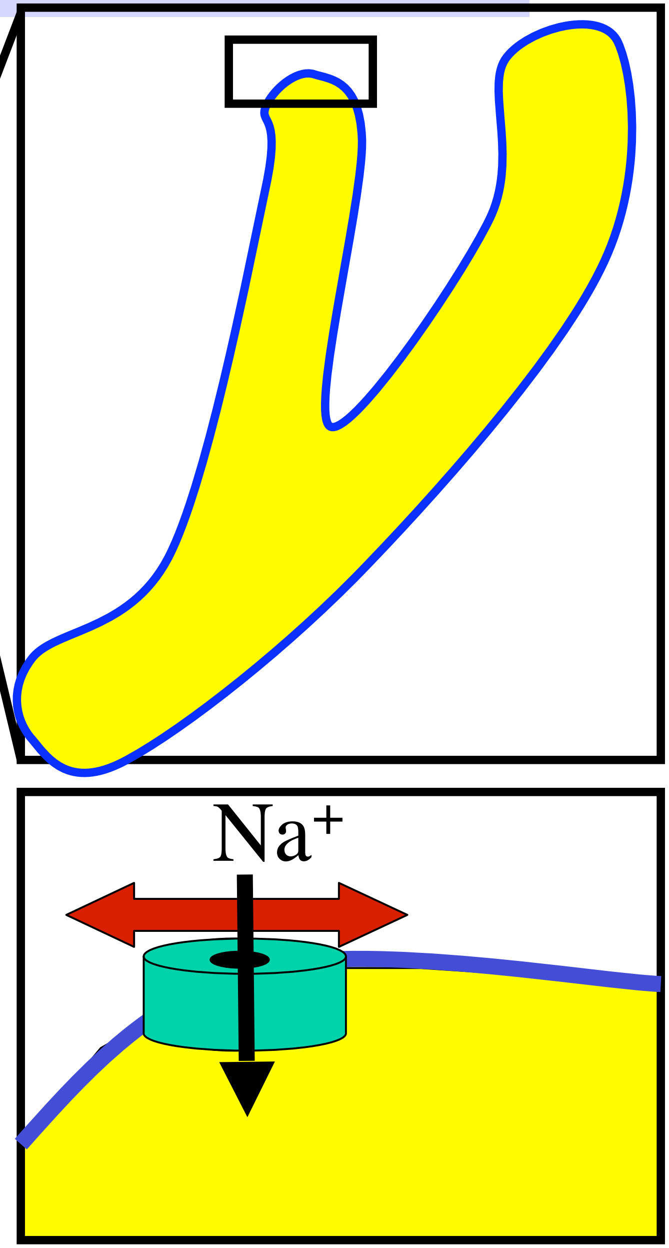
$\sigma_2 = E_2\,\varepsilon$

$\sigma = E_2\,\varepsilon + E_1\,\varepsilon\,(1 - e^{-E1t/\mu})$

$+ E_3\,\varepsilon\,(1 - e^{E3t/\mu})$

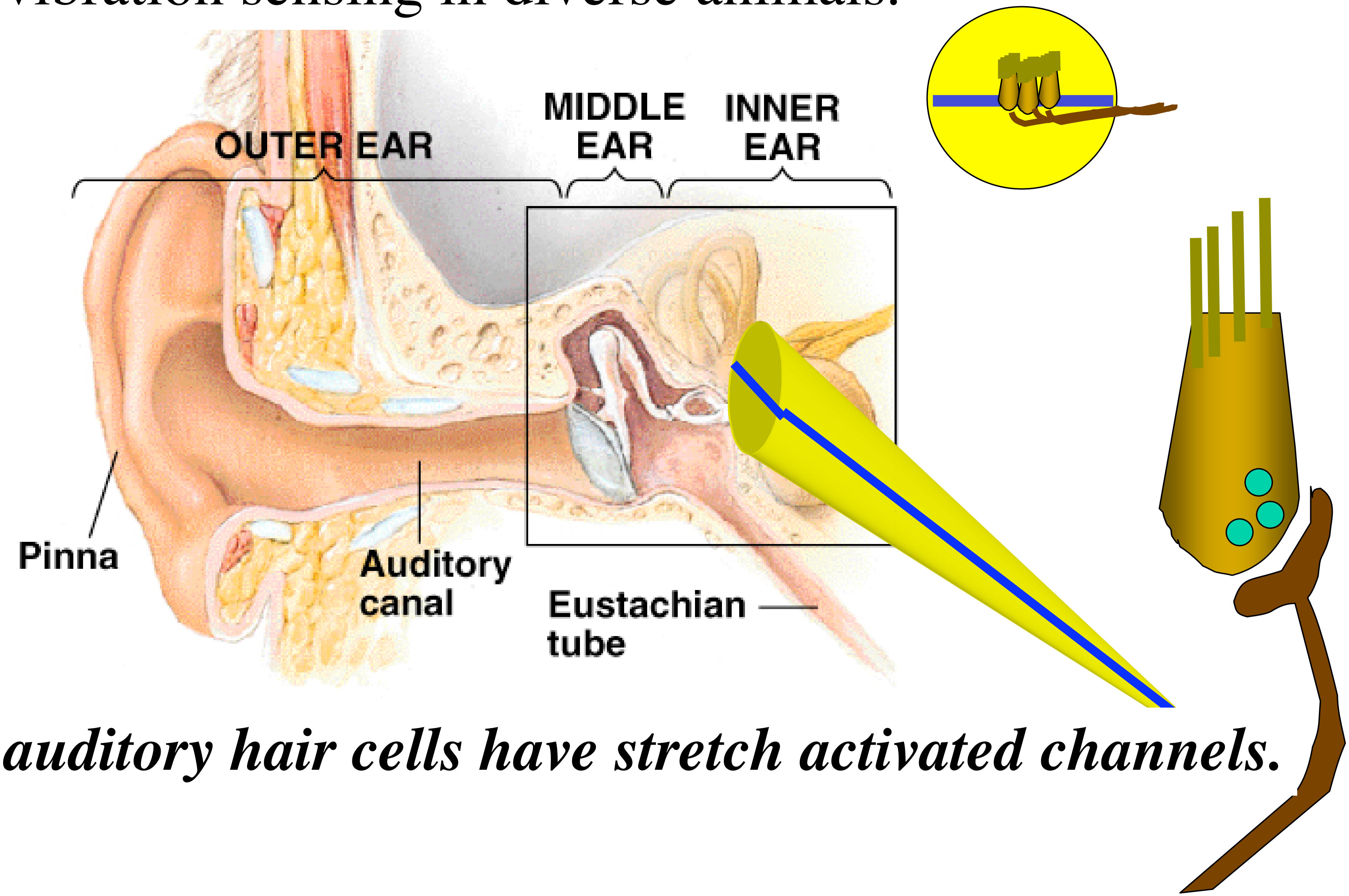
Mechanoreception: detecting motions and forces



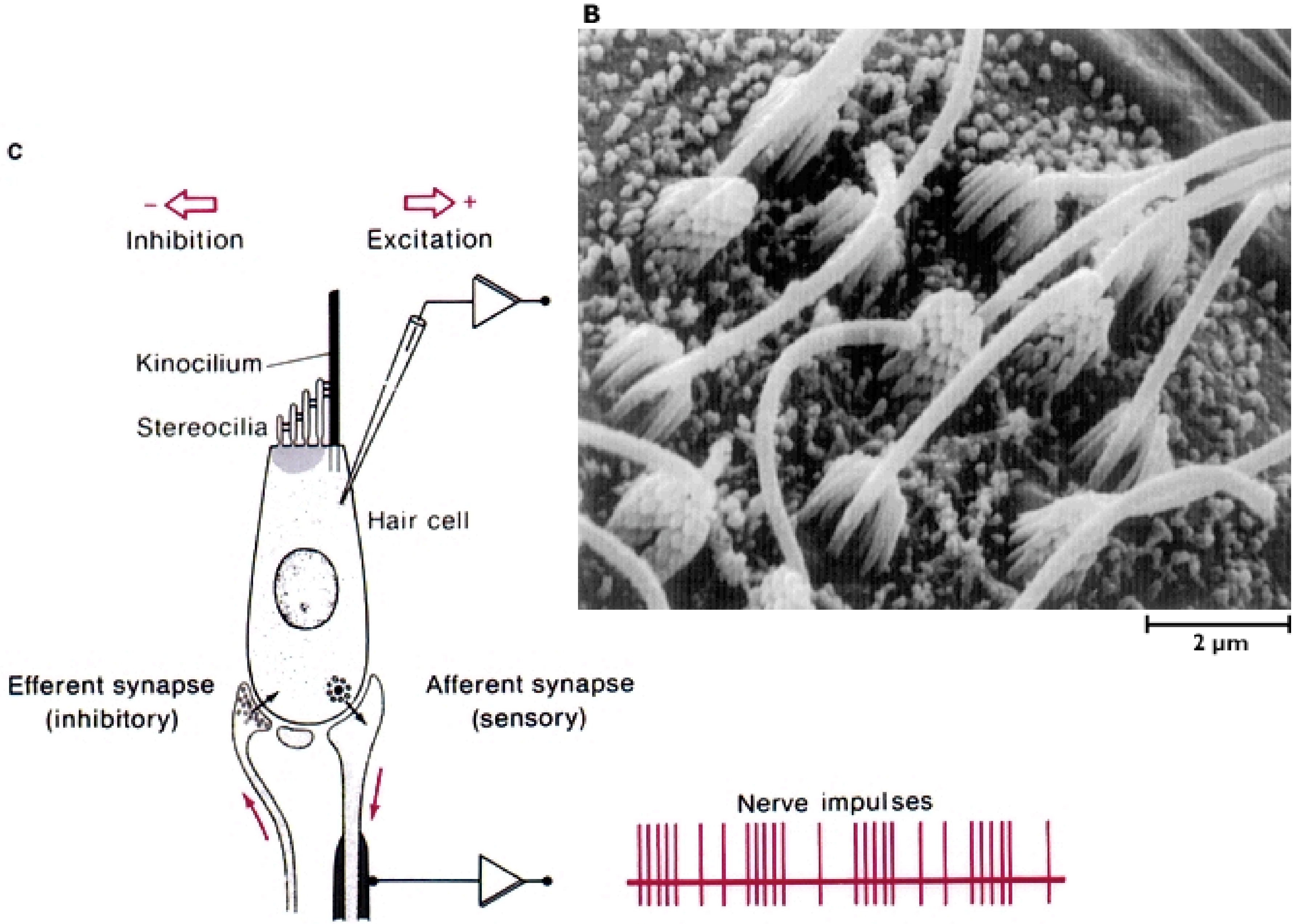


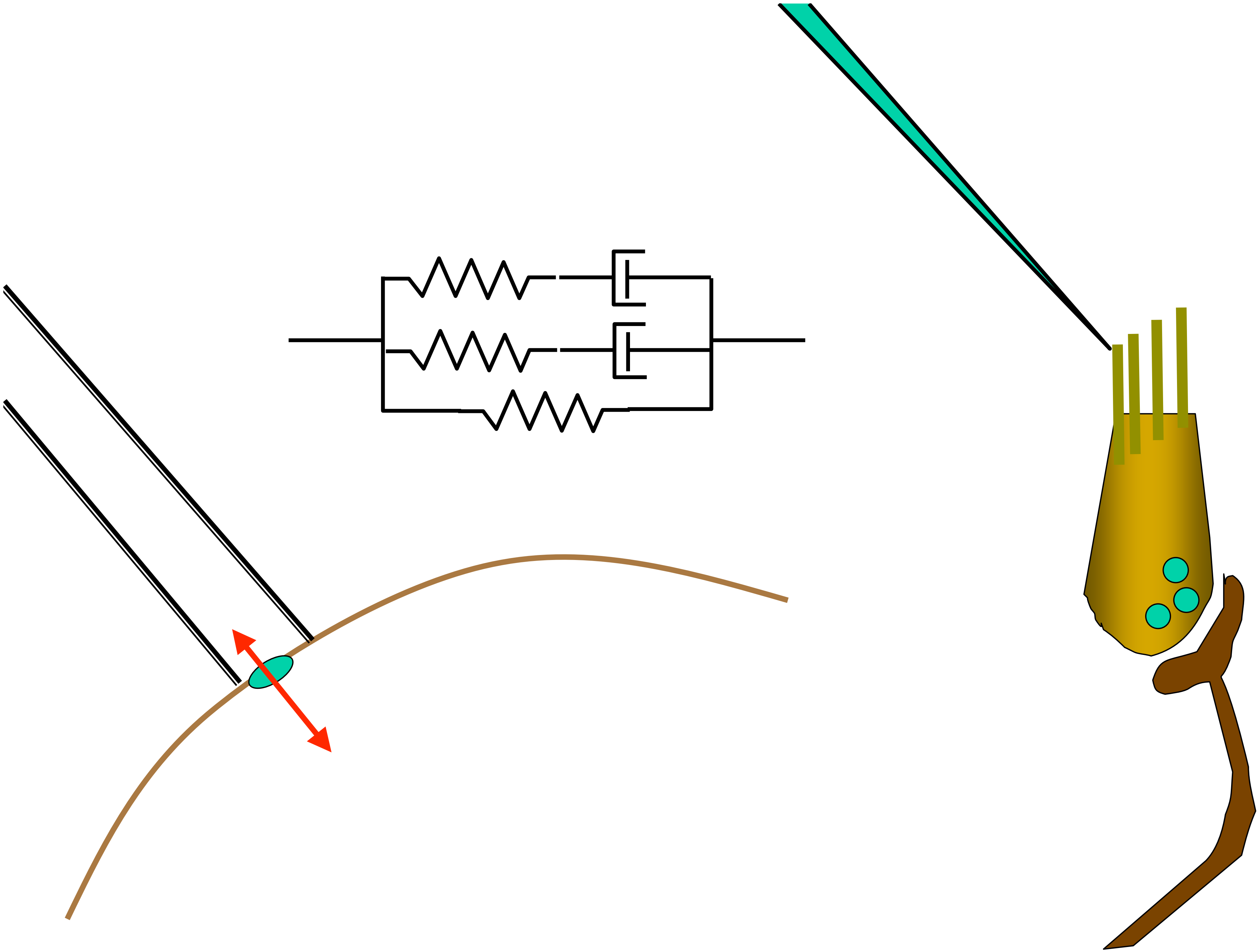
Stretch activated channels can transduce motion and forces into changes in cell potential.

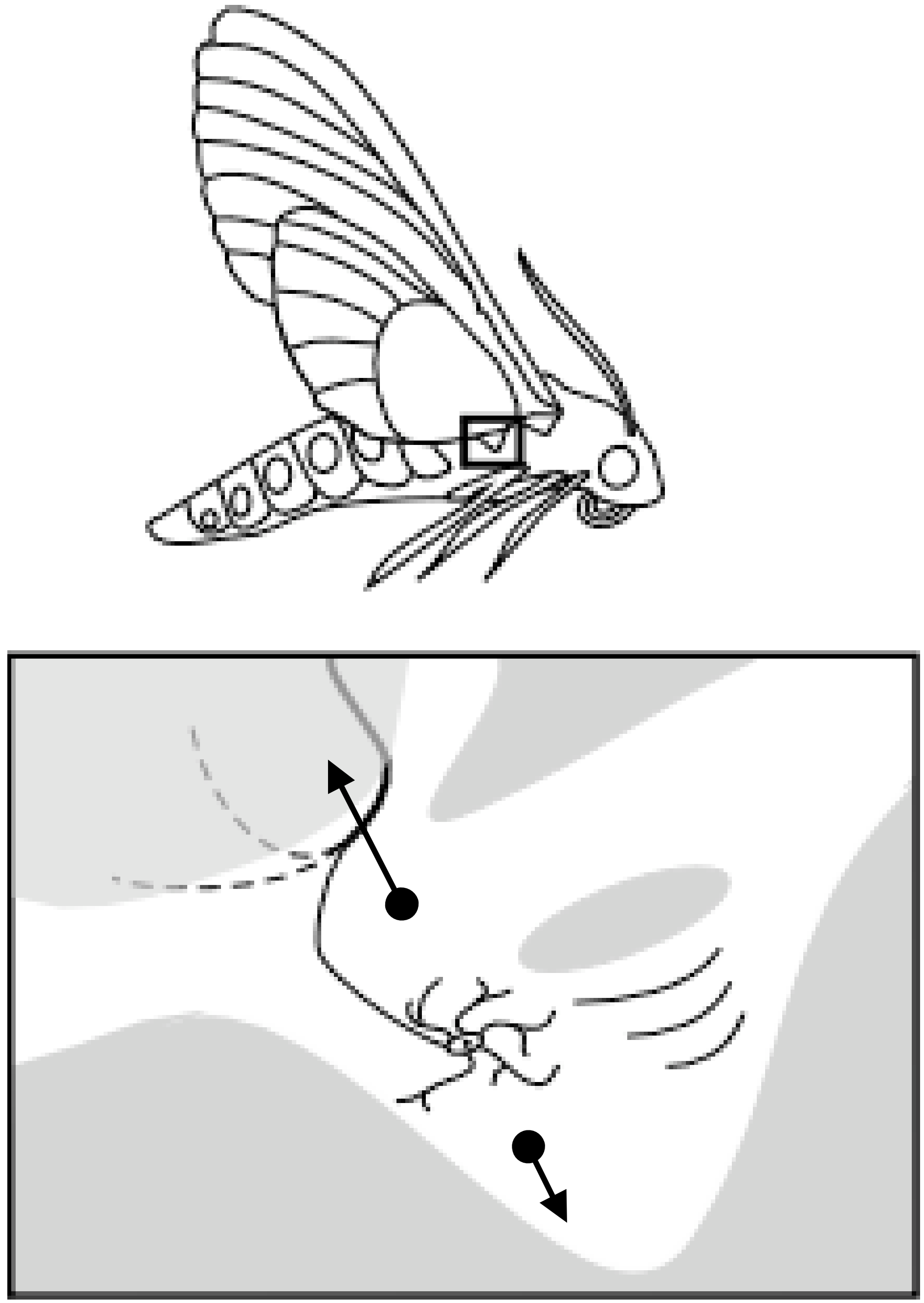
Stretch activated channels underlie hearing, balance, vibration sensing in diverse animals.



auditory hair cells have stretch activated channels.

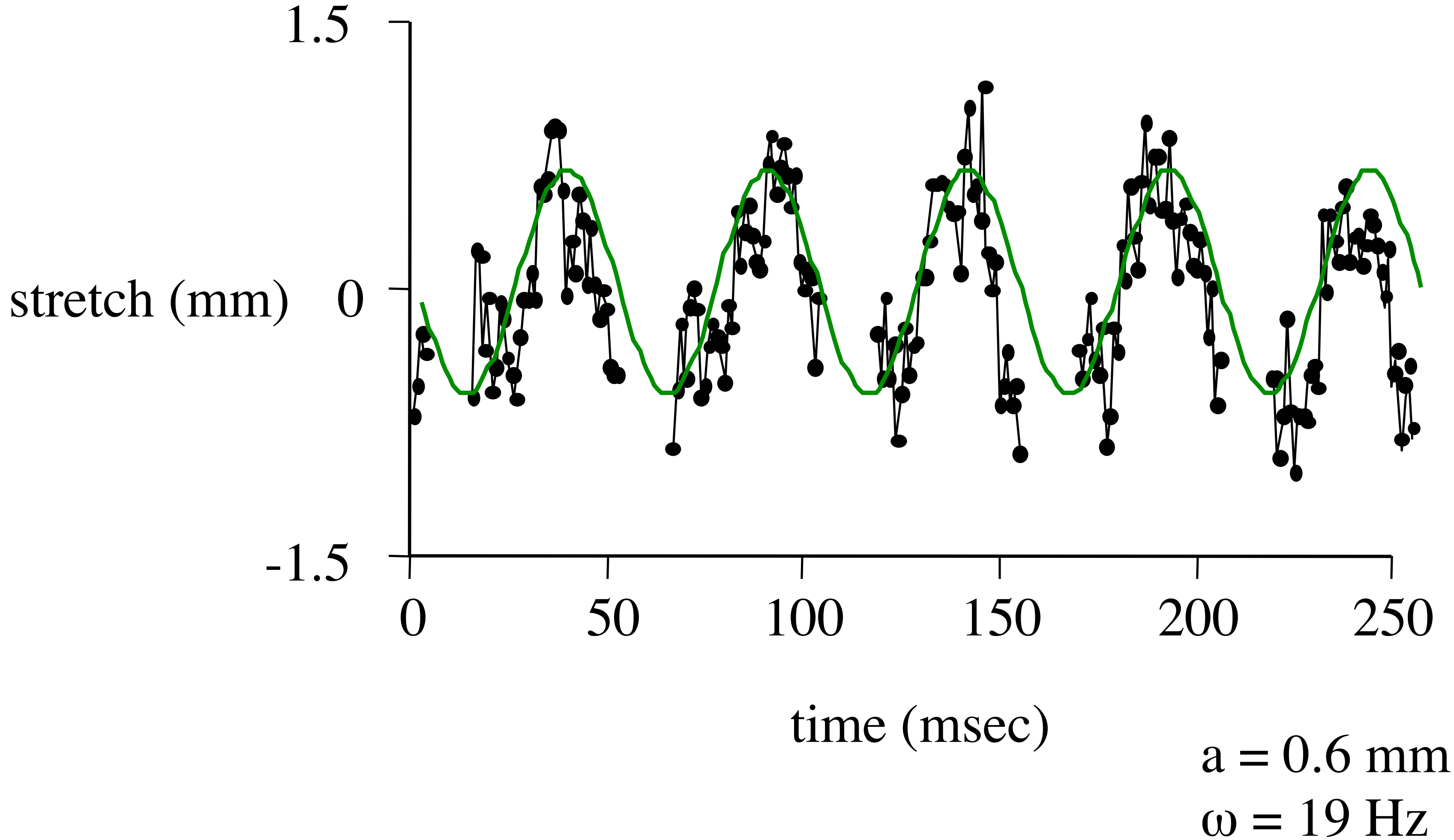

B
2 μm
C
Inhibition
Excitation
Kinocilium
Stereocilia
Hair cell
Efferent synapse
(inhibitory)
Afferent synapse
(sensory)
Nerve impulses

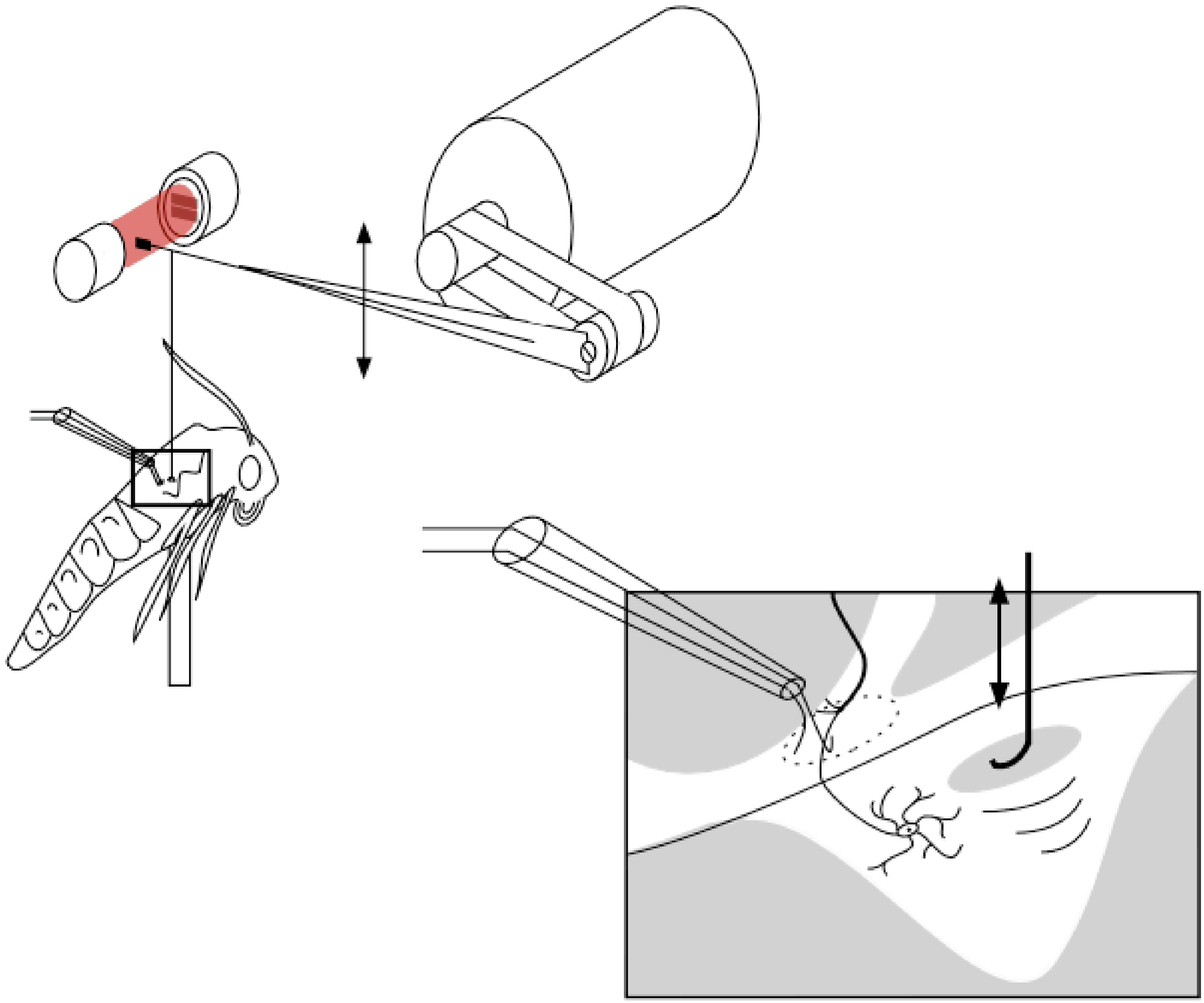


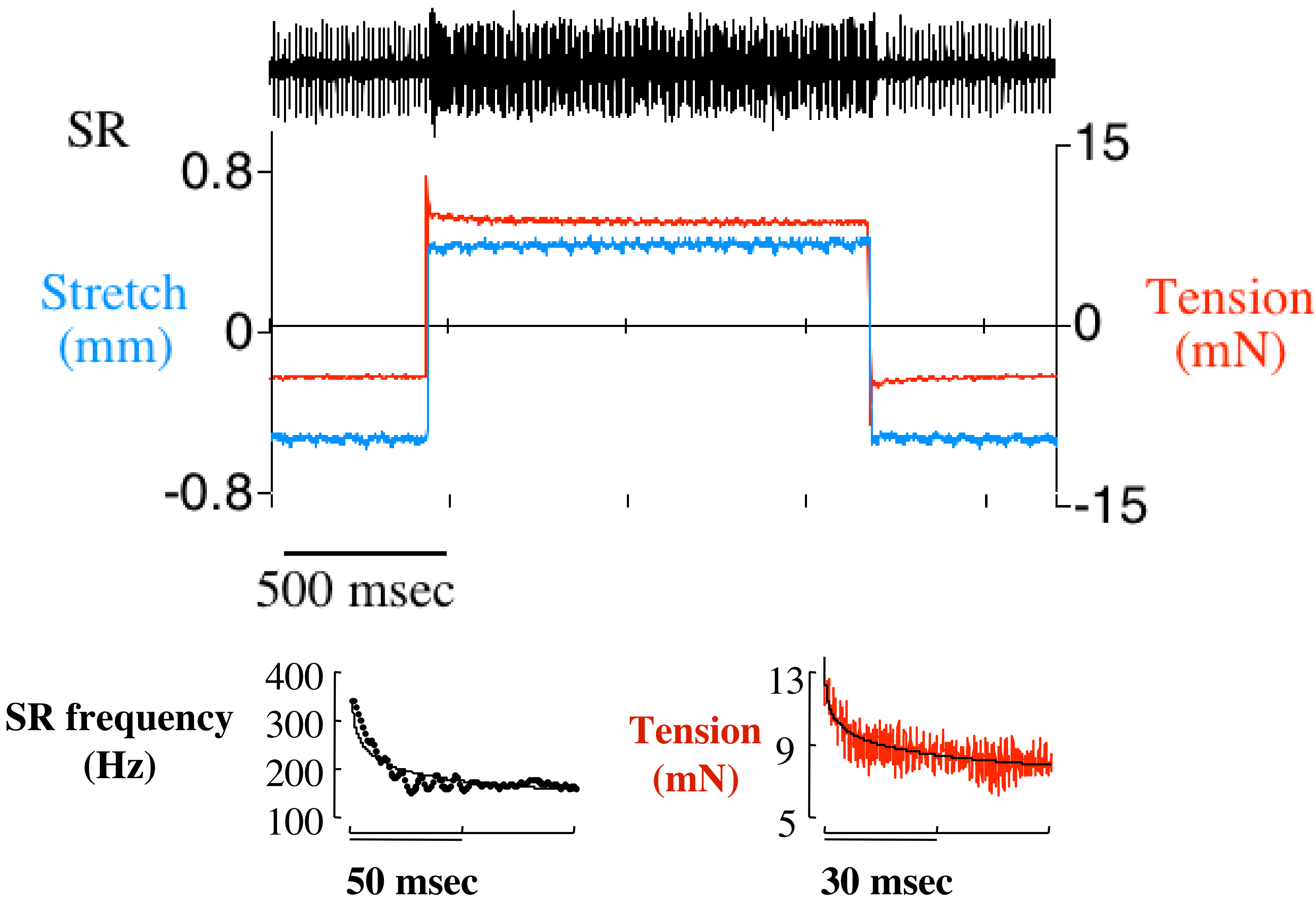


M. Frye
UW &
UCB

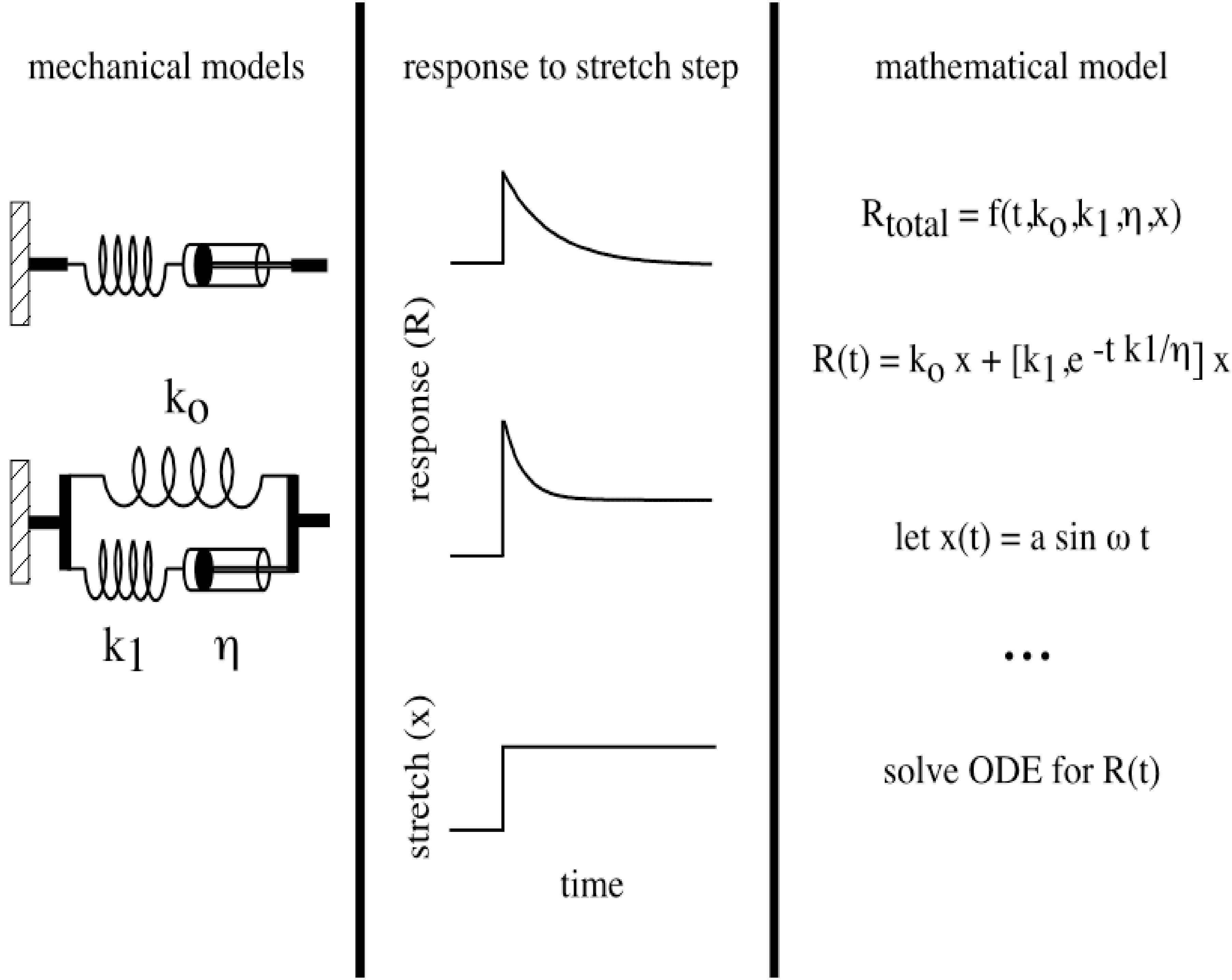
Sinwave captures *in vivo* wing hinge deformation

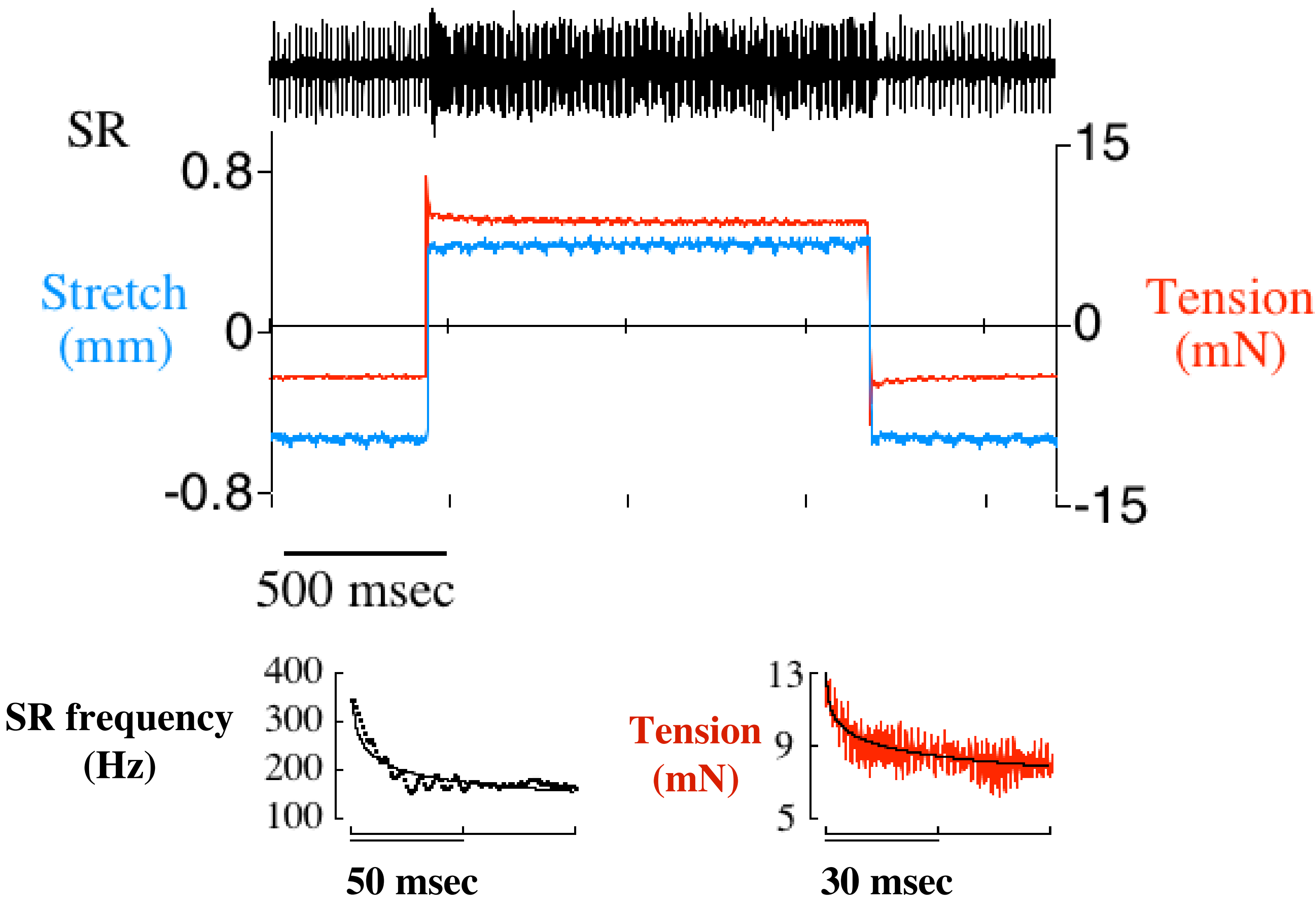






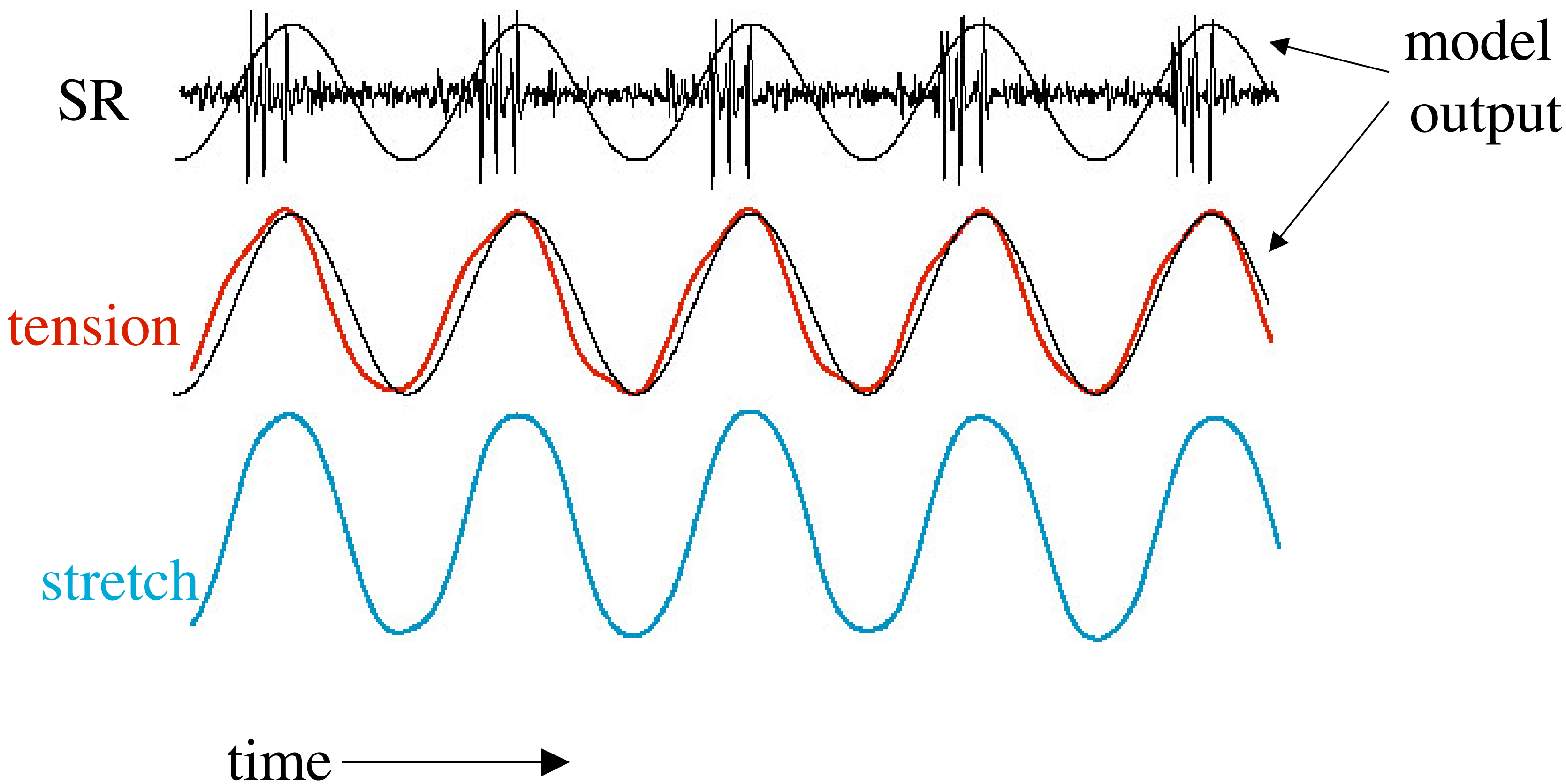
SR
0.8
0
-0.8
15
0
-15
Stretch (mm)
Tension (mN)
500 msec
SR frequency (Hz)
400
300
200
100
50 msec
Tension (mN)
13
9
5
30 msec

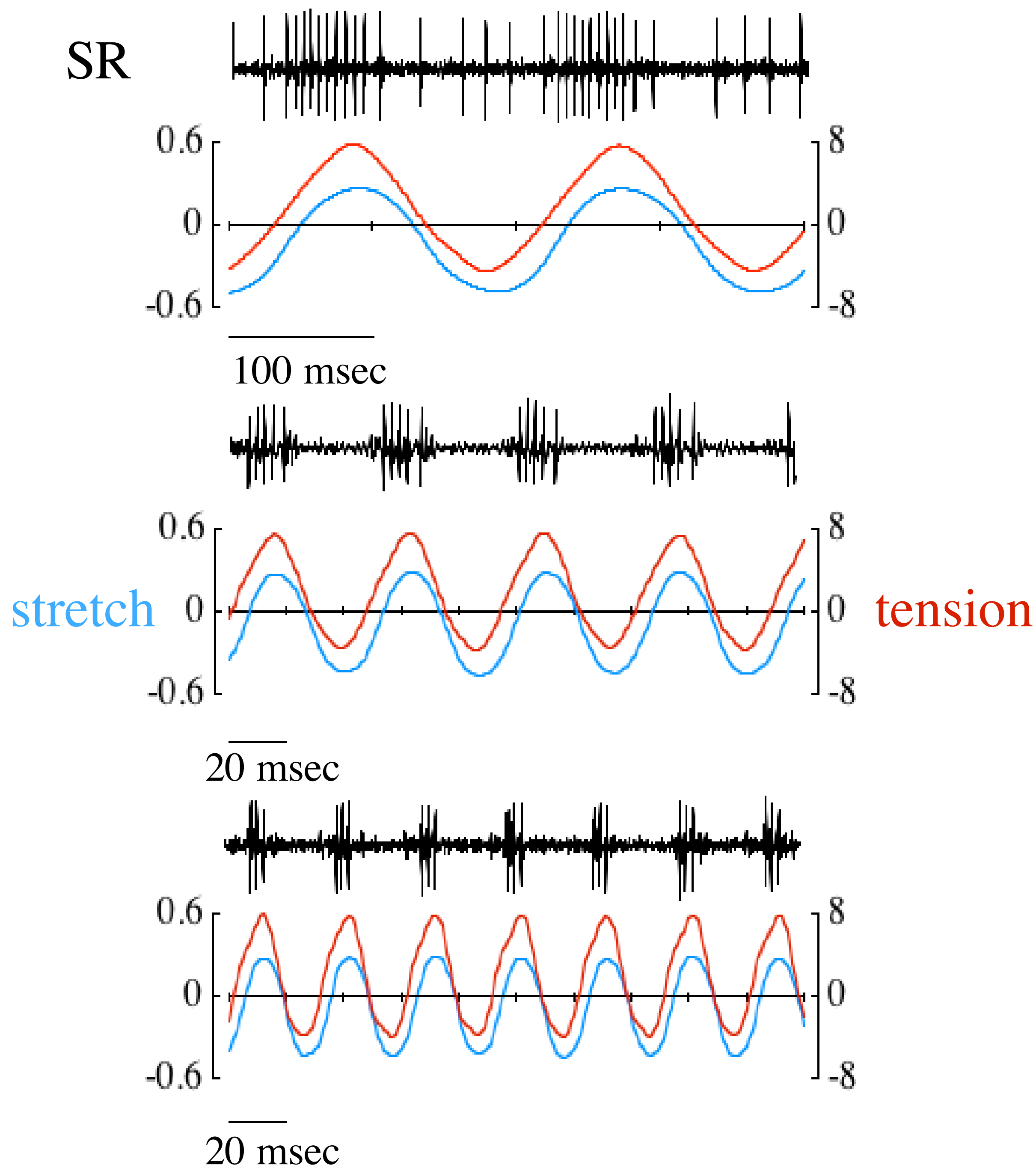

mechanical models
response to stretch step
mathematical model
k_o
k_1
η
response (R)
stretch (x)
time
$R_{total} = f(t, k_o, k_1, \eta, x)$
$R(t) = k_o\, x + [k_1, e^{-t\, k1/\eta}]\, x$
let $x(t) = a \sin \omega t$
...
solve ODE for R(t)



SR
0.8
0
-0.8
Stretch (mm)
15
0
-15
Tension (mN)
500 msec
SR frequency (Hz)
400
300
200
100
50 msec
Tension (mN)
13
9
5
30 msec

Mechanical model predicts tension, but not SR firing dynamics




SR
0.6
0
-0.6
8
0
-8
100 msec
stretch
tension
20 msec
20 msec

Biology 427 Biomechanics

Lecture 10. Shape and stress: architecture in biology.

- Recap material properties
- Cross-sectional shape: The second moment of area (I)
- The flexural stiffness of a structure (EI)
- Buckling and twisting
- Failure and safety factors: many ways to break up.

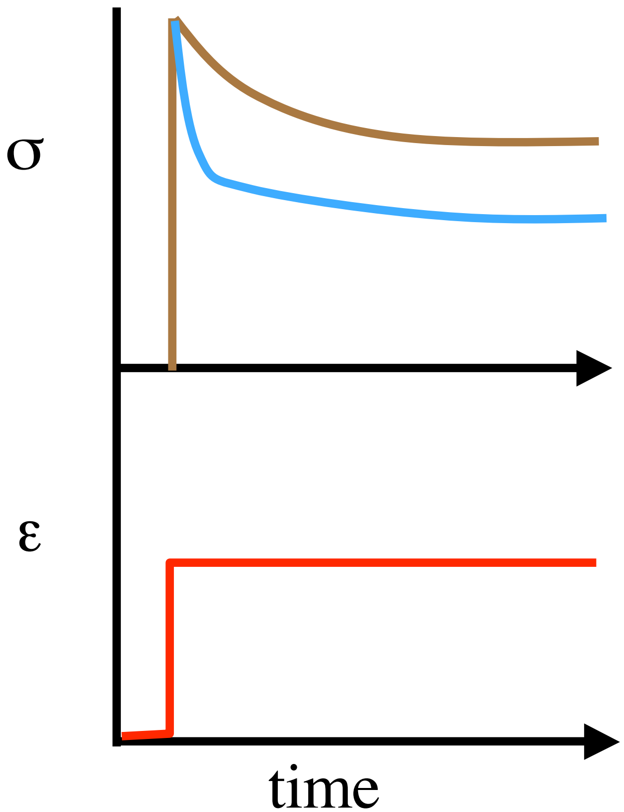
Project proposals: due Friday, February 6

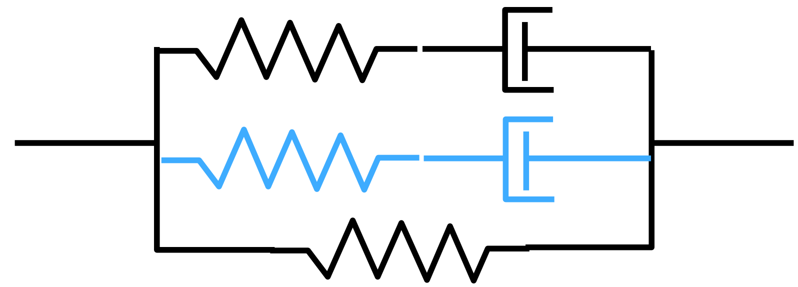
Proposals should be <u>no more than</u> *3 double-spaced pages, and should address the following:*

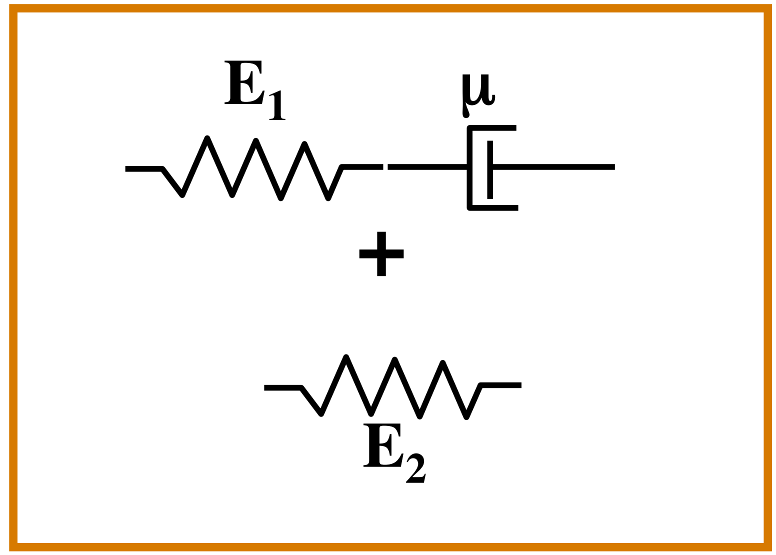
- *What is your question?*
- *Why is your question important/interesting?*
- *What is known about your question? (give background from literature*/web searches)*
- *How will you develop a quantitative analysis of your problem? (you do not need to provide any equations in the proposal, but should explain the quantitative approach/steps you will take)*

- ***Read "Advice for preparing projects" on the webpage!**

Stress relaxation in tendon, ligament, chitin ….







Standard linear solid

$\sigma_1 = E_1\,\varepsilon\,(1 - e^{E1t/\mu})$

$\sigma_2 = E_2\,\varepsilon$

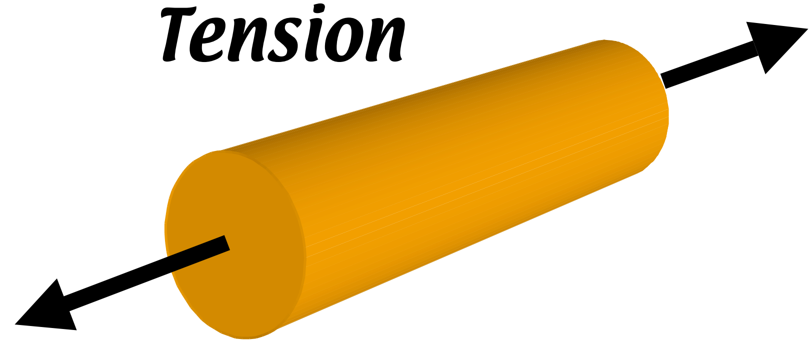
$\sigma = E_2\,\varepsilon\ + E_1\,\varepsilon\,(1 - e^{E1t/\mu})$

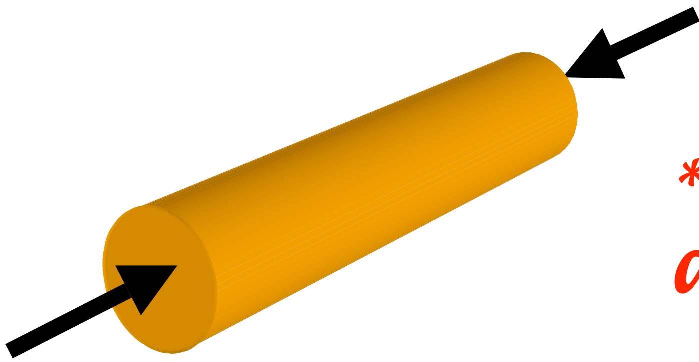
$+ E_3\,\varepsilon\,(1 - e^{E3t/\mu})$

In tension, the behavior of a structure depends only on material properties and cross-sectional area (not shape!):

$$\sigma = E\varepsilon = E\,\Delta L/L$$
$$\Delta L = \sigma L/E = FL/AE$$

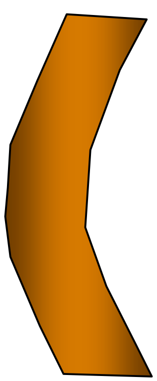
Tension

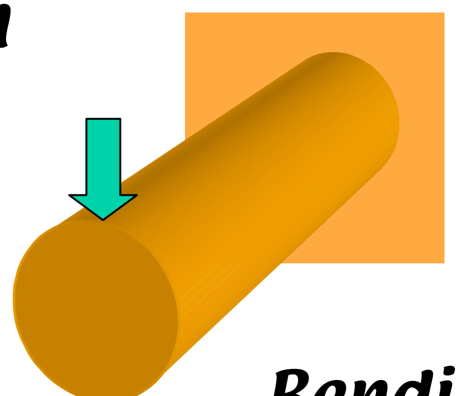




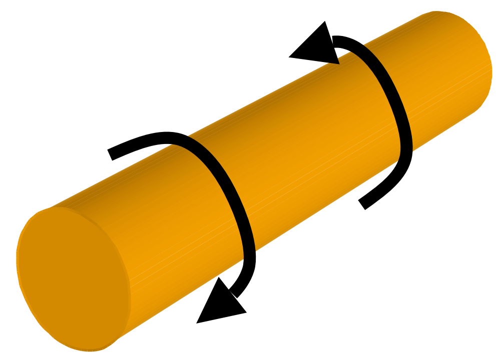
***Responses to other types of loads depend on shape**

Compression --> can lead to buckling





Bending

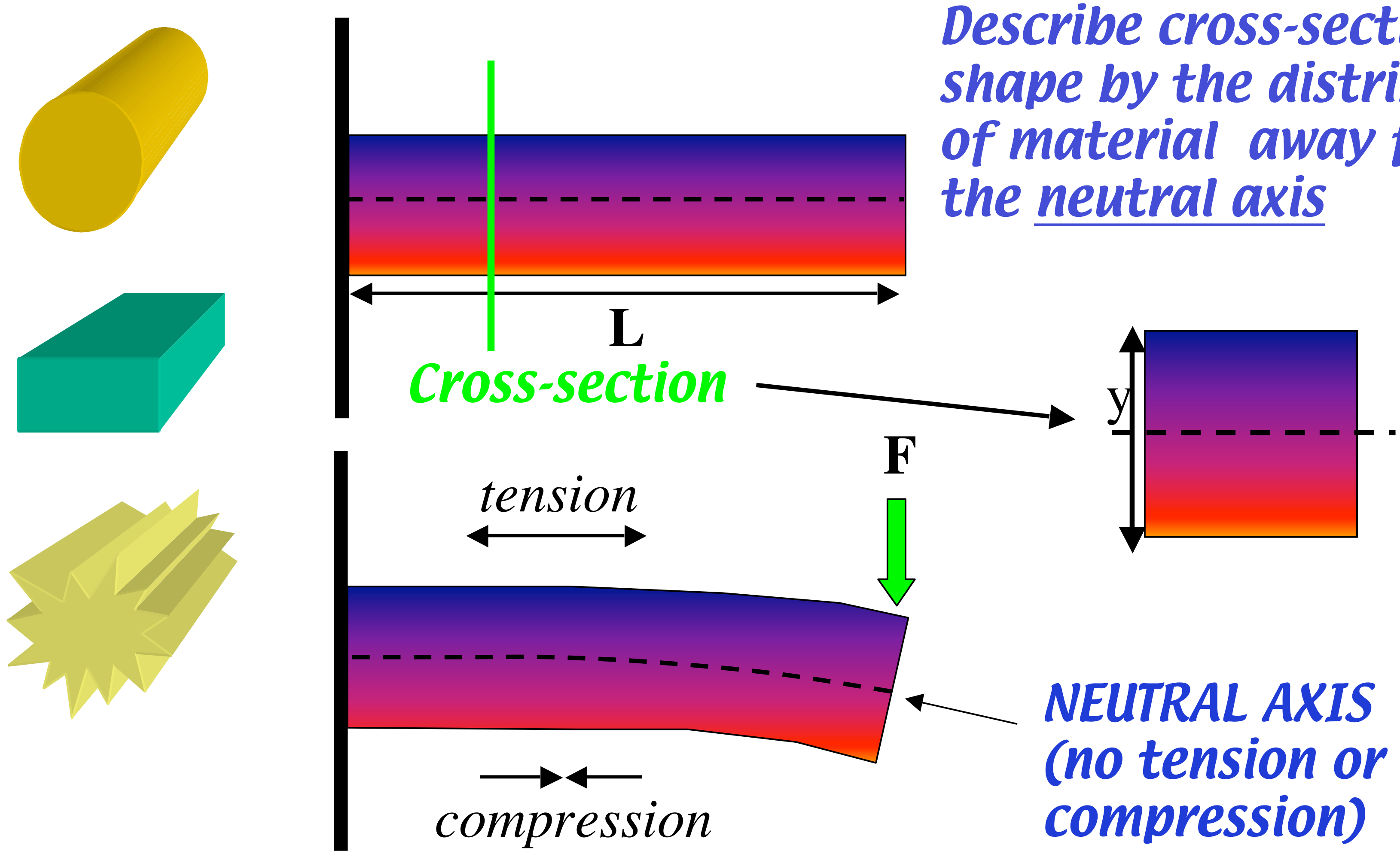


Torsion

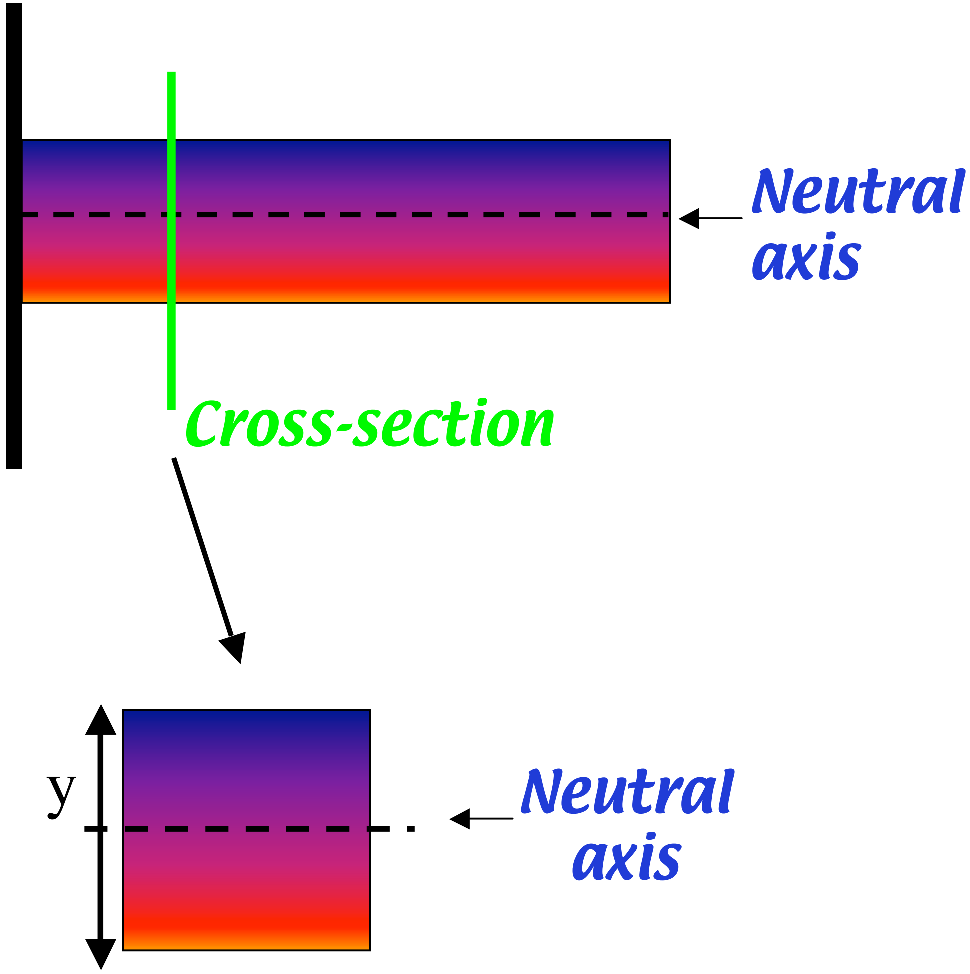
How can we quantify the shape of a structure?

Beam theory treats structures like simple beams with a cross-sectional shape and a length L

Describe cross-sectional shape by the distribution of material away from the <u>neutral axis</u>

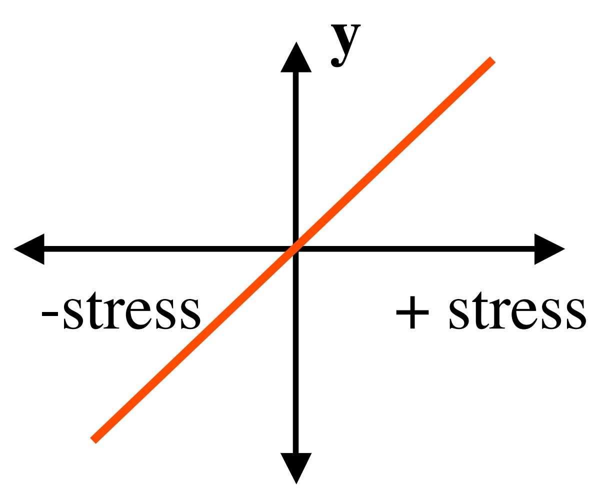


Cross-sectional shape: second moment of area

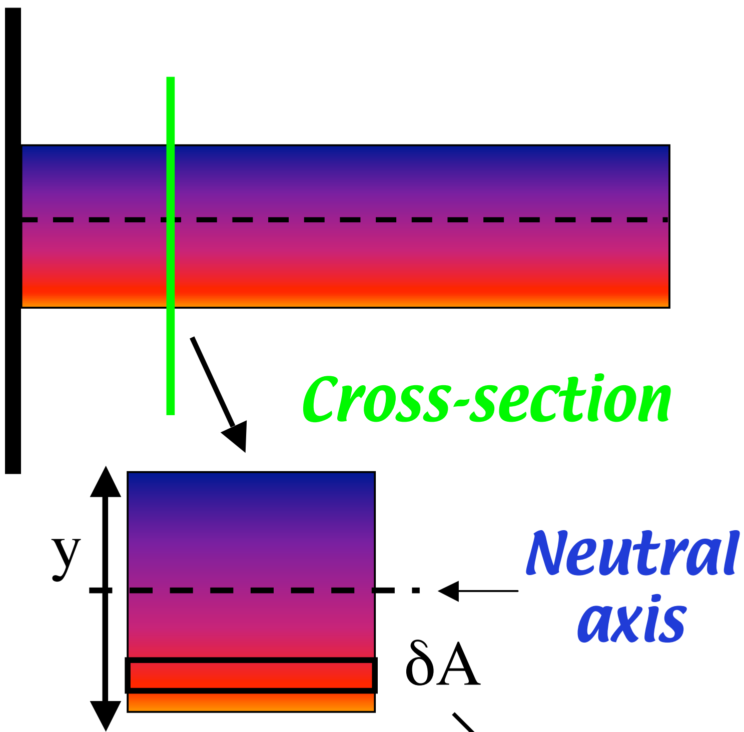


y = distance from neutral axis

Stress during bending (tensile or compressive) rises with distance from the neutral axis

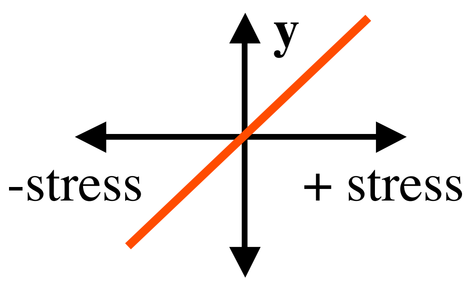


Cross-sectional shape: second moment of area



y = distance from neutral axis

small piece of area at a certain distance y from the neutral axis



$\sigma \propto \kappa y$
where κ is a constant

Find force and moment acting on a small piece: **($F = \sigma A$)**

Force on $\delta A = \sigma_y \, \delta A$

Moment about neutral axis: ($M=Fy$)

$$\delta M = y(\sigma_y \, \delta A)$$

Find the total moment about the neutral axis:

$$M = \int \sigma \, y \, dA \qquad (\sigma \propto \kappa y)$$

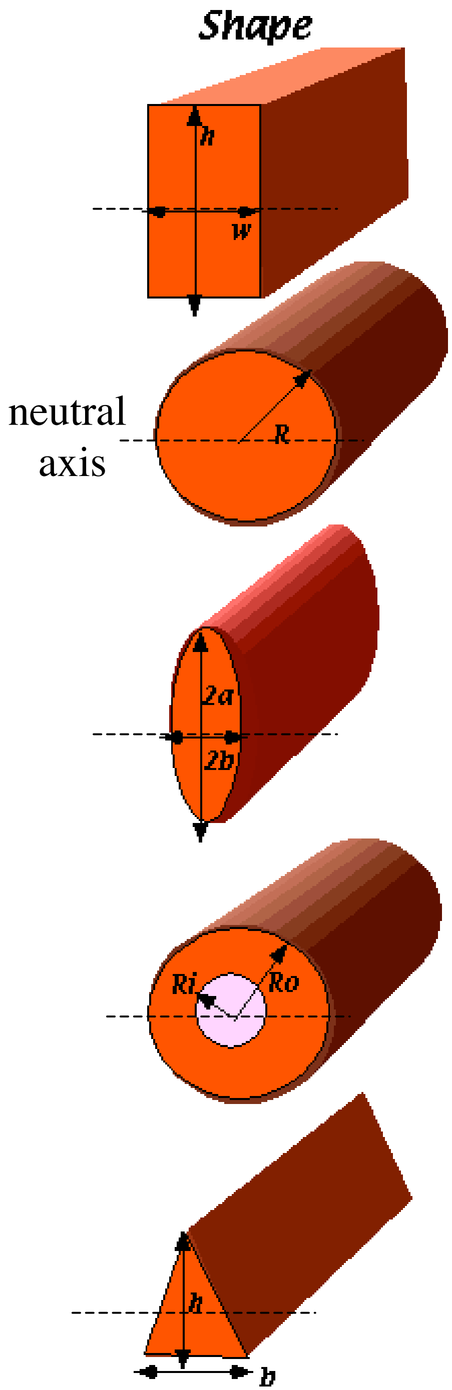
$$M = \int \kappa \, y^2 \, dA = \kappa \int y^2 \, dA$$

$$M = \sigma / y \int y^2 \, dA = I \, \sigma / y$$

I *is the second moment of area*

$$I = \int y^2 \, dA$$

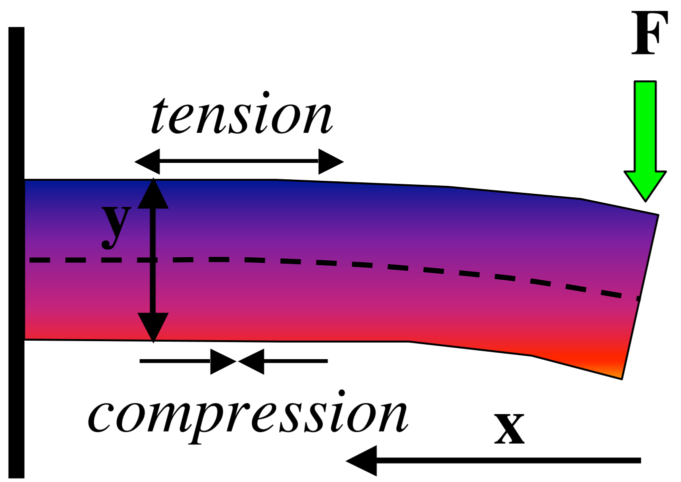
Second moment of area for some beam shapes



Area	I
$w\,h$	$w\,h^3/12$
πR^2	$\pi R^4/4$
$\pi\, a\, b$	$\pi\, a3\, b/4$
$\pi\,(R_o^2 - R_i^2)$	$\pi\,(R_o^4 - R_i^4)/4$
$b\,h/2$	$bh^3/36$

"height" of object contributes more strongly to I than width ("higher" object has more area distributed away from neutral axis)

Predicting overall bending behavior of a structure: EI -- Flexural stiffness



$$M = \sigma/y \int y^2\, dA = I\ \sigma/y$$

$$M = F\, x$$

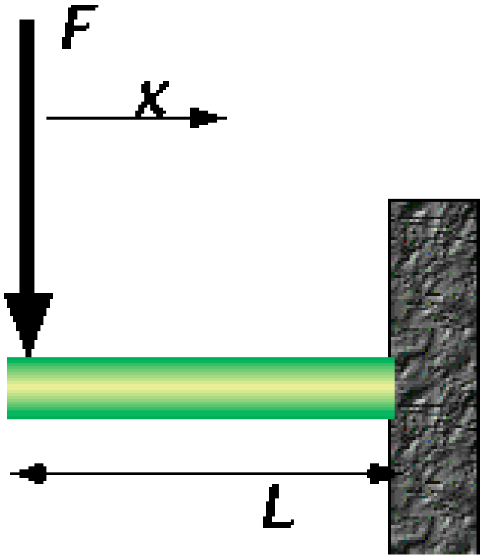
$$F\, x = I\, \sigma/y$$

$$\sigma = F\, x\, y/\, I$$

$\sigma = E\, \varepsilon$ $\qquad$ $\varepsilon = F\, x\, y/(E\, I)$

EI describes the overall behavior of the beam due to BOTH material and structural properties

Predicting overall bending behavior of a structure: EI -- Flexural stiffness



Point load F at end of beam

Moment $M = Fx$

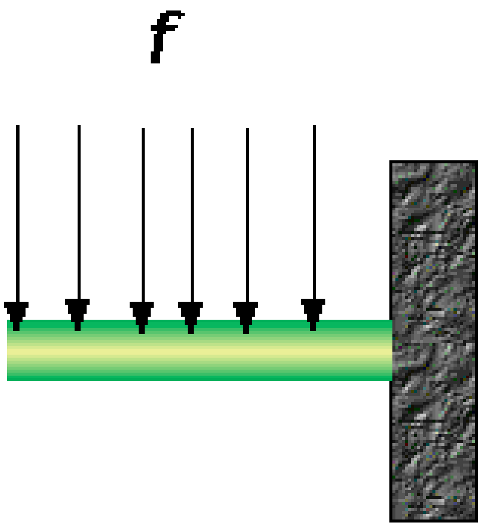
Maximum moment = FL

Deflection of beam at point x

$y = F(x^3-3L^2x+2L^3)/(6EI)$

Maximum deflection (at tip):

$\boxed{y_{max} = FL^3/(3EI)}$



Uniformly distributed load $f = FL$

Moment $M = Fx^2/3L$

Maximum moment = $FL/2$

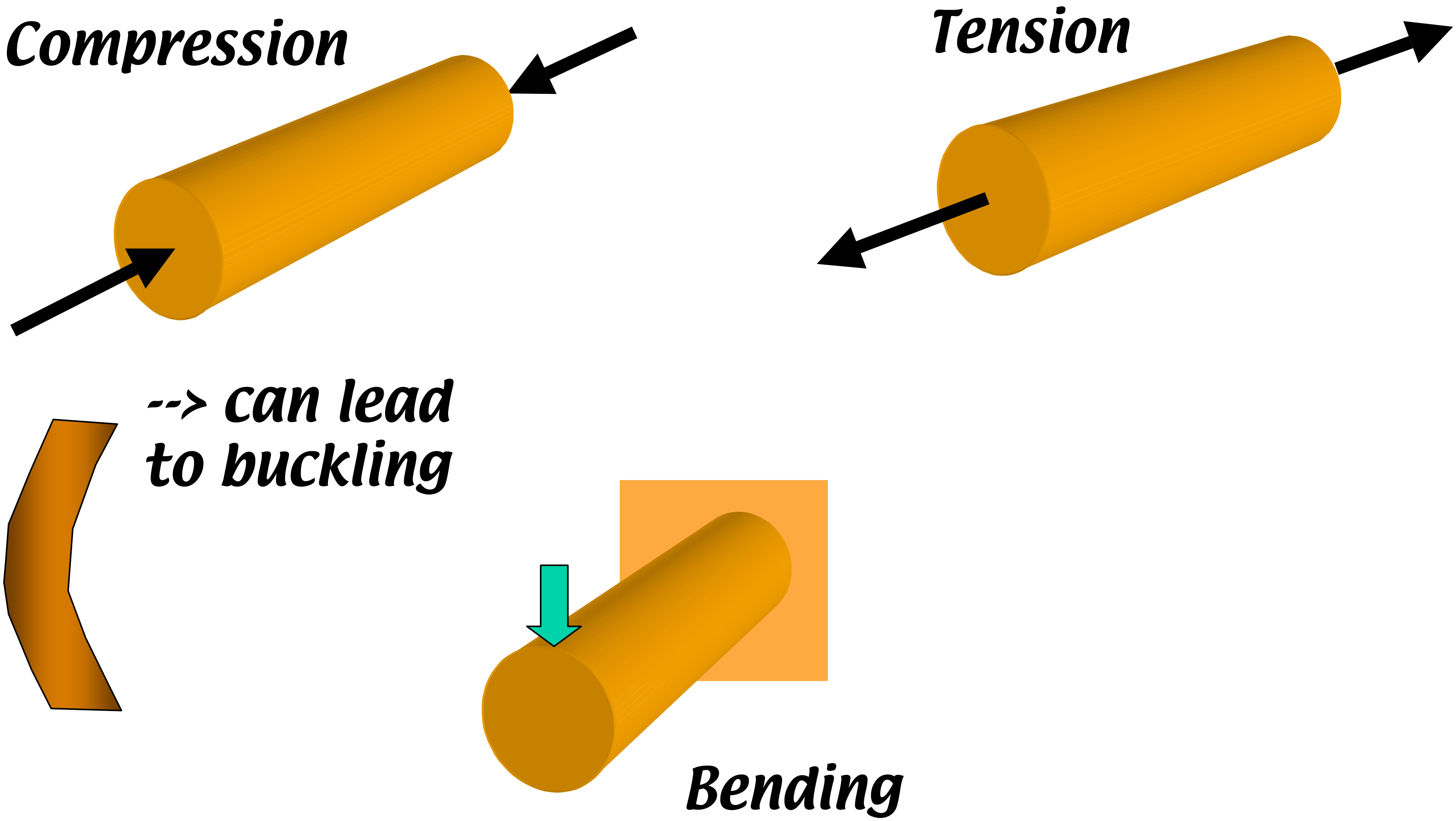
Deflection of beam at point x

$y = F(x^4-4L^3x+3L^4)/(24EIL)$

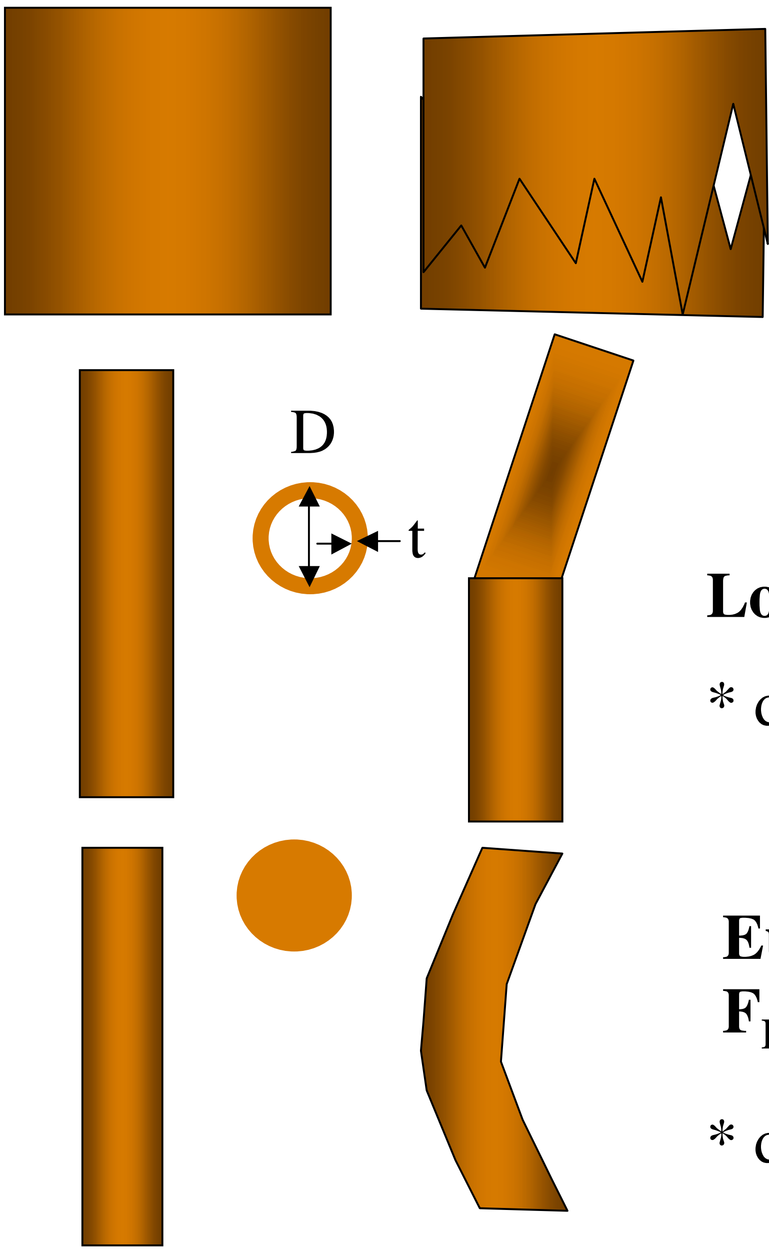
Maximum deflection (at tip):

$\boxed{y_{max} = FL^3/(8EI)}$

How do beams repond to compression?



Size and shape matter in compression too!



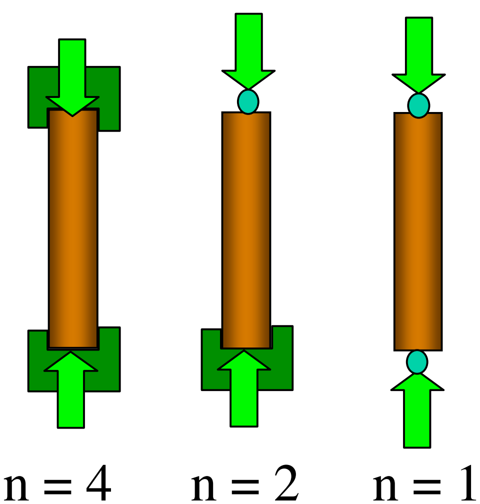
Compressive failure (σ_{max})

Local buckling $\sigma_L = k\,E\;t/D \quad (k \sim 0.7)$

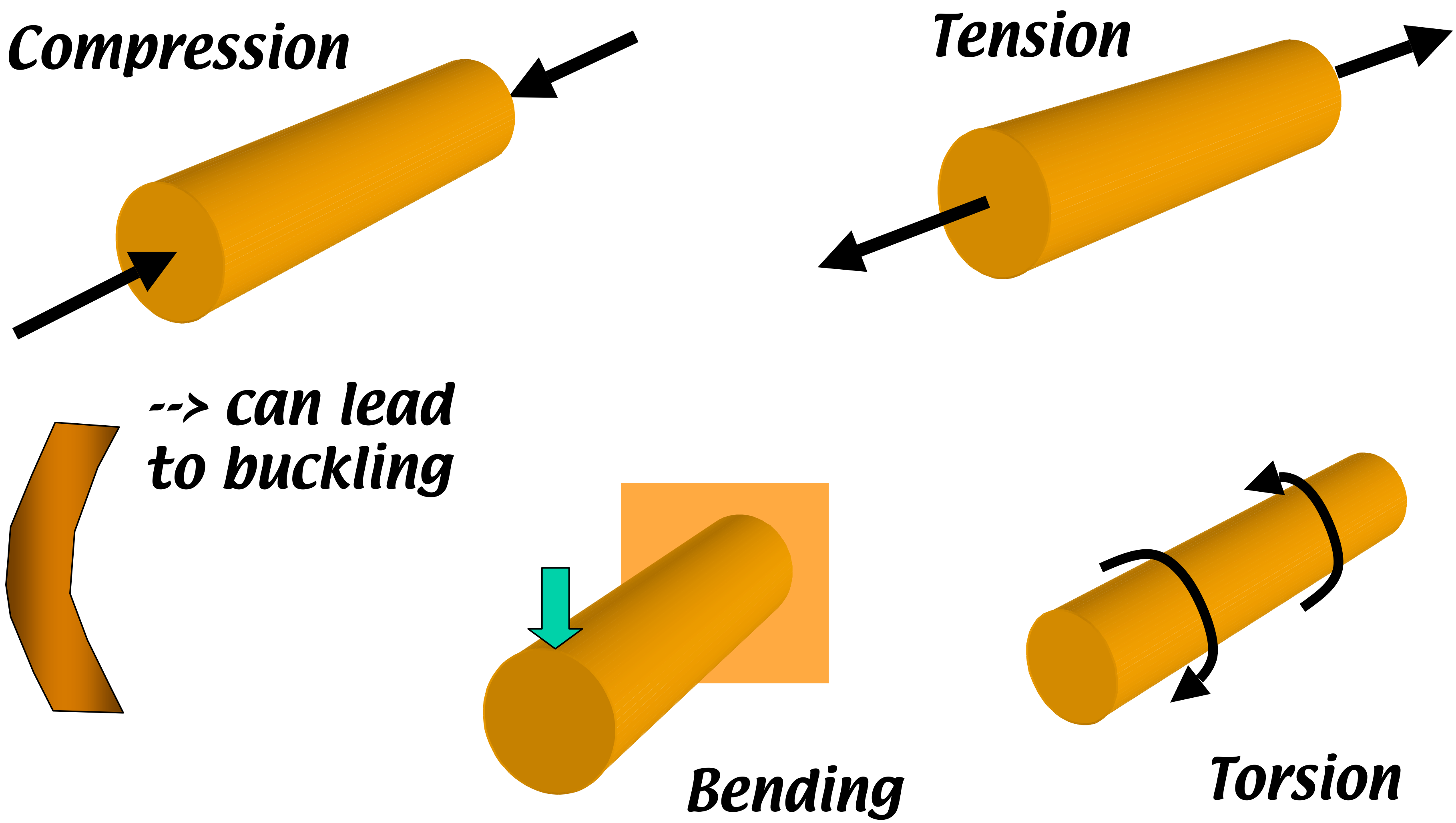
* depends on E, wall thickness and diameter

Euler buckling
$F_E = n\,\pi^2\,E\,I/L^2$

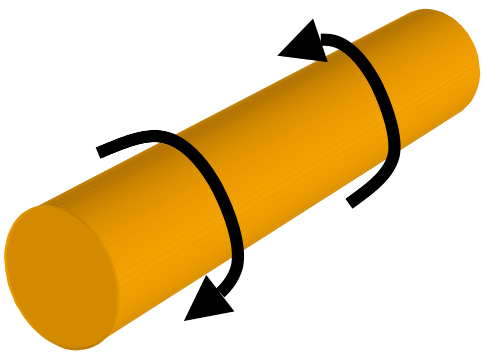
* depends on EI
and length



How do beams repond to torsion?



Torsion involves tensile forces (outside) and compressive forces (inside), as well as shear forces



Torsion

shear top and bottom in opposite directions

$\tau = G\gamma$ (like $\sigma = E\varepsilon$)

shear stress — shear modulus — shear strain

Torsional stiffness (like flexural stiffness) depends on both material properties (G) and structural properties (J):

$$\boxed{\theta = FrL/GJ}$$

J = second polar moment of area

$J = \pi/2\ (r_o^4 - r_i^4)$

where θ is angle of twist, F is tangential applied force, and r is radius

A huge collection of material and structural properties stiffness (E), strength (σ_{max}), flexural stiffness (EI), torsional stiffness (GJ), critical buckling force (F_E)...

To what extent does the design of biological materials and structures help them withstand various forces?

Safety factor *: SF = $\sigma_{max}/\sigma_{expected}$ *(for tensile/ compressive failure)*

	Bones		Tendon
	Tensile	Compressive	
σ_{max}, Strength (Mpa)	172	284	84
σ_{ex} Dog jumping	68-80(2-3)	100(2.8)	84(1.0)
σ_{ex} Kangaroo hopping	60(3)	90(3.2)	40-80(1-2)
σ_{ex} Elephant running	45-69(2.5-4)	57-85(3.3-5)	
σ_{ex} Man weight lifting		(1 - 1.7)	
σ_{ex} Goose flying		50(6)	

safety factor

*Alexander, R.McN. 1981. Sci. Prog. Oxf. 67:109-120

What about safety factors for other types of loading?

Structure	Load/Failure Mode	Factor
Cuttlefish, buoyancy chambers	pressure	1.3 - 1.4
Squid, shell (pen)	bending	1.3 - 1.4
Spider, dragline	tension	1.5
Reptilian hindlimbs	bending	5.5 - 10.8
Reptilian hindlimbs	torsion	3.9 - 5.4
Mammalian bones (general)	bending	2 - 6
Horse, leg bones in galloping	bending	4.8
Ostrich, leg bones in running	bending	2.5
Bird wing bones	bending	2.2
Bat (microchiropteran) wing bones	bending	1.4
Bat (megachiropteran) wing bones	bending	3.9
Pigeon (wing) humerus	bending	3.5
Pigeon (wing) humerus	torsion	1.9
Pigeon wing feather shaft	bending	6 - 12
Tree trunks	critical buckling	about 4
Stems, annual plants	critical buckling	about 2
Garlic, grown in windy field	critical buckling	1.78
Garlic, grown in greenhouse	critical buckling	1.03

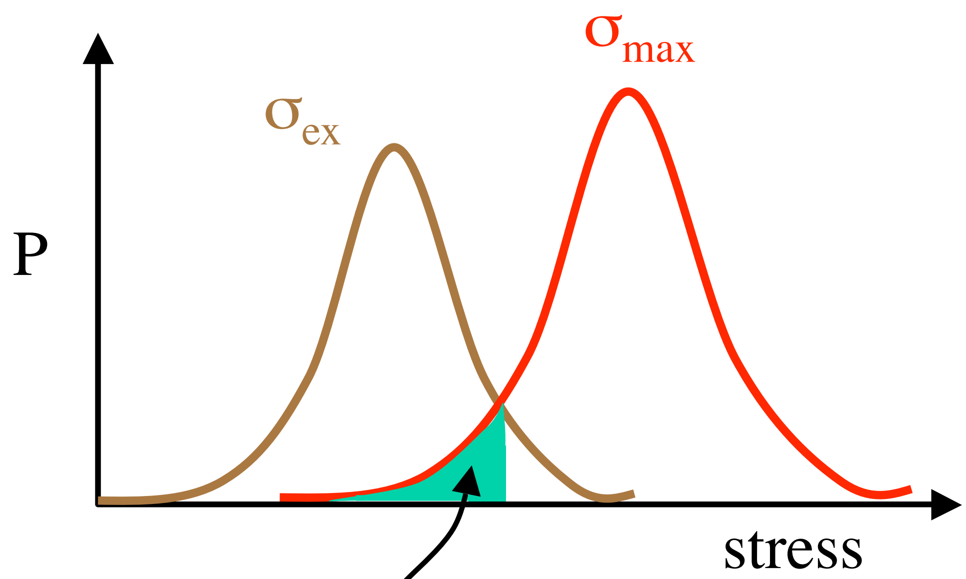
lower SF in predictable environments?

SF differs for different types of loading - related to types of loads normally encountered?

Higher SF in longer-lived organisms?

Higher SF when exposed to a high stress environment?

But … there is variation in real biological materials and loads may be unpredictable (they may vary from the expected values).



$P(\sigma_{ex} > \sigma_{max})$ --> breakage!

<u>probability of breakage</u>:
0.0004/year human humerus
0.0006/year human femur
0.02/life either humerus/femur
0.07/life viverids humerus/femur
0.4/life gibbons humerus/femur
0.5/life red deer antlers
0.5/life spider webs

• distribution of stresses experienced

• distribution of strengths (in a structure or population)

Alexander suggested minimizing

$$\Phi(n) = P(n)*F + G(n) + U(n)$$

P(n)*F = probability of failure*cost of failure
G(n) = cost of producing structure
U(n) = cost of using structure

Biology 427 Biomechanics

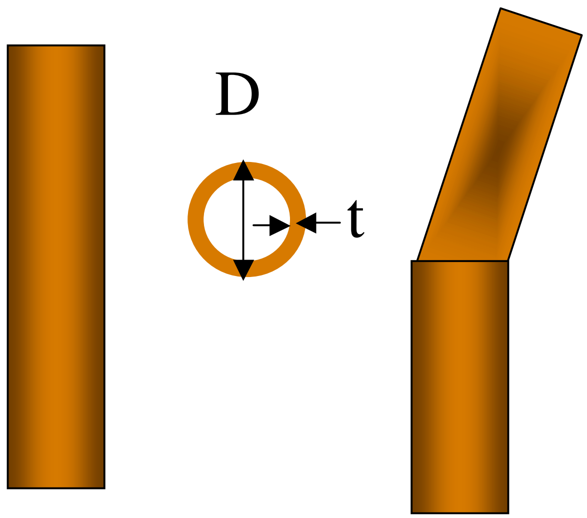
Lecture 11. More on shape and stress: architecture in biology and the design of biological structures

- *Recap flexural stiffness, I, and beam examples*
- *Stress distributions in hip bones and tree limbs*
- *The design of mammalian long bones*
- *Design for selective failure: ripping fingernails*

Paper Proposal due Next Friday

Talk to Tom or Stacey about your project!

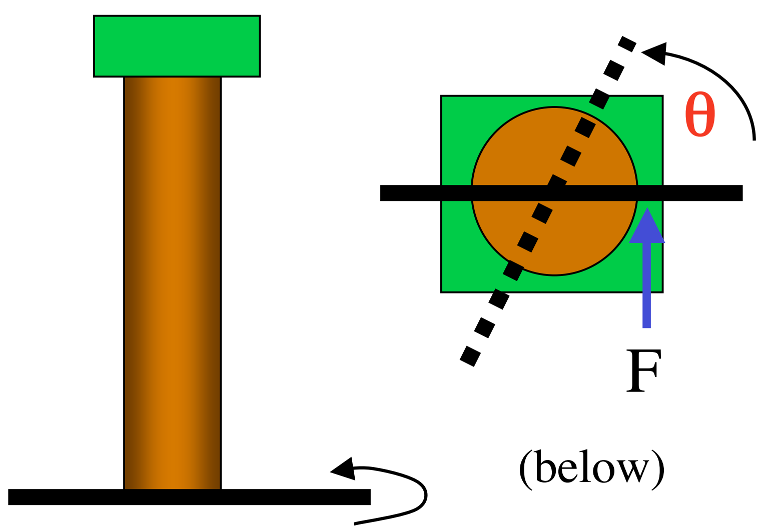
Buckling diameter, torsion measurements……



Local buckling $\sigma_L = k\,E\ \ t/D \quad (k \sim 0.7)$

* depends on E, <u>wall thickness</u> and <u>diameter</u>

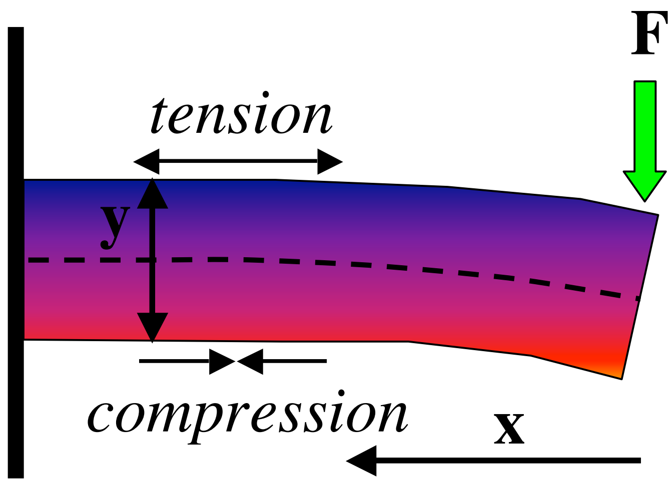
D = outer diameter



Torsion $\theta = F\,r\,L\,/\,G\,J$

where θ is angle of twist, F is tangential applied force, and r is radius

Predicting overall bending behavior of a structure: EI -- Flexural stiffness



Are any of these equations time-dependent?

$M = \sigma/y \int y^2\, dA = I\ \sigma/y$

$M = F\, x$

$F\, x = I\, \sigma/y$

$\sigma = F\, x\, y/ I$

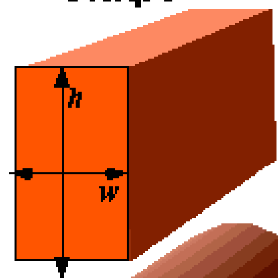
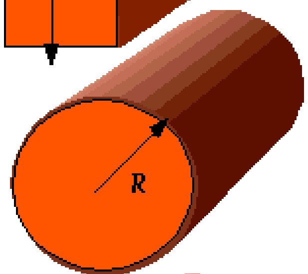
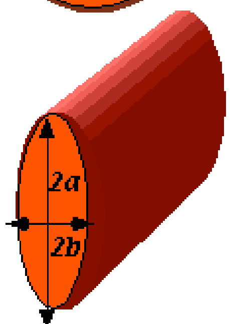
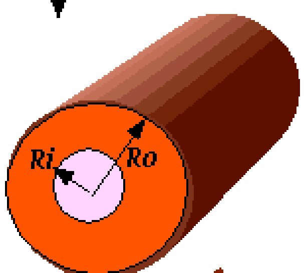
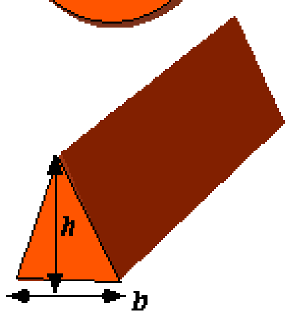
$\sigma = E\, \varepsilon$ $\quad$ $\varepsilon = F\, x\, y/(E\, I)$

Yes!

- *E can vary with time (viscoelastic) or force (non-linear)*
- *I can also change with time if structure deforms (squishes)*

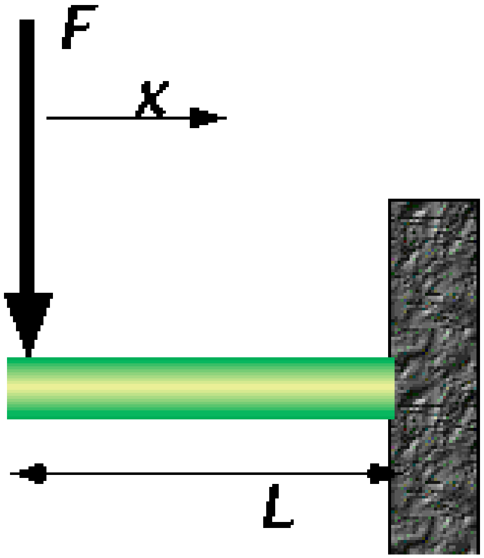
***EI** describes the overall behavior of the beam due to BOTH material and structural properties*

Second moment of area for some beam shapes

Shape	Area	I
	$w\,h$	$w\,h^3/12$
	$\pi\,R^2$	$\pi\,R^4/4$
	$\pi\,a\,b$	$\pi\,a3\,b/4$
	$\pi\,(R_o^2 - R_i^2)$	$\pi\,(R_o^4 - R_i^4)/4$
	$b\,h/2$	$bh^3/36$

see class web page

Predicting overall bending behavior of a structure: EI -- Flexural stiffness

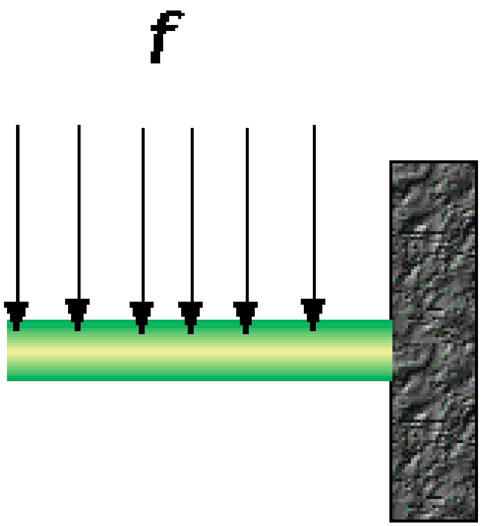


Point load F at end of beam
Moment $M = Fx$
Maximum moment = FL
Deflection of beam at point x

$$y = F(x^3 - 3L^2x + 2L^3)/(6EI)$$

Maximum deflection (at tip):

$$\boxed{y_{max} = FL^3/(3EI)}$$



Uniformly distributed load $f = FL$
Moment $M = Fx^2/3L$
Maximum moment = $FL/2$
Deflection of beam at point x

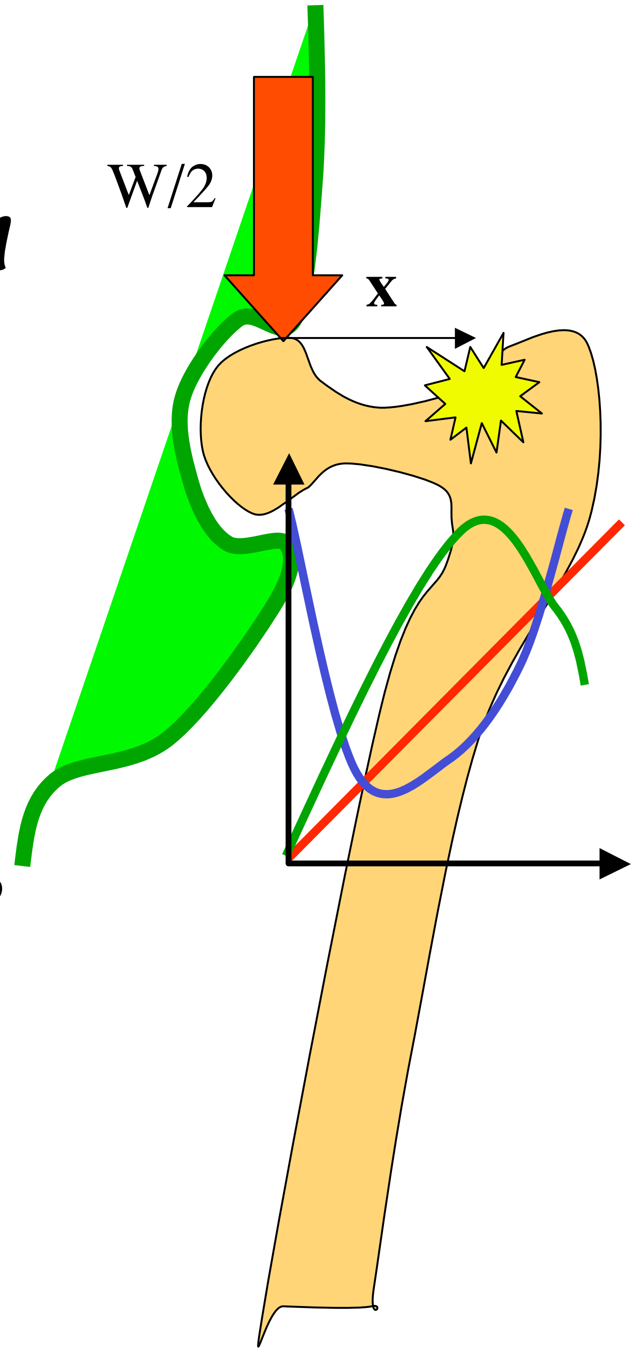
$$y = F(x^4 - 4L^3x + 3L^4)/(24EIL)$$

Maximum deflection (at tip):

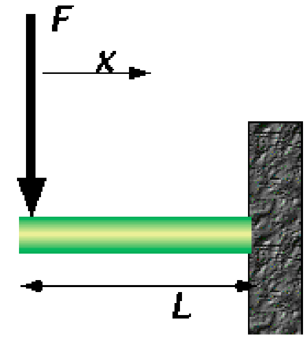
$$\boxed{y_{max} = FL^3/(8EI)}$$

Stress distributions in biological beams: hip

Bending will cause tension and compression, but most bones are stronger in compression



$$\sigma = M\, y/I$$



Moment $M = Fx$

Where do you think the tensile stress is greatest?

$$\boxed{\sigma = F\, x\, y/I}$$

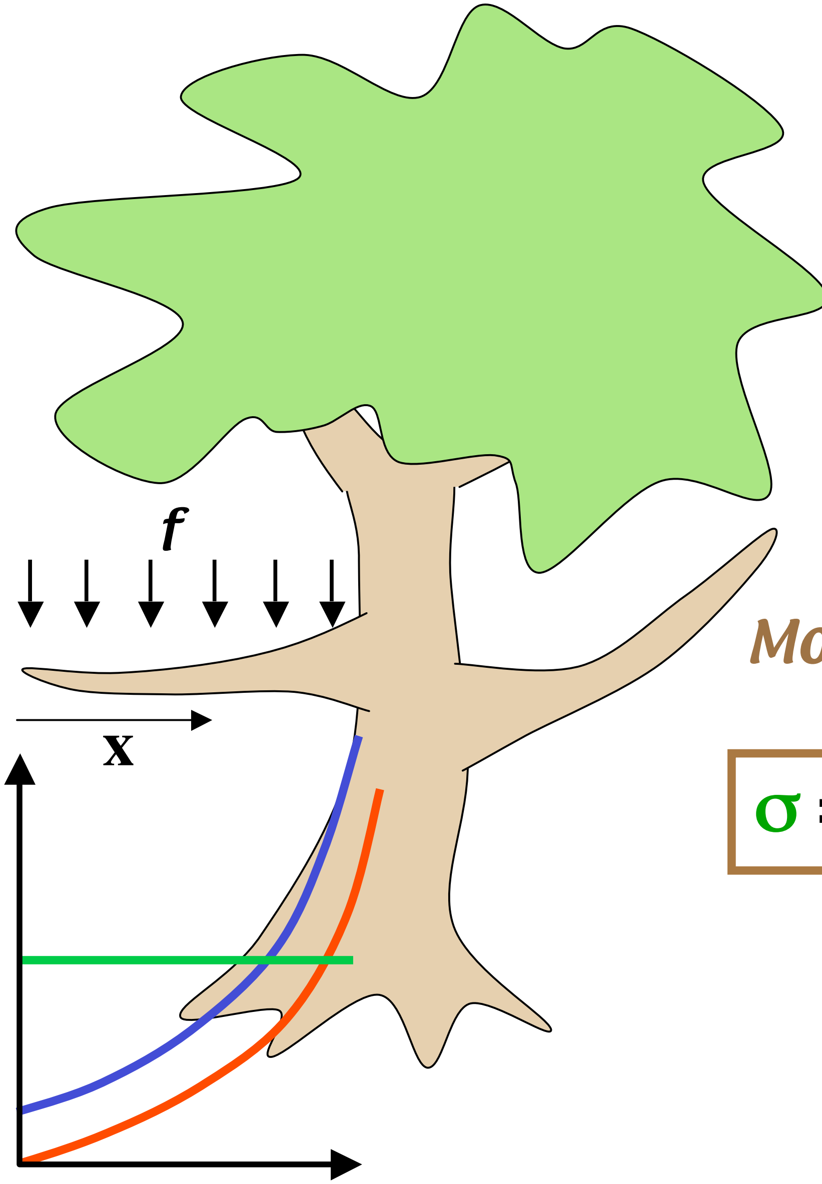
Where is the most likely zone for failure?

$$I = \pi R^4/4$$

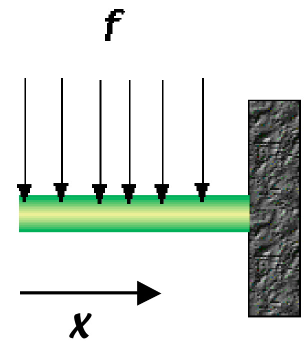
Stress distributions in biological beams: tree branches

Tree branches support their own weight, plus the weight of leaves, fruit, etc.

How does the design of branches affect the distribution of bending stresses?



$$\sigma = M\, y/I$$



Moment $M = Fx^2/3L$

$$\boxed{\sigma = F\, x^2\, y/\, 3\, L\, I}$$

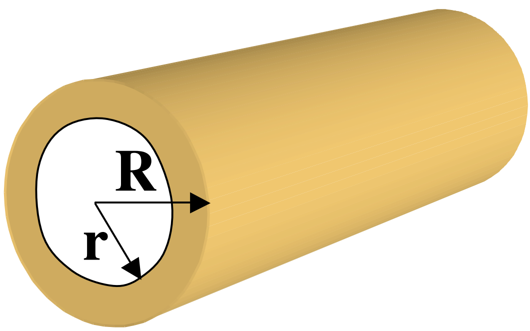
$$I = \pi R^4/4$$

Mammalian long bone design:

How do structural features affect stress on bones?

cost of structure — probability of failure* cost of failure — cost to produce — cost to use

minimize: $\boxed{\Phi(n) = \underbrace{P(n)*F}_{\text{thick, big bones } (k \to 0)} + G(n) + U(n)}$



$$\boxed{\sigma = F \times y/I}$$

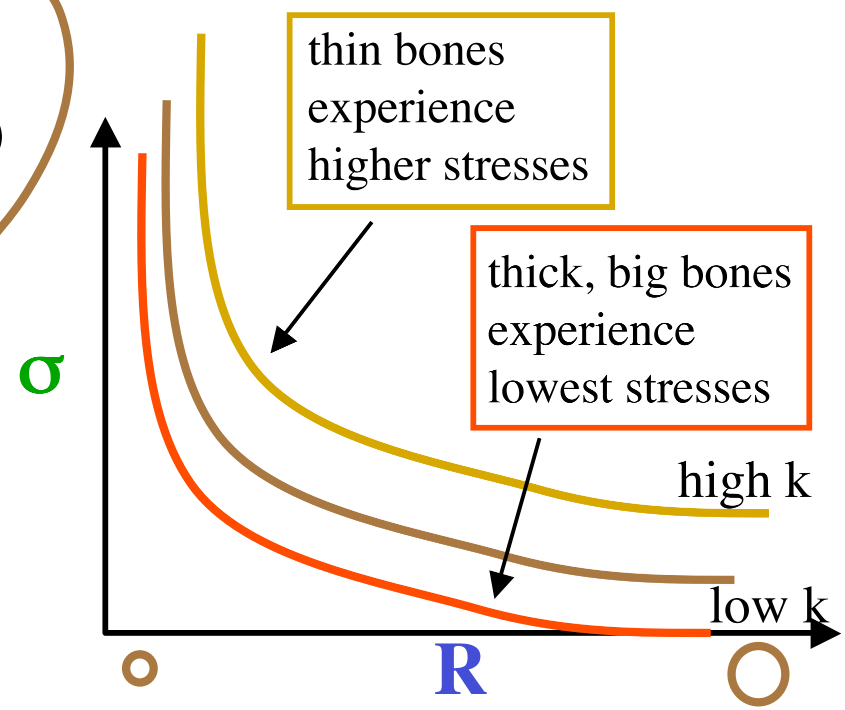
$$I = \pi R^4/4 - \pi r^4/4$$

k = r/R (thinness ratio; *thin*: k -> 1)

$$r = kR$$

$$I = \pi R^4 (1 - k^4)/4$$

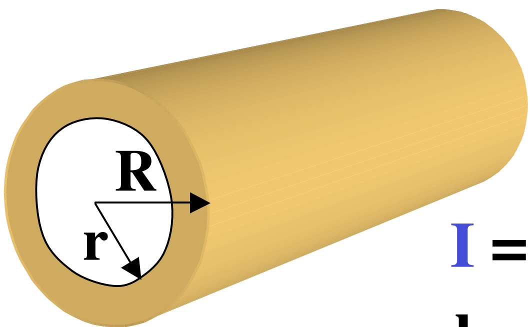
$$\boxed{\sigma = F \times y/ (\pi R^4 (1 - k^4)/4)}$$



But this assumes that we can use as much material as we want……

cost of structure | probability of failure* cost of failure | cost to produce | cost to use

minimize: $\boxed{\Phi(n) = \underbrace{P(n)*F + G(n)}_{\text{thick, big bones } (k \to 0)} + U(n)}$



$\boxed{\sigma = F \times y/I}$

$I = \pi R^4 (1 - k^4)/4$

$k = r/R$

$V = \pi R^2 L - \pi (kR)^2 L = \pi R^2 L (1 - k^2)$

$R^2 = V/(\pi L (1 - k^2))$

$R^4 = V^2/(\pi L (1 - k^2))^2$

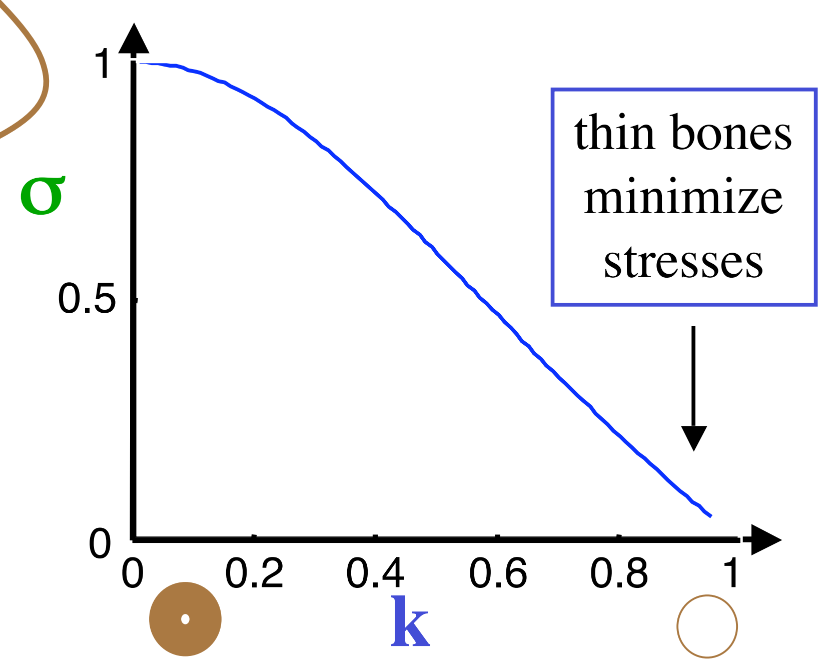
$\sigma \sim 1/I \sim 1/ R^4 (1 - k^4)$

big radius minimizes stresses

$\sigma \sim 1/ (V^2/(\pi L (1 - k^2))^2) (1 - k^4)$

$\boxed{\sigma \sim (1 - k^2)^2/ (1 - k^4)}$

For a given volume of bone, how best might we distribute it?



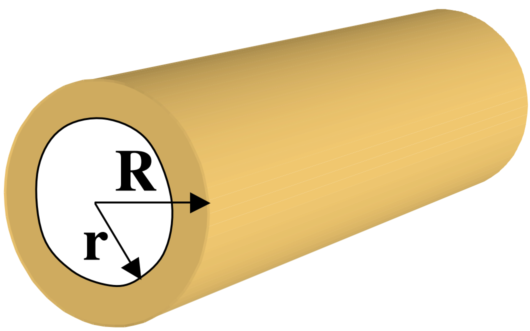
For a given volume of bone, how best might we distribute it?

cost of structure | probability of failure* cost of failure | cost to produce | cost to use

minimize: $\boxed{\Phi(n) = P(n)^{*}F + G(n) + U(n)}$

thick, big bones (k -> 0)

thin, wide bones (k -> 1)



$\boxed{\sigma = F \times y/I}$

$k = r/R$

Really wide thin bones (high k) would minimize stresses for a given volume of bone

What are values of k in mammalian long bones?

Values for k in terrestrial mammals

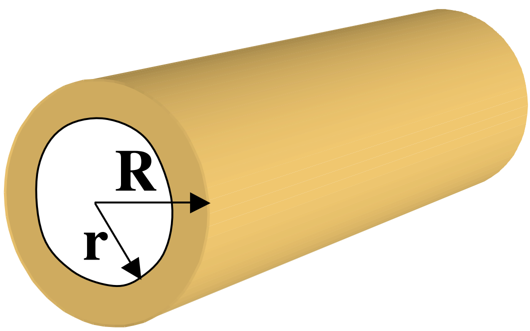
Bone	Hare	Fox	Lion	Camel	Buffalo	Swan
femur	0.57	0.63	0.56	0.62	0.54	0.60
humerus	0.55	0.59	0.57	0.66	0.51	0.92

What is the best design for minimum weight?

Compute the mass per unit length of bone

cost of structure | probability of failure* cost of failure | cost to produce | cost to use

minimize: $$\boxed{\Phi(n) = \underbrace{\underbrace{P(n)^*F}_{\text{thick, big bones } (k \to 0)} + G(n) + U(n)}_{\text{thin, wide bones } (k \to 1)}}$$



$$\boxed{\sigma = F \times y/I}$$

$$k = r/R$$

We will set the limiting condition of the size of bone that just avoids breaking

$$\sigma_{max} = F \times y/I$$
$$= My/I = M\,R/I$$
$$I = \pi R^4(1-k^4)/4$$

$$\boxed{\sigma_{max} = M\,R/(\pi R^4(1-k^4)/4)}$$

$$\boxed{R = \{4\,M/(\pi\,\sigma_{max}(1-k^4)\}^{1/3}}$$

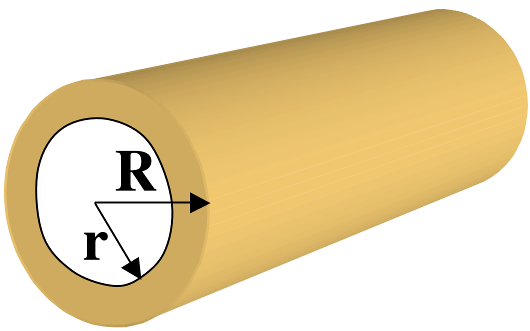
This is the radius needed for a bone to just avoid breaking under a given bending load

What is the best design for minimum weight?

Compute the mass per unit length of bone

cost of structure | probability of failure* cost of failure | cost to produce | cost to use

minimize: $$\boxed{\Phi(n) = \underbrace{\underbrace{P(n)*F}_{\text{thick, big bones (k -> 0)}} + G(n)}_{\text{thin, wide bones (k -> 1)}} + U(n)}$$



$$\boxed{\sigma = F \times y/I}$$

$k = r/R$

$$R = \{4\,M/(\pi\,\sigma_{max}(1-k^4)\}^{1/3}$$

mass = area*length*density ($m = A\,L\,\rho$)

$$m_{bone}/L = A\,\rho_{bone} = \rho_{bone}\,\pi\,R^2(1-k^2)$$

$$= \rho_{bone}\pi\,\{4M/(\pi\,\sigma_B(1-k^4)\}^{2/3}(1-k^2)$$

This is the mass per unit length of bone that will just avoid breaking

What is the best design for minimum weight?

cost of structure

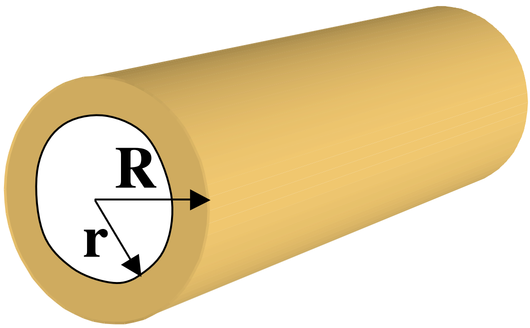
probability of failure* cost of failure

cost to produce

cost to use

minimize: $\boxed{\Phi(n) = \underbrace{\underbrace{P(n)*F + G(n)}_{\text{thick, big bones } (k \to 0)}\ + U(n)}_{\text{thin, wide bones } (k \to 1)}}$

Compute the mass per unit length of bone



$\boxed{\sigma = F \times y/I}$

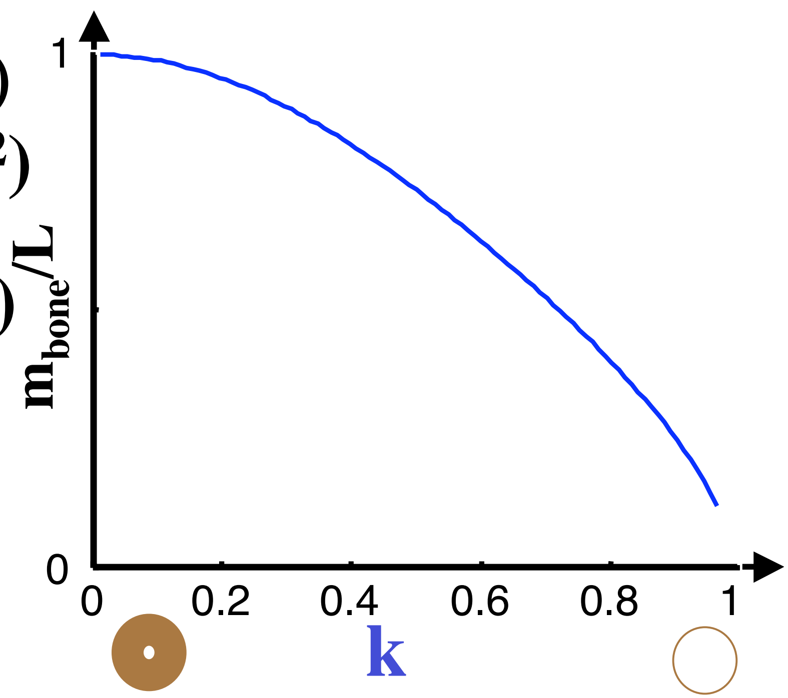
$k = r/R$

$R = \{4\,M/(\pi\,\sigma_{max}(1-k^4)\}^{1/3}$

mass = area*length*density $(m = A\,L\,\rho)$

$m_{bone}/L = A\,\rho_{bone} = \rho_{bone}\,\pi\,R^2(1-k^2)$

$= \rho_{bone}\pi\,\{4M/(\pi\,\sigma_B(1-k^4)\}^{2/3}(1-k^2)$



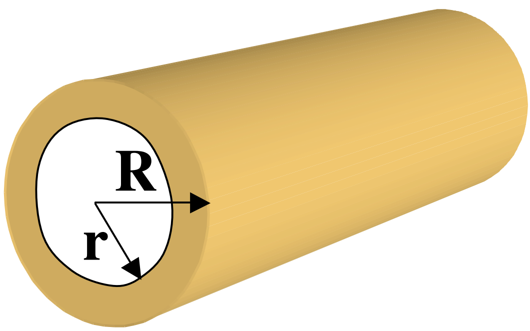
This is the mass per unit length of bone that will just avoid breaking

But, bone is not an empty tube.....the marrow inside has weight too

* $\rho_{marrow} = \rho_{bone}/2$

cost of structure | probability of failure* cost of failure | cost to produce | cost to use

minimize: $\boxed{\Phi(n) = \underbrace{\underbrace{P(n)*F}_{\text{thick, big bones } (k \to 0)} + G(n) + U(n)}_{\text{thin, wide bones } (k \to 1)}}$



$\boxed{\sigma = F \times y/I}$

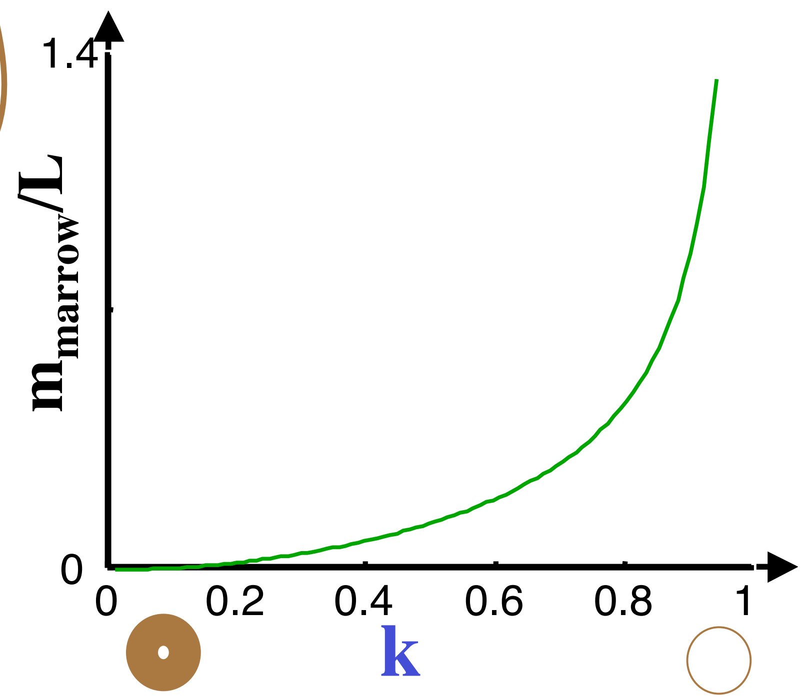
$k = r/R$

$R = \{4\ M/(\pi\ \sigma_{max}(1-k^4)\}^{1/3}$

$m_{bone}/L = \rho_{bone}\ \pi\ R^2(1-k^2)$

$m_{marrow}/L = \rho_{marrow}\ \pi\ r^2 = \rho_{bone}\ \pi\ R^2 k^2/2$

$$\boxed{m_{marrow}/L = \rho_{bone}\ \pi\ \{4\ M/(\pi\ \sigma_B(1-k^4)\}^{2/3}\ k^2/2}$$



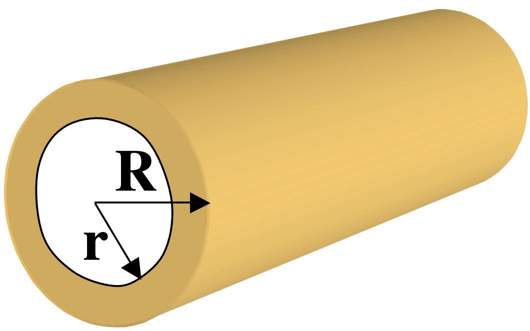
This is the mass of marrow per unit length

Compute the TOTAL mass per unit length of bone that just avoids breaking.....

cost of structure | probability of failure* cost of failure | cost to produce | cost to use

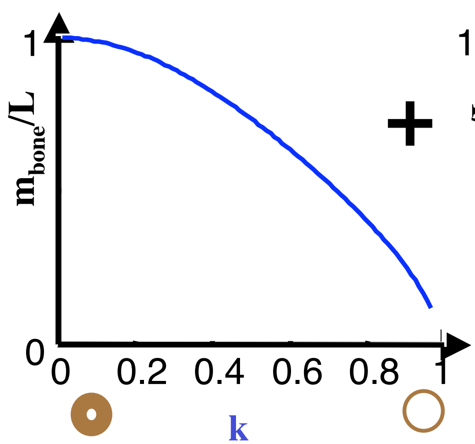
minimize: $\boxed{\Phi(n) = \underbrace{P(n)*F}_{\text{thick, big bones } (k \to 0)} + G(n) + U(n)}$

thin, wide bones (k -> 1)



$\boxed{\sigma = F \times y/I}$

$k = r/R$



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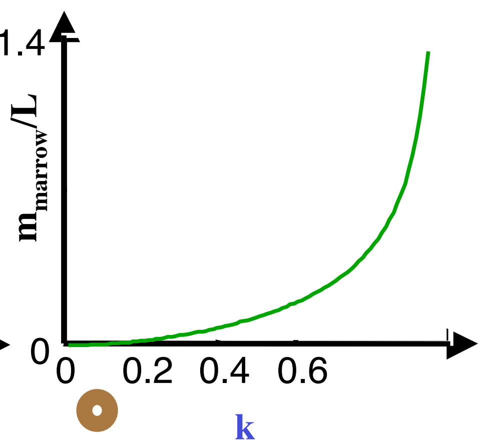
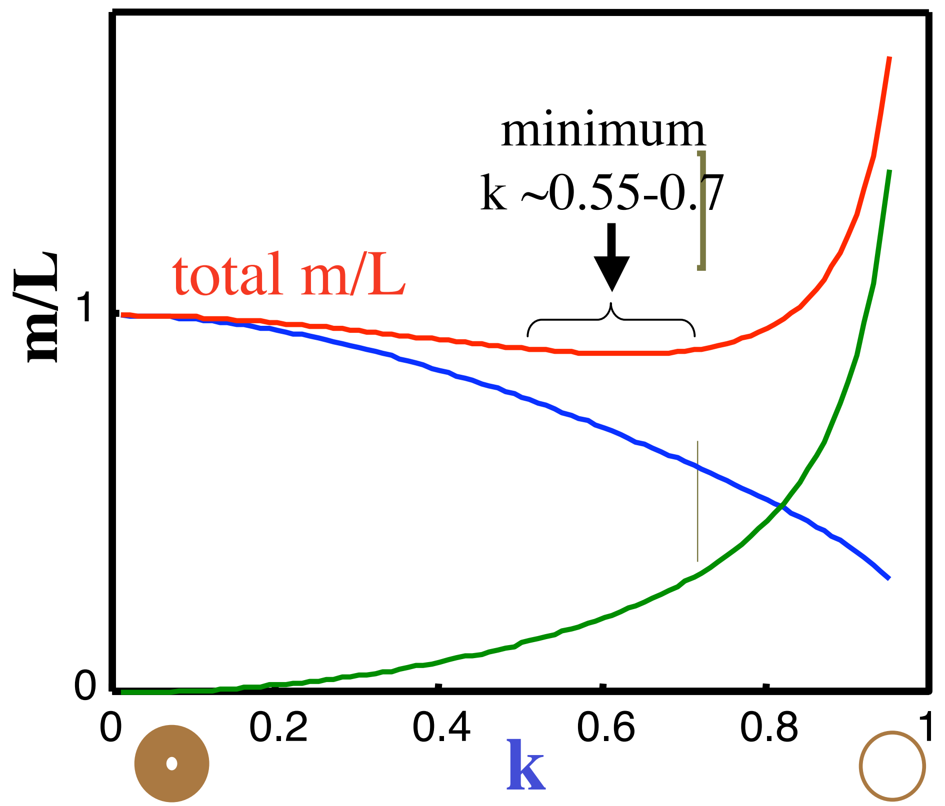




Values of k for terrestrial long bones are close to our predictions

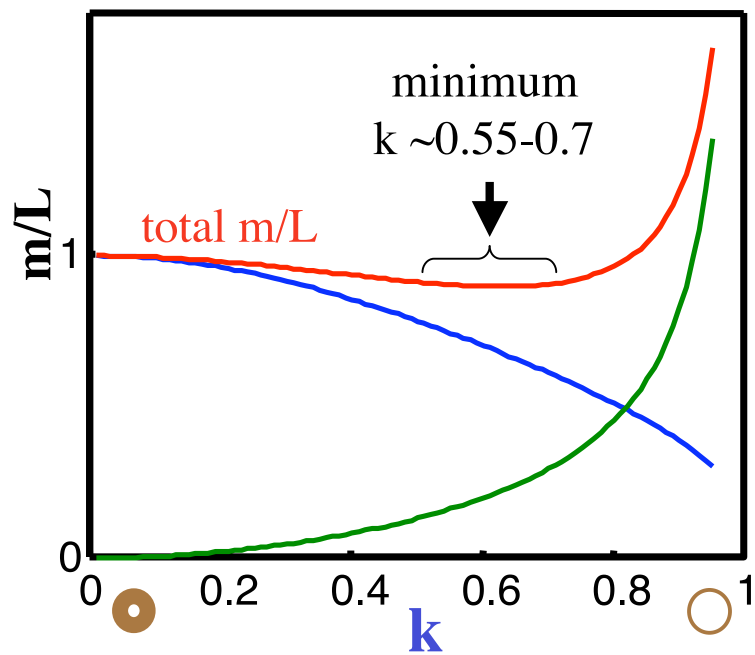
cost of structure — probability of failure* cost of failure — cost to produce — cost to use

minimize: $\boxed{\Phi(n) = P(n)^*F + G(n) + U(n)}$

thick, big bones (k -> 0)

thin, wide bones (k -> 1)

k ~0.55-0.7



What about swan humerus?

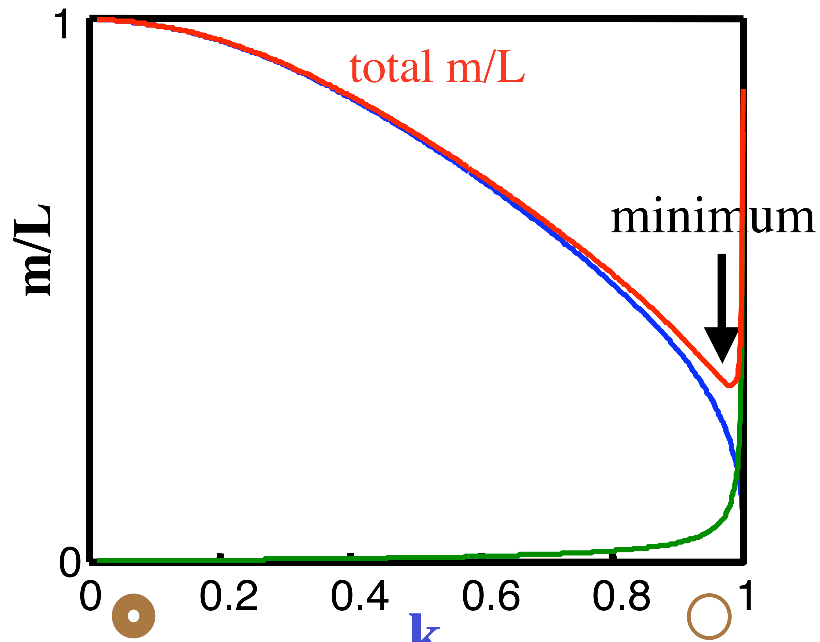
Values for k in terrestrial mammals

Bone	Hare	Fox	Lion	Camel	Buffalo	Swan
femur	0.57	0.63	0.56	0.62	0.54	0.60
humerus	0.55	0.59	0.57	0.66	0.51	0.92

Values of k for terrestrial long bones are close to our predictions

$$\text{minimize: } \Phi(n) = P(n)^*F + G(n) + U(n)$$

cost of structure → $\Phi(n)$; probability of failure* cost of failure → $P(n)^*F$; cost to produce → $G(n)$; cost to use → $U(n)$



Swan humerus (wing bone) is filled with air sacs instead of marrow

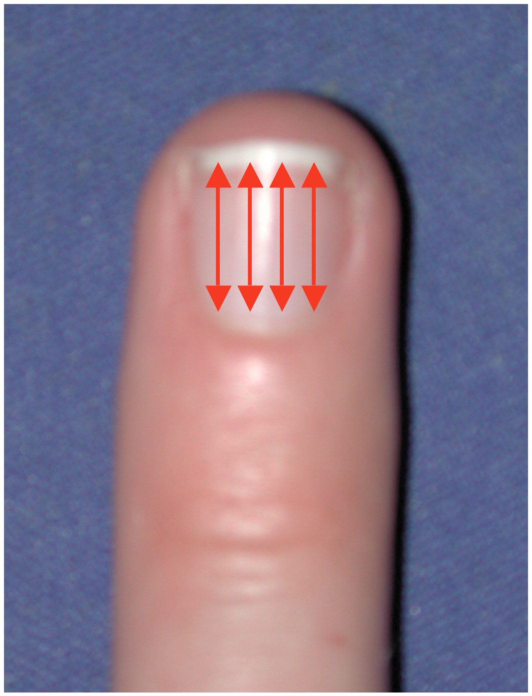
Membranes of air sacs, vasculature, and air have some mass:

$$\rho_{airsac} \approx \rho_{marrow}/50$$

Values for k in terrestrial mammals

Bone	Hare	Fox	Lion	Camel	Buffalo	Swan
femur	0.57	0.63	0.56	0.62	0.54	0.60
humerus	0.55	0.59	0.57	0.66	0.51	0.92

Design for selective failure: anisotropy and ripping fingernails

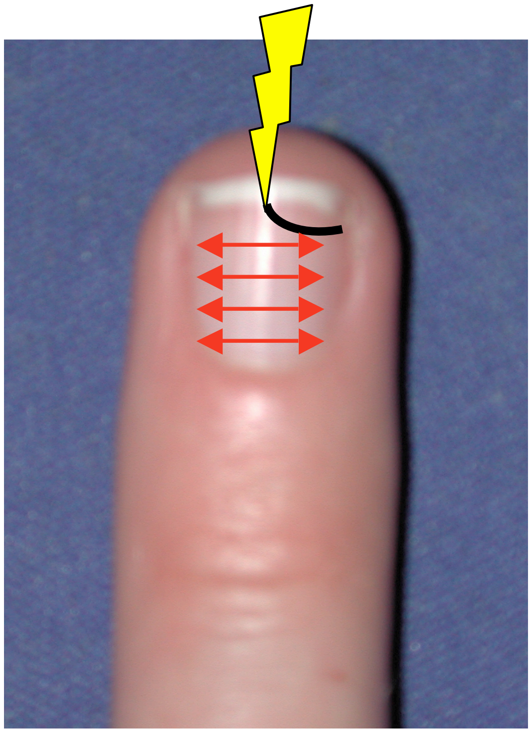


Nails need to be able to resist upward bending forces and prevent damage to vulnerable nail bed at base

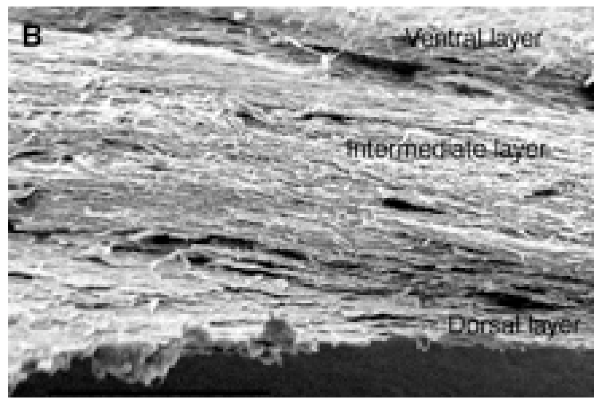
Keratin fibers can be aligned in one direction to provide material ANISOTROPY (different properties in one direction vs. another)

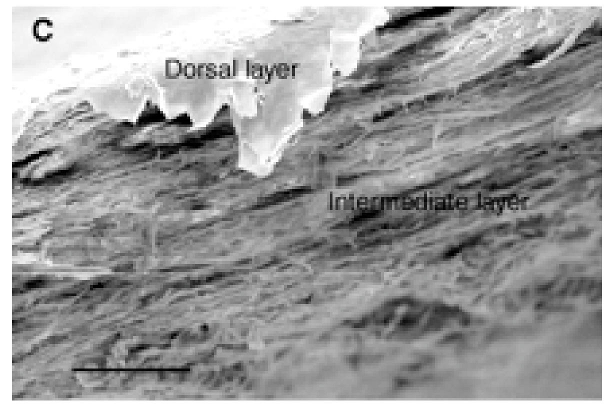
Longitudinally oriented keratin fibers would provide more stiffness to upward bending, but this might propagate tears directly to the nail bed

Design for selective failure: anisotropy and ripping fingernails



Nails need to be able to resist upward bending forces and prevent damage to vulnerable nail bed at base





How do tears propagate in fingernails?

Thick middle layer of nails has transversely-oriented keratin fibers that deflect tears to the side of the nail, providing MATERIAL ANISTROPY (makes easier to tear to side than to base)

Concave shape provides STRUCTURAL ANISOTROPY (makes nails harder to bend upward)

*Farren et al. (2004)

Biology 427 Biomechanics

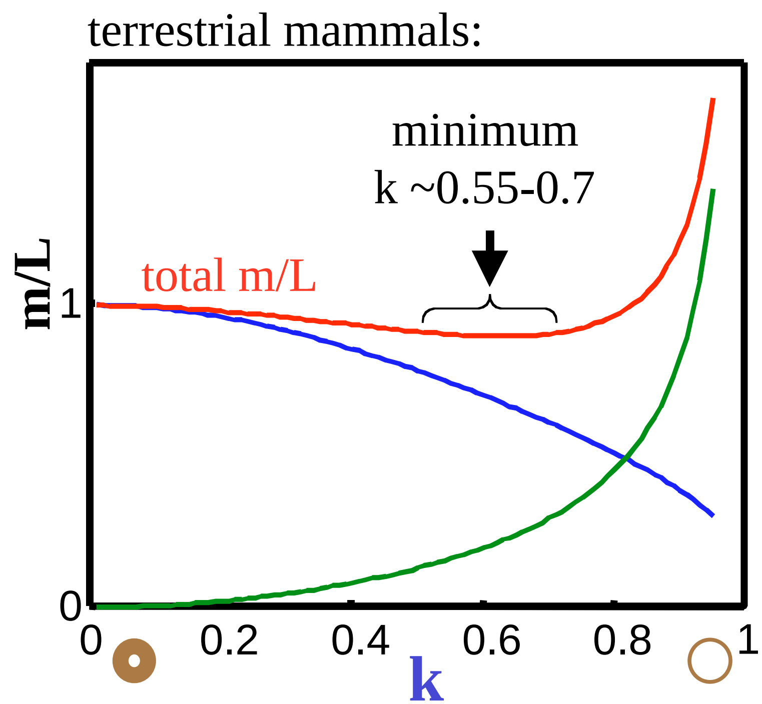
Lecture 12. Less simple structures: dealing with anisotropy, inhomogeneity, and scaling in biological structures.

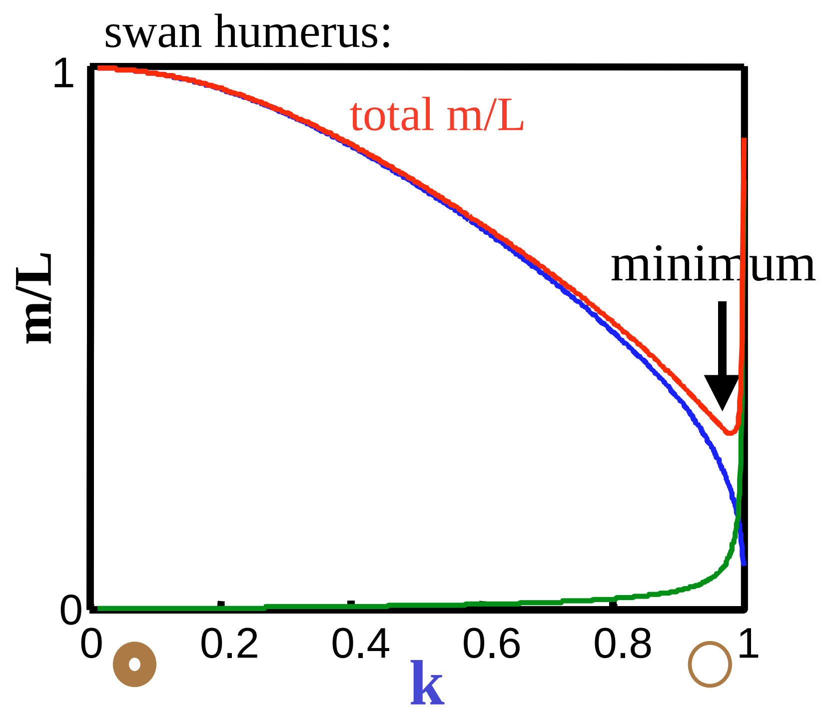
- *Recap design of mammalian long bones and ripping fingernails*
- *Insect wing venation patterns and structural anisotropy*
- *Putting together the pieces: using finite element models to understand complex structures*
- *Bending in just the right places: inhomogeneity in biological structures*

Values of k for long bones are close to our predictions for a strong structure that minimizes mass per unit length

cost of structure | probability of failure* cost of failure | cost to produce | cost to use

(n) = P(n)*F + G(n) + U(n)





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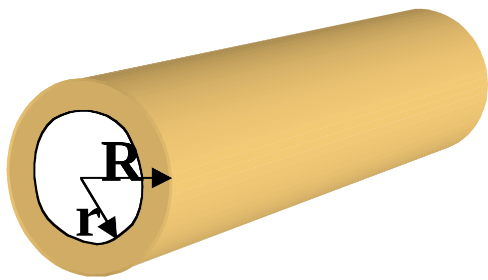
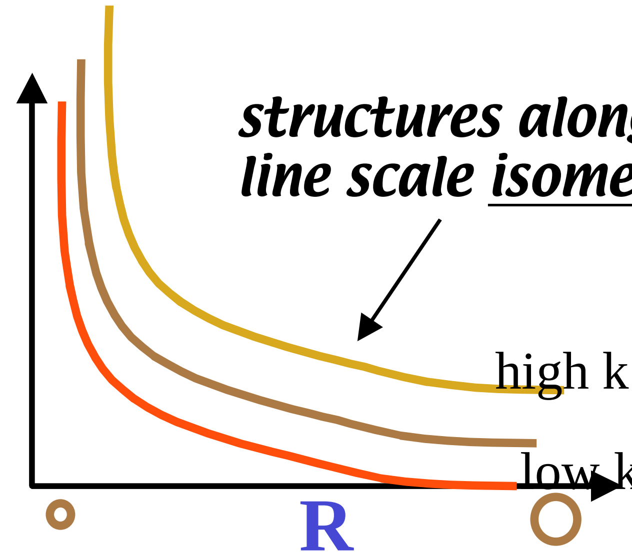
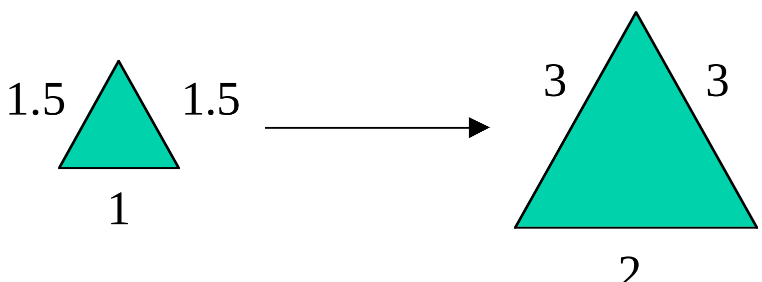
Scaling in biological structures: a simple matter of getting bigger?



k = r/R



isometric scaling: structures retain same relative proportions as they become larger



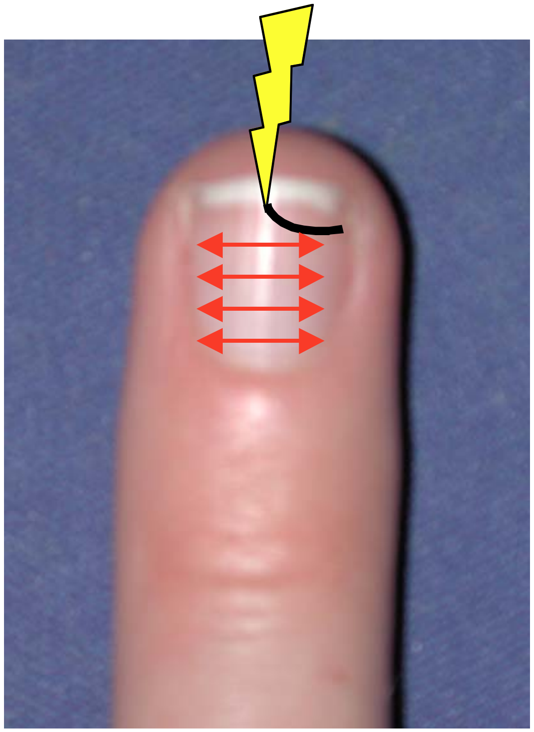
**structures may be scaled to provide a similar functional performance (i.e. similar degree of bending) instead of remaining geometrically similar*

** Many biological structures do not scale isometrically because forces do not scale isometrically*

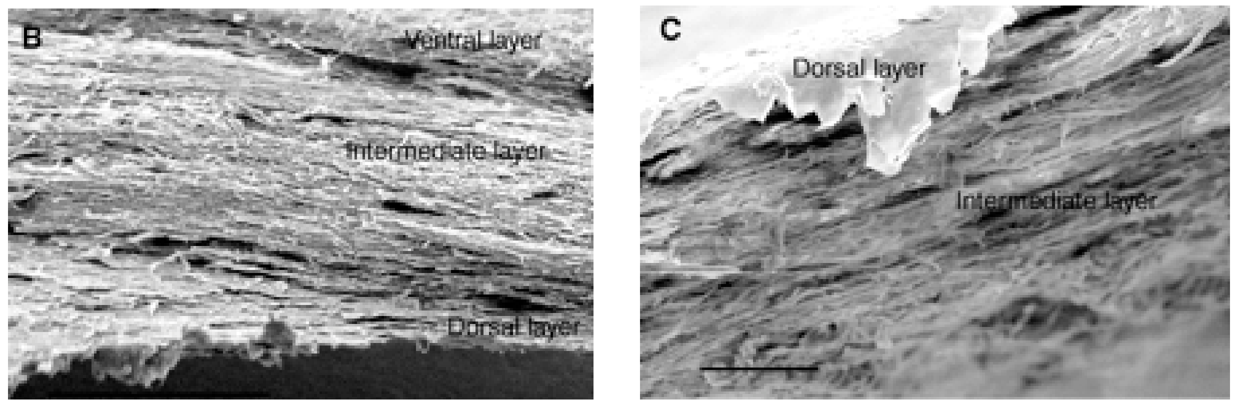
i.e. Euler buckling: $F_E = n\ ^2 E\ I/L^2$

critical buckling force goes as $1/L^2$, so longer columns will buckle at a lower relative force --> elephant vs. mouse legs

Design for selective failure: anisotropy and ripping fingernails



Nails need to be able to resist upward bending forces and prevent damage to vulnerable nail bed at base



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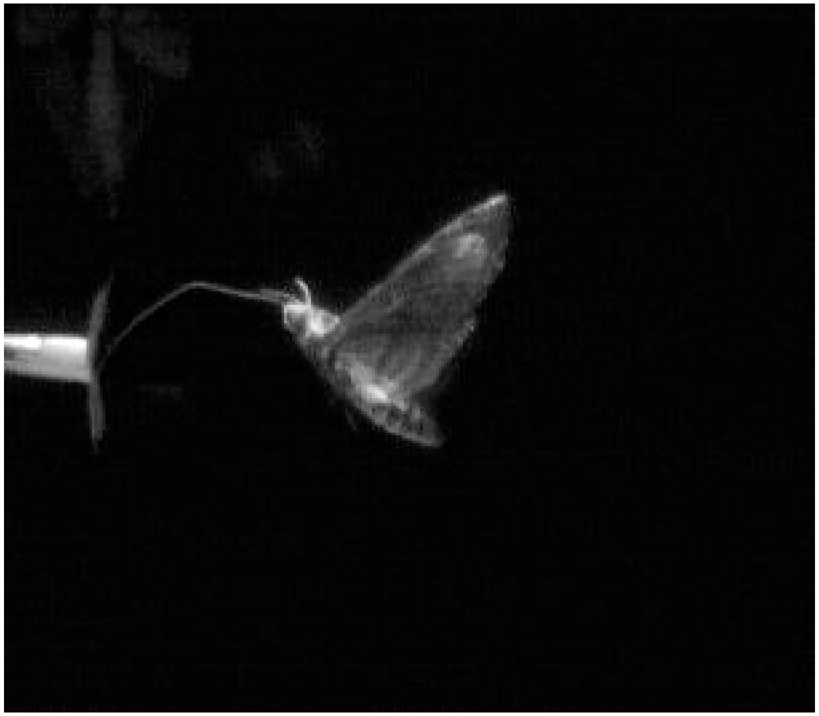
Concave shape provides STRUCTURAL ANISOTROPY (makes nails harder to bend upward)

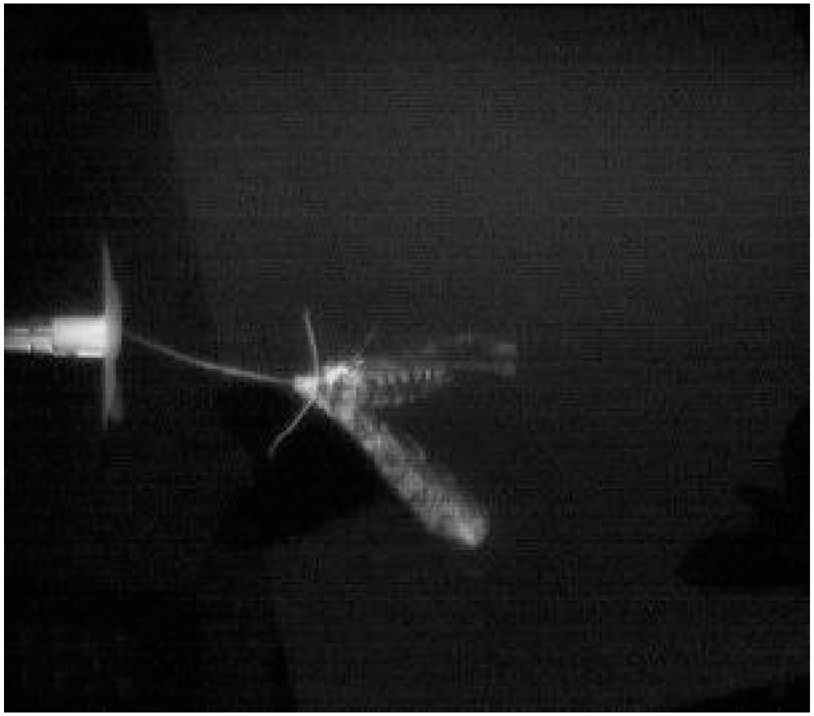
*Farren et al. (2004)

Insect wings are flexible structures that are flapped through the air up to several hundred times per minute!

Insect wings deformations may affect force generation and aerodynamic efficiency

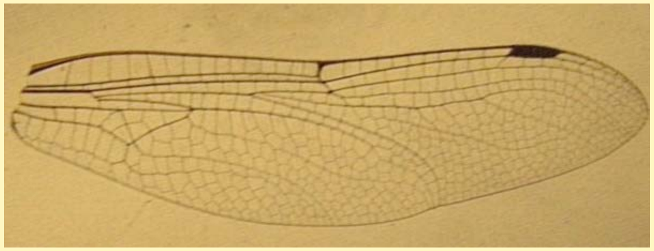
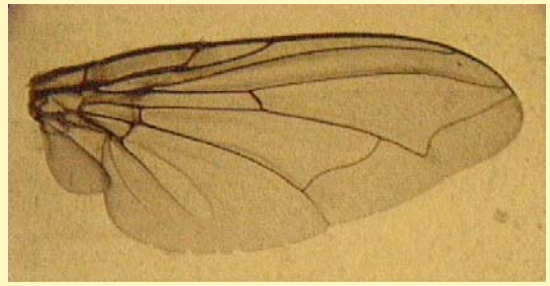
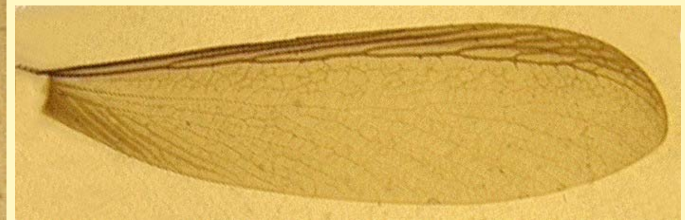
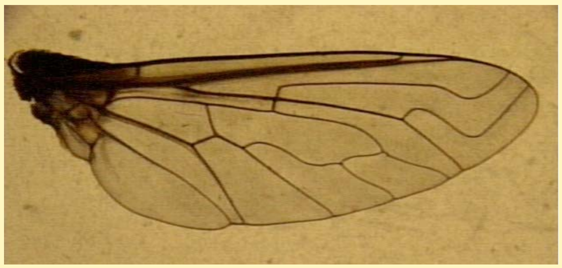
How are wing deformations controlled during flight?

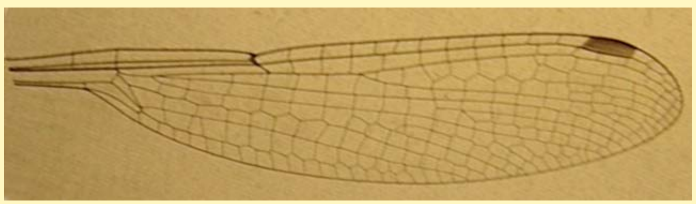
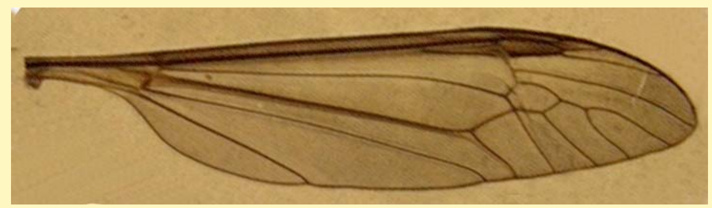
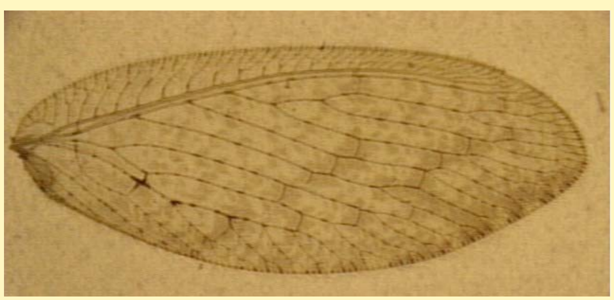




Does insect wing structure control passive shape changes?

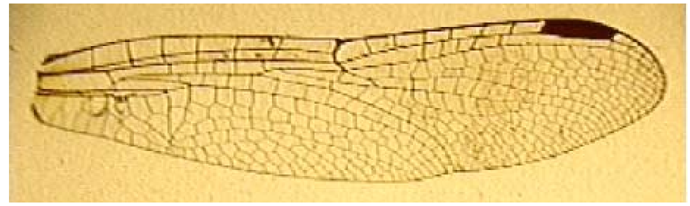

Manduca

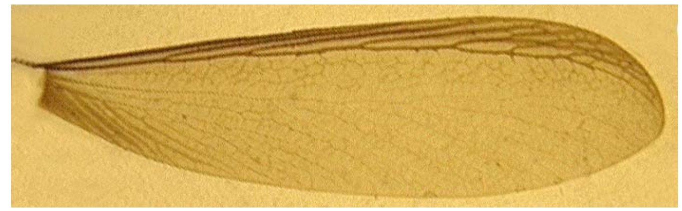




Is insect wing stiffness related to venation pattern?

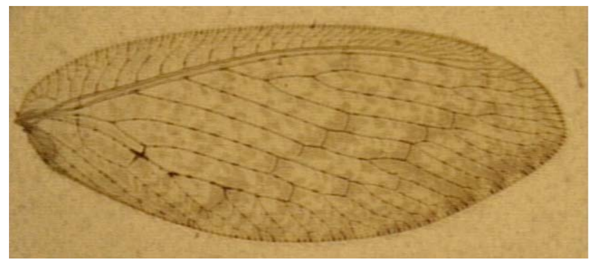
— Odonata (dragonflies, damselflies)



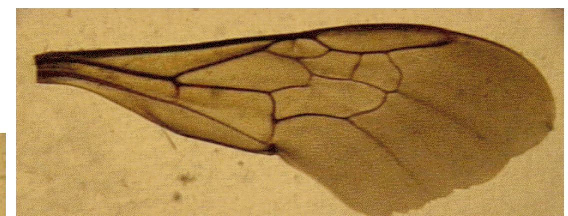
— Isoptera (termites)



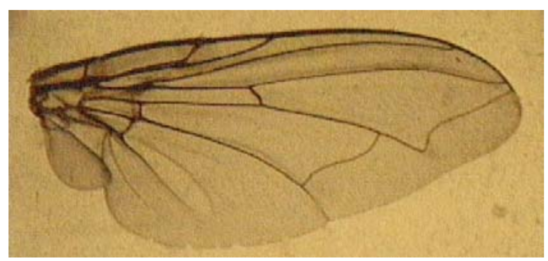
— Neuroptera (lacewings)



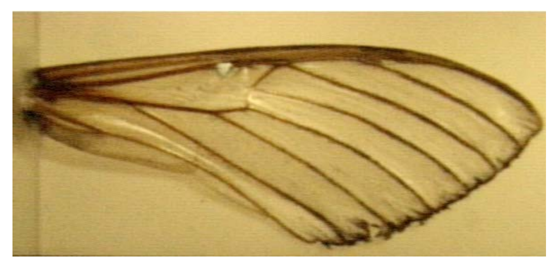
— Hymenoptera (bees,wasps)



— Diptera (flies)

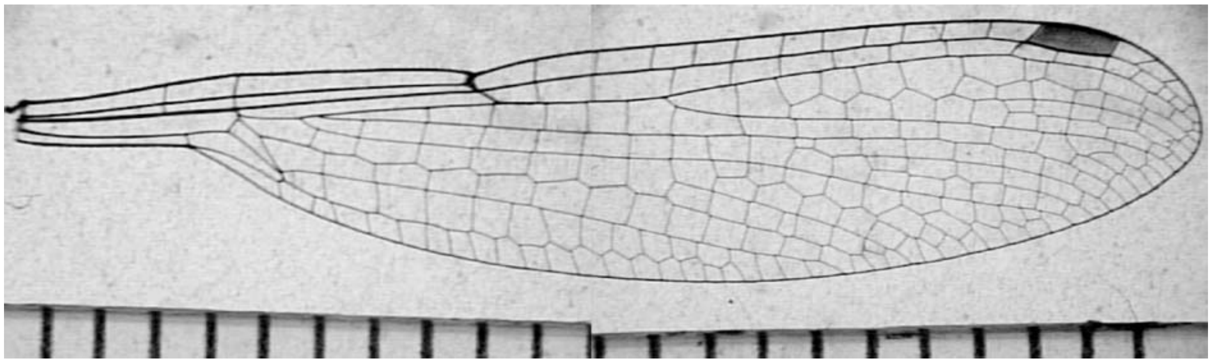


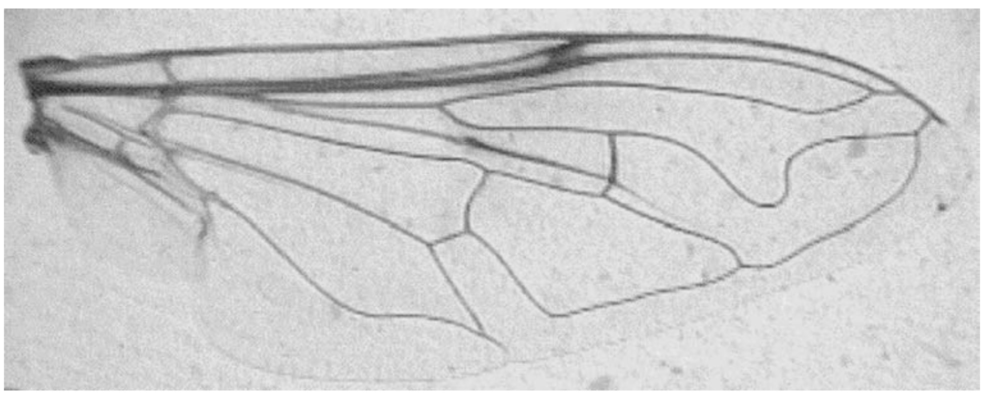
— Lepidoptera (moths, butterflies)



Quantify wing venation pattern in 16 species from 6 orders

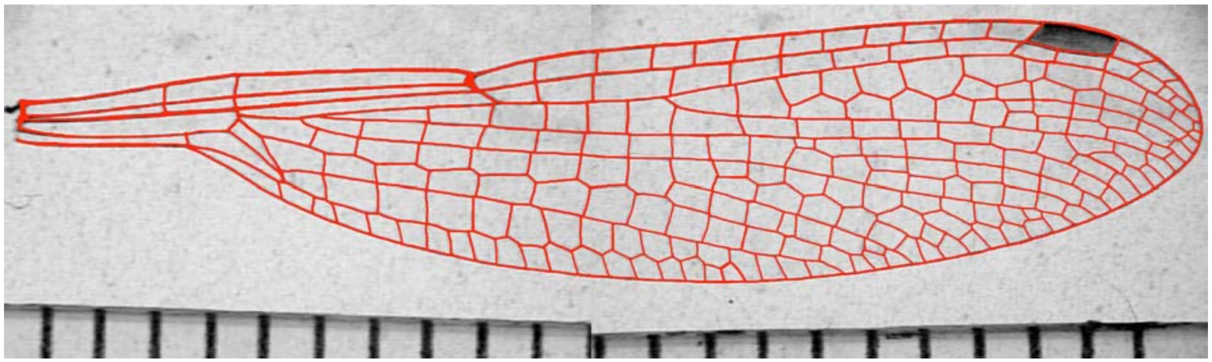
Lestes spp. (damselfly)

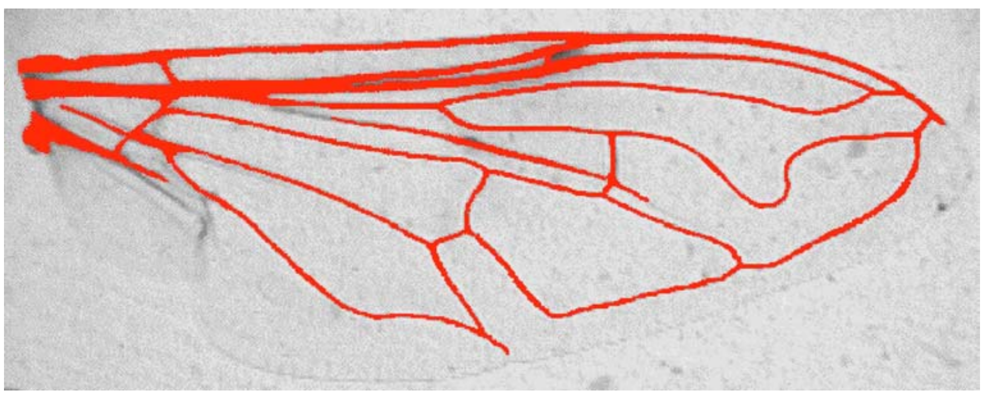




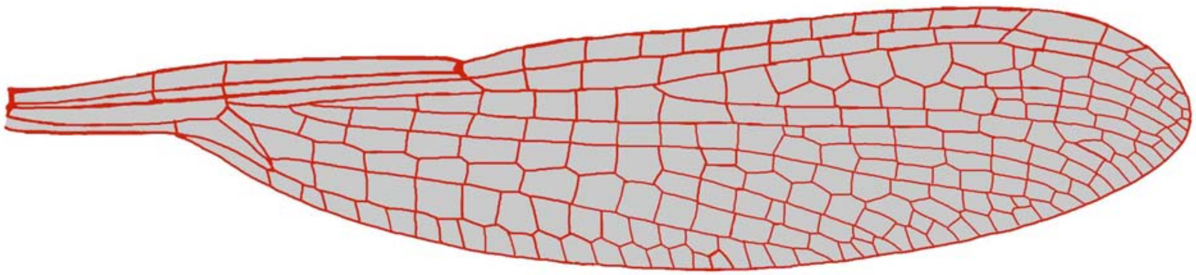
Calliphora spp. (blow fly)

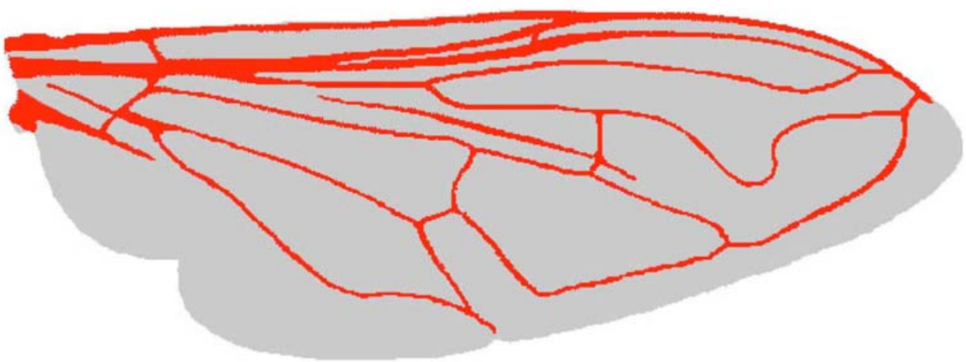
Quantify wing venation pattern in 16 species from 6 orders





Quantify wing venation pattern in 16 species from 6 orders





percent vein area

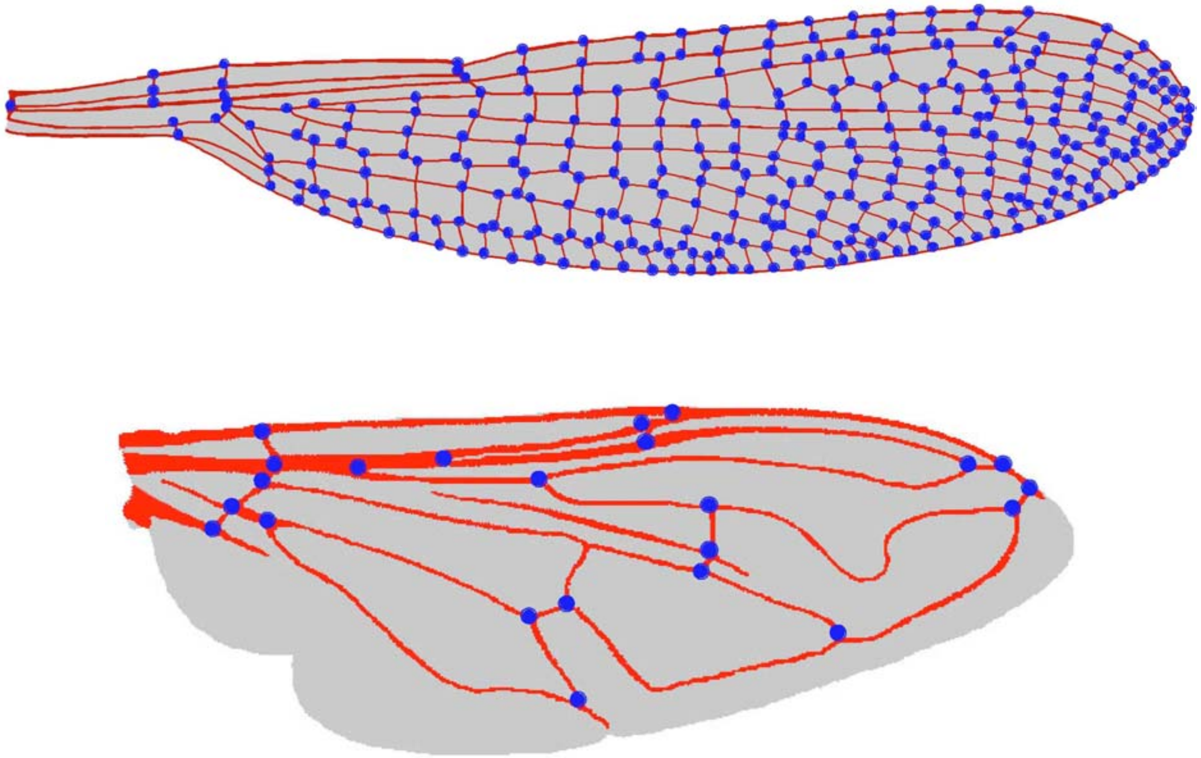
(vein area/total area)

+

vein thickness/span

((vein area/total length)/span)

Quantify wing venation pattern in 16 species from 6 orders



percent vein area

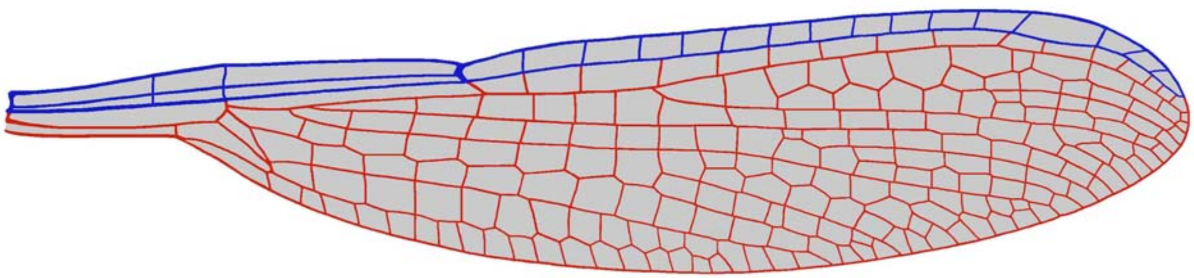
(vein area/total area)

vein thickness/span

((vein area/total length)/span)

log(vein intersections)

Quantify wing venation pattern in 16 species from 6 orders



percent vein area

(vein area/total area)

vein thickness/span

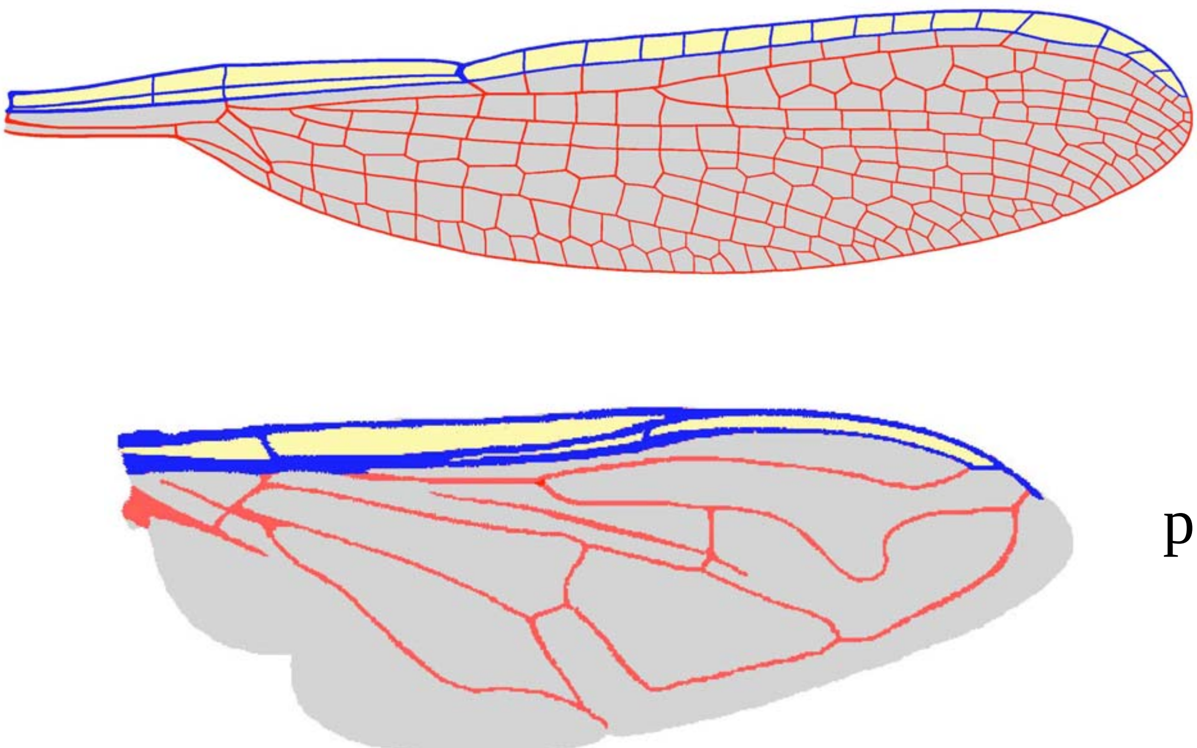
((vein area/total length)/span)

log(vein intersections)

percent leading edge veins

(lead vein area/total vein area)

Quantify wing venation pattern in 16 species from 6 orders



percent vein area

(vein area/total area)

vein thickness/span

((vein area/total length)/span)

log(vein intersections)

percent leading edge veins

(lead vein area/total vein area)

leading edge vein density

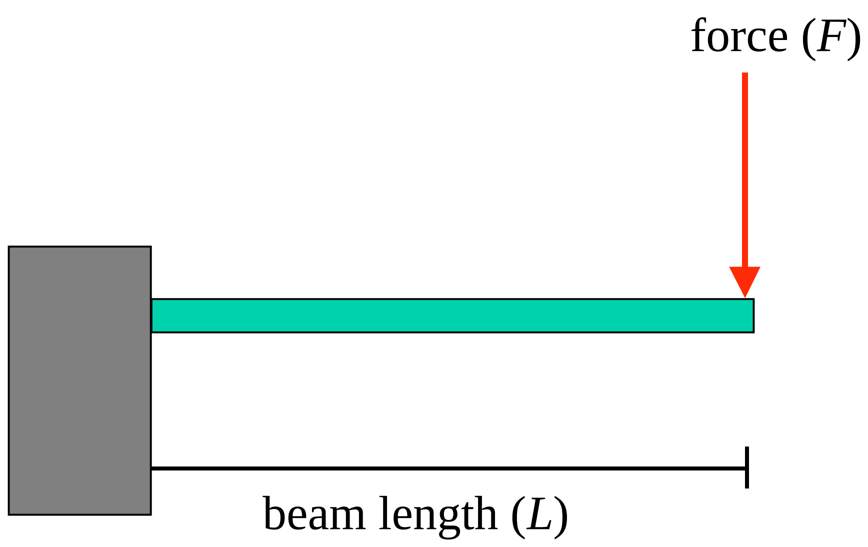
(lead vein area/total lead area)

Treat the wing as a 2-D beam to measure overall stiffness

*Flexural stiffness (*EI*)

E = Young's modulus (material properties)

I = second moment of area
(cross-sectional geometry)



Treat the wing as a 2-D beam to measure overall stiffness

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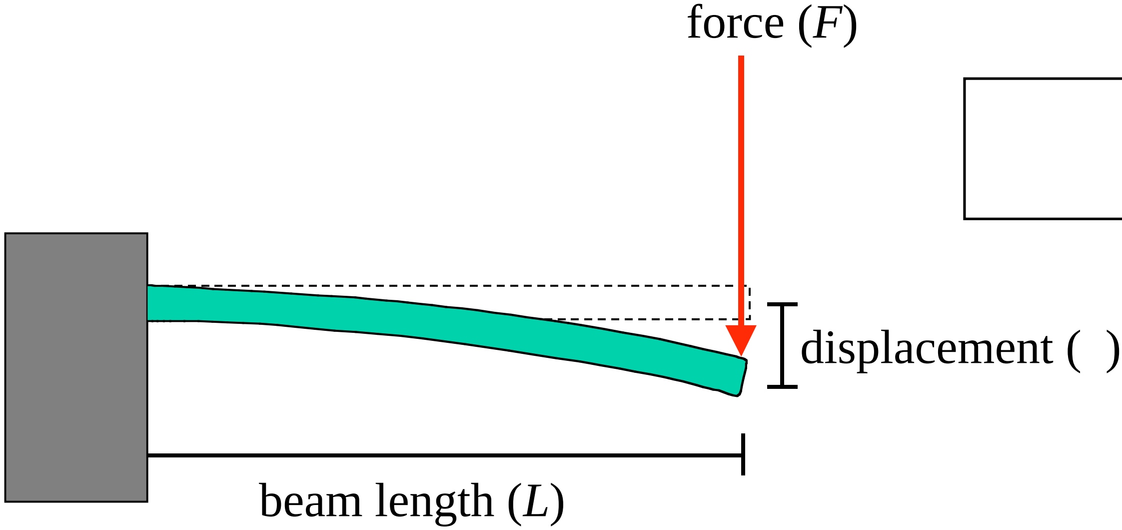
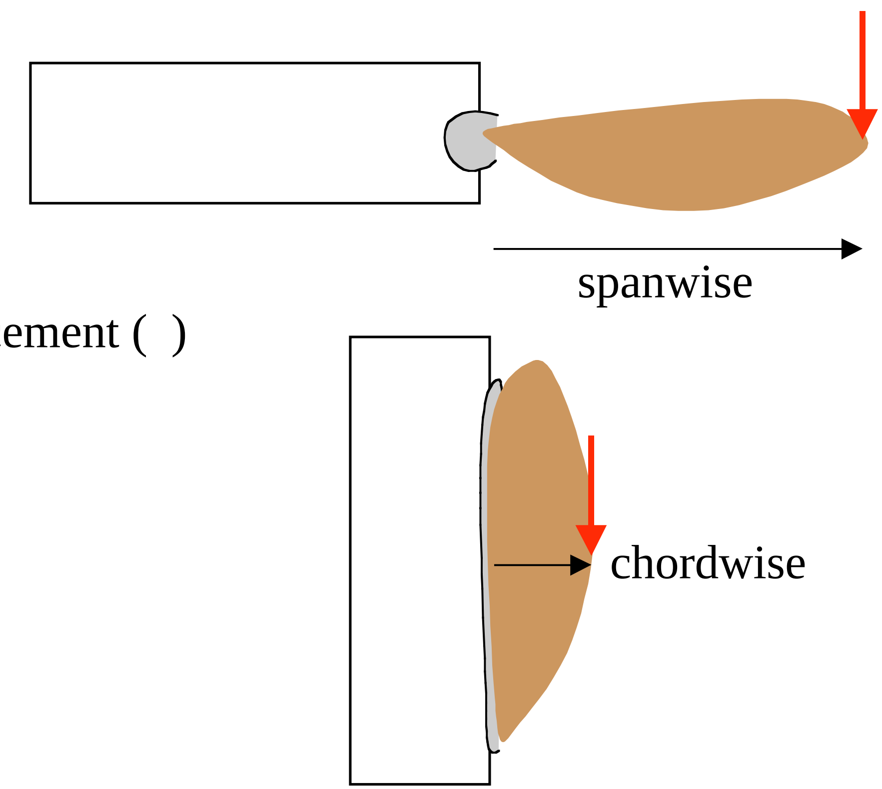
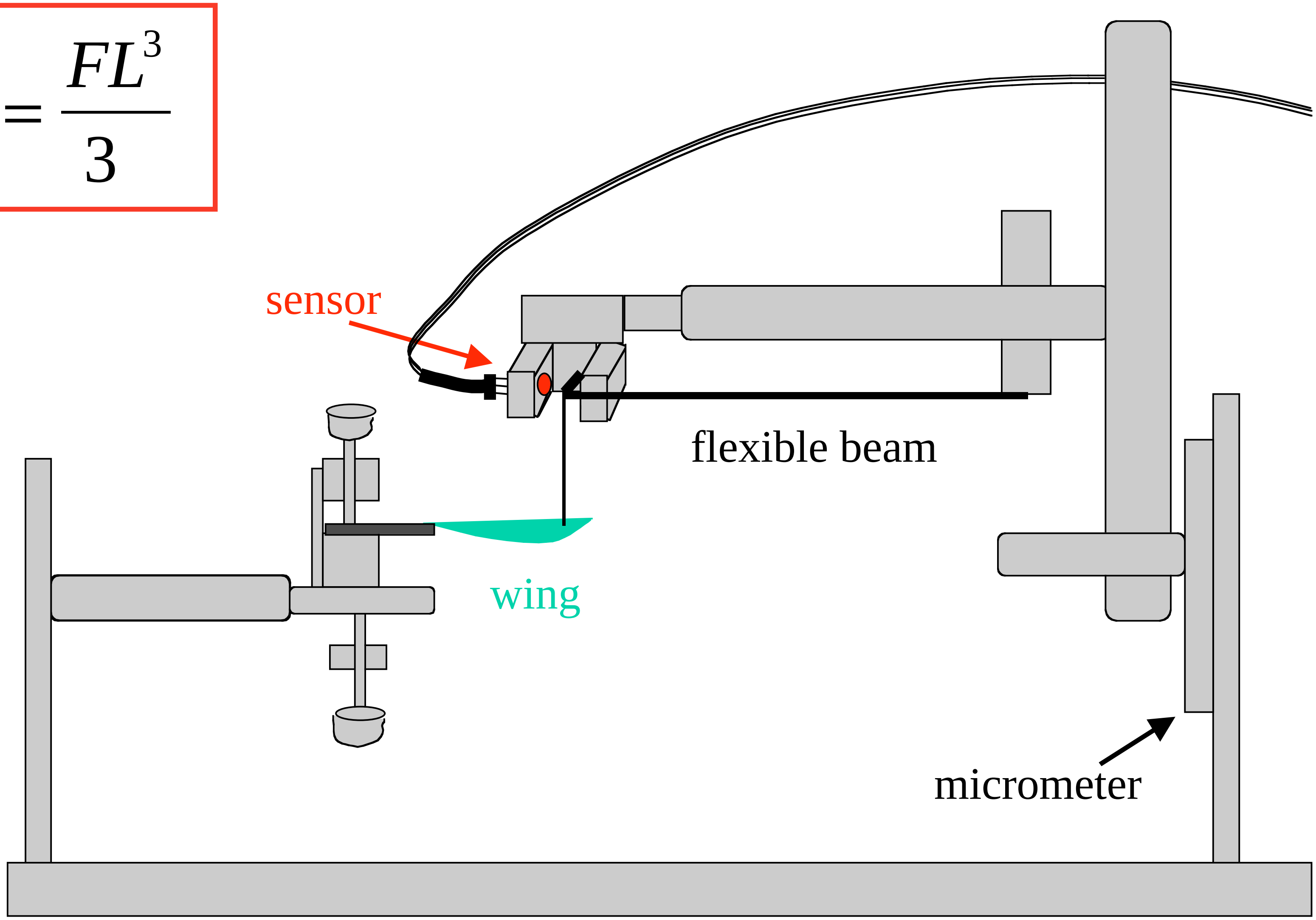
$$EI = \frac{FL^3}{3}$$





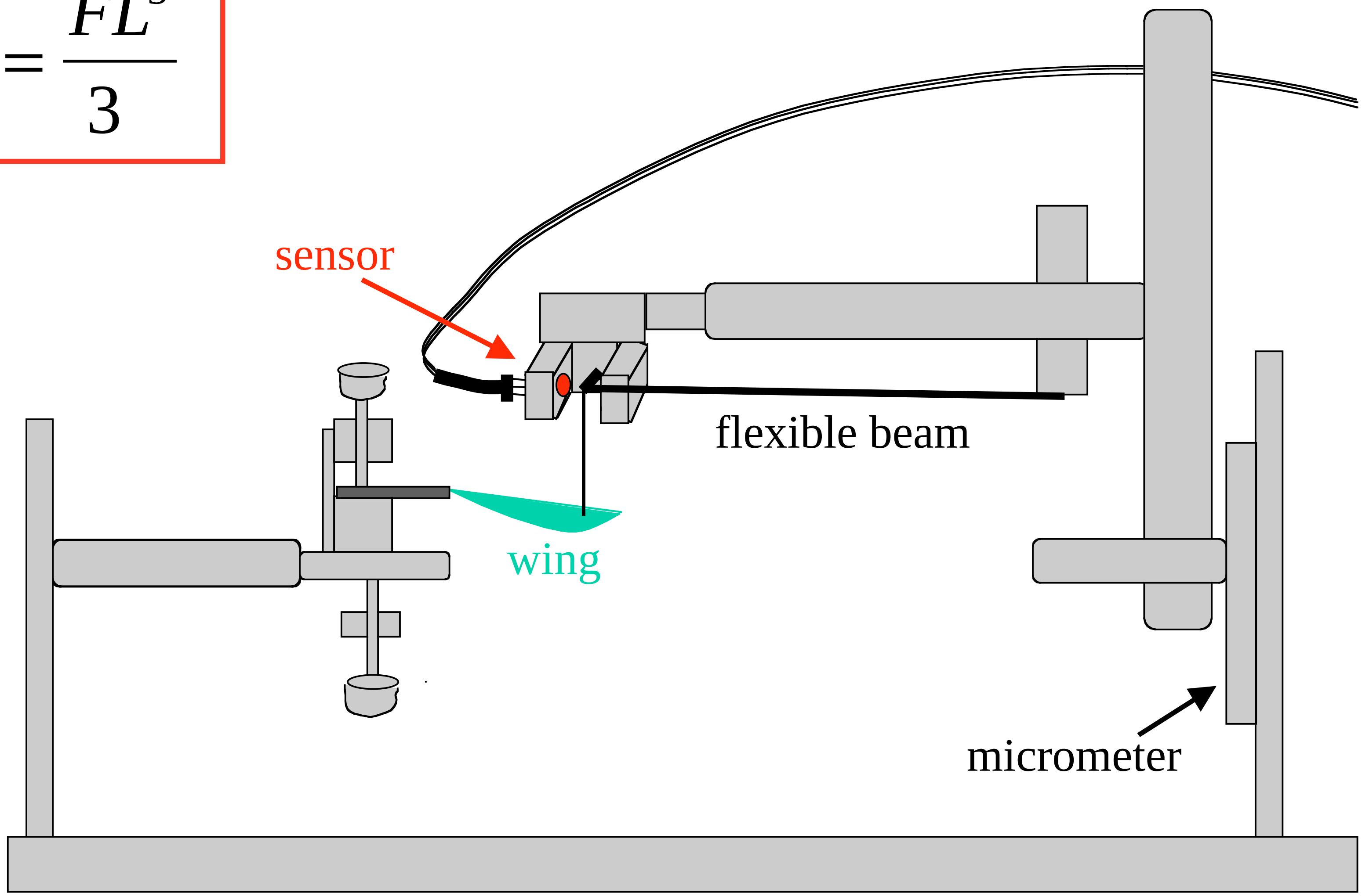
Measuring force and wing tip displacement

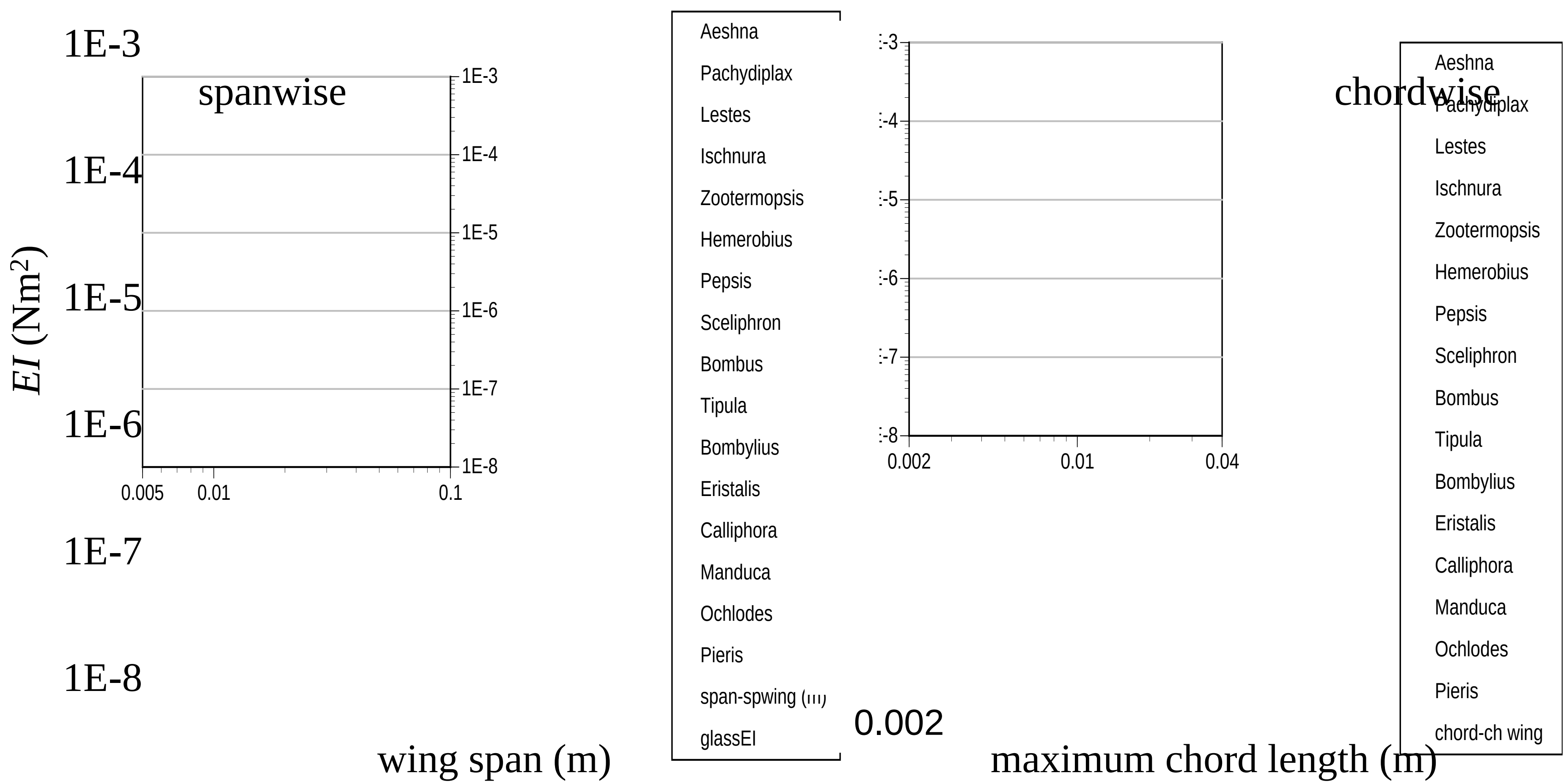
$$EI = \frac{FL^3}{3}$$



Measuring force and wing tip displacement

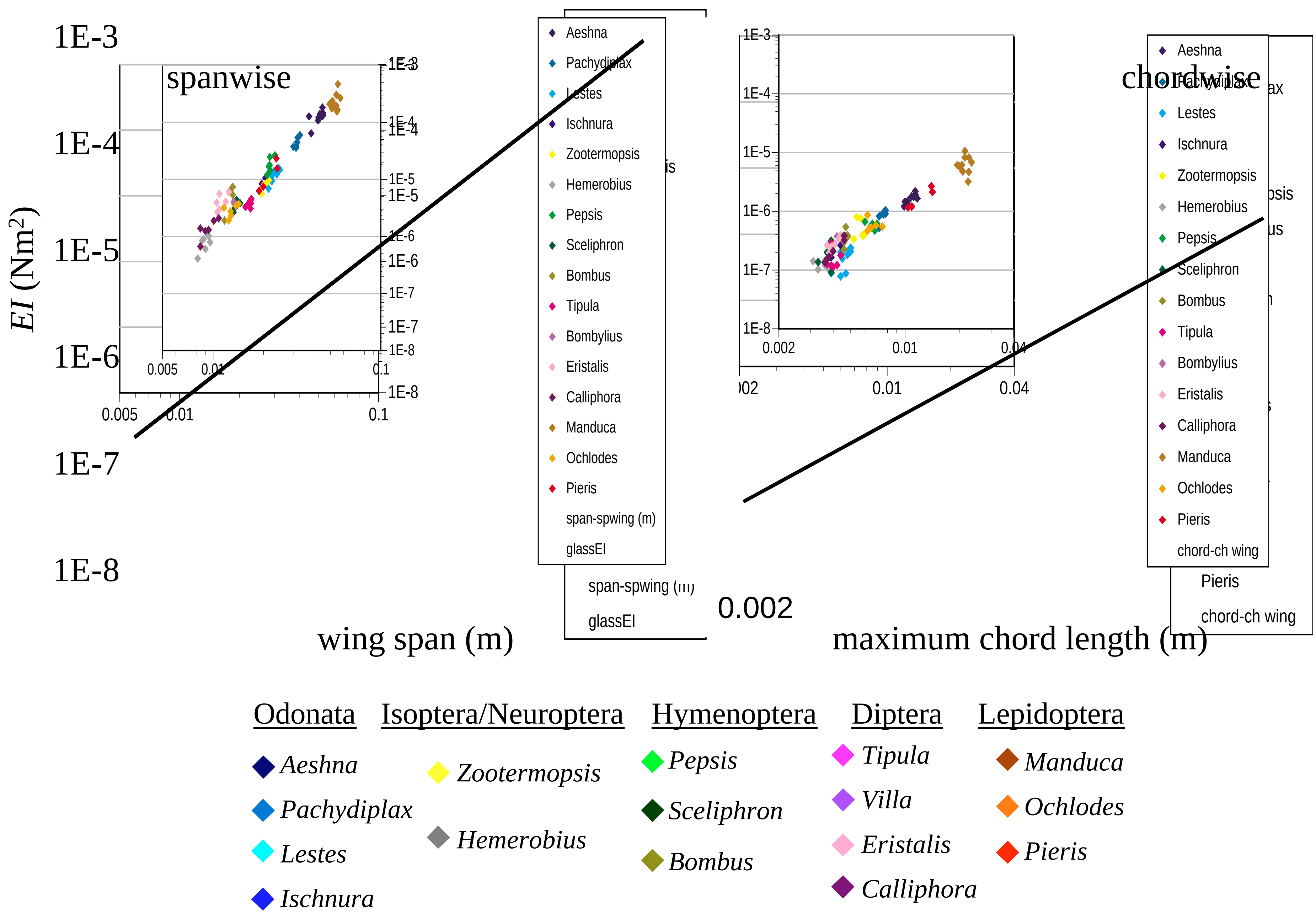
$$\boxed{EI = \frac{FL^3}{3}}$$



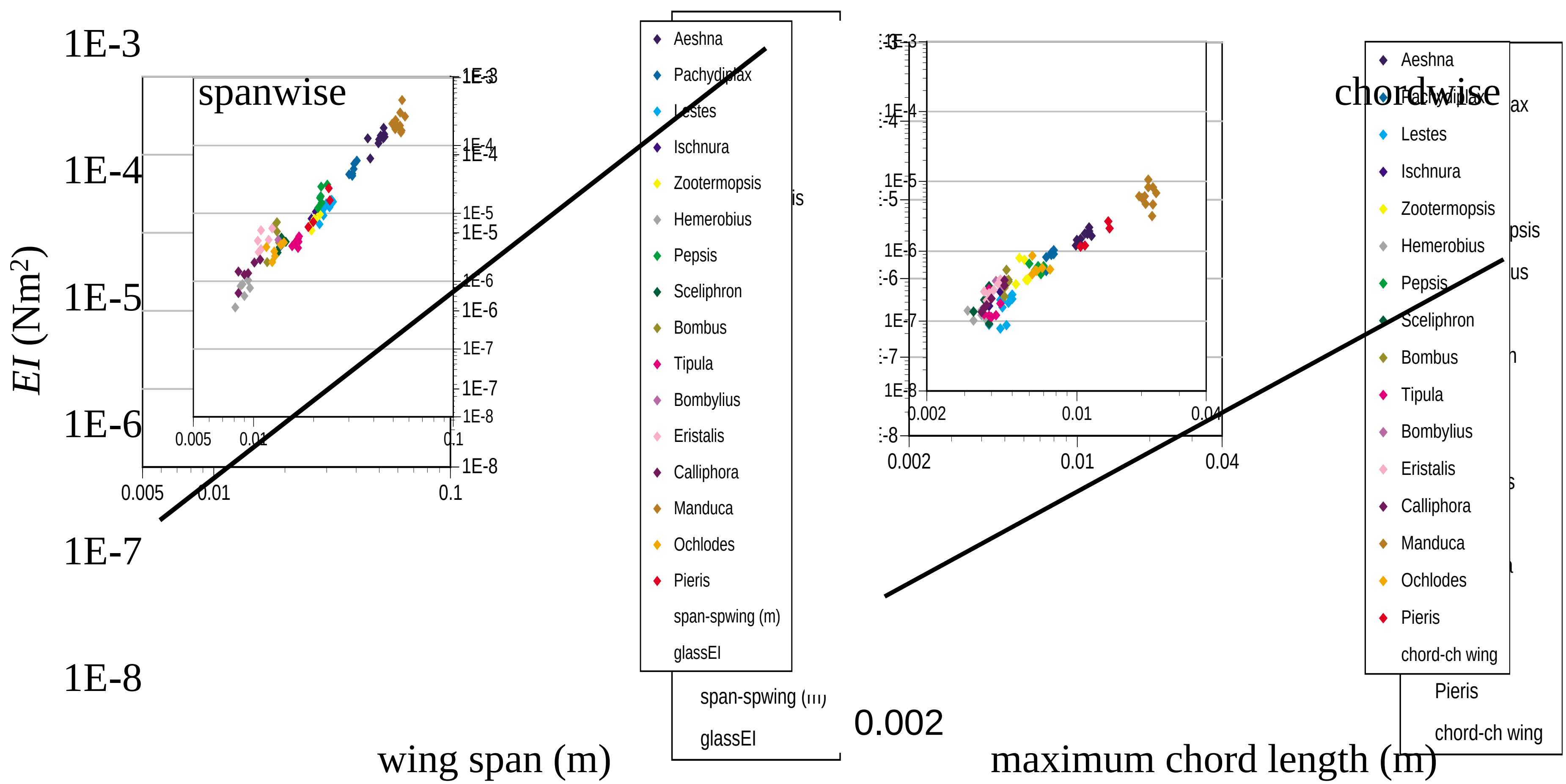


spanwise
chordwise
EI (Nm2)
1E-3
1E-4
1E-5
1E-6
1E-7
1E-8
0.005
0.01
0.1
0.002
0.04
Aeshna
Pachydiplax
Lestes
Ischnura
Zootermopsis
Hemerobius
Pepsis
Sceliphron
Bombus
Tipula
Bombylius
Eristalis
Calliphora
Manduca
Ochlodes
Pieris
span-spwing
glassEI
chord-ch wing
wing span (m)
maximum chord length (m)

Spanwise and chordwise stiffness increase with wing size

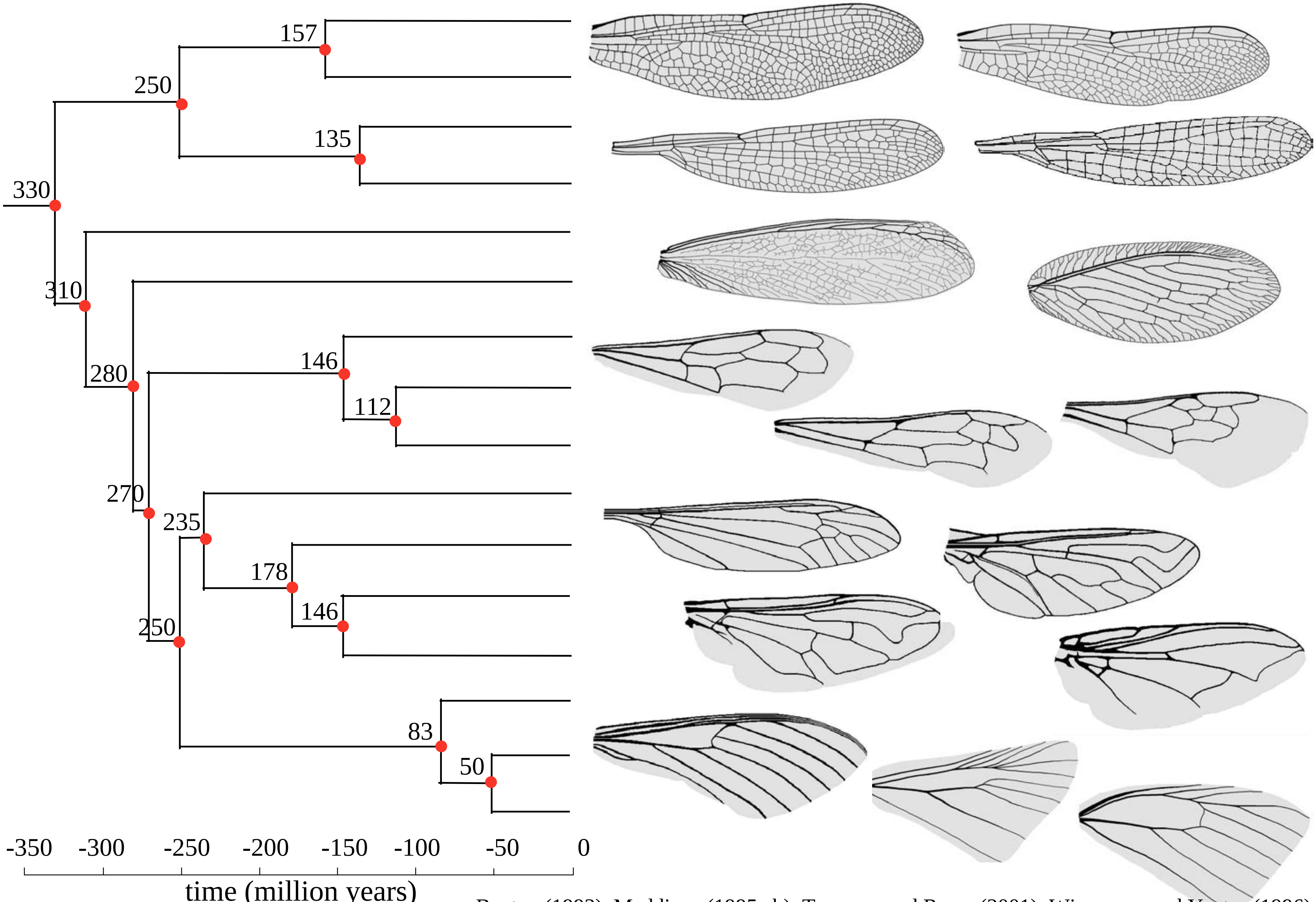


Is a species' position above or below this relationship related to venation pattern?



Odonata	Isoptera/Neuroptera	Hymenoptera	Diptera	Lepidoptera
Aeshna	*Zootermopsis*	*Pepsis*	*Tipula*	*Manduca*
Pachydiplax	*Hemerobius*	*Sceliphron*	*Villa*	*Ochlodes*
Lestes		*Bombus*	*Eristalis*	*Pieris*
Ischnura			*Calliphora*	

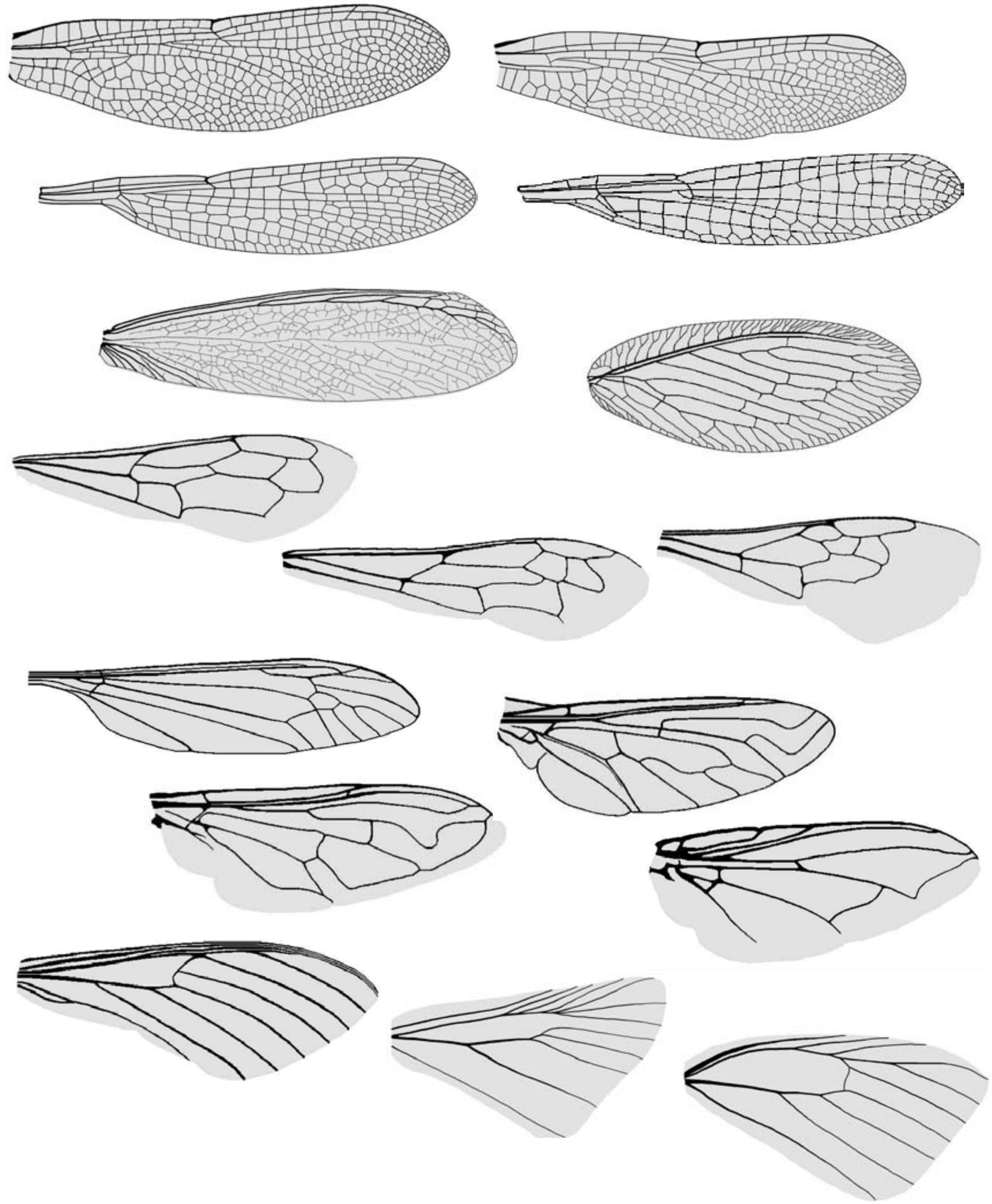
Calculate independent contrasts of wing stiffness residuals and venation traits



Benton (1993), Maddison (1995a,b), Trueman and Rowe (2001), Wiegmann and Yeates (1996)

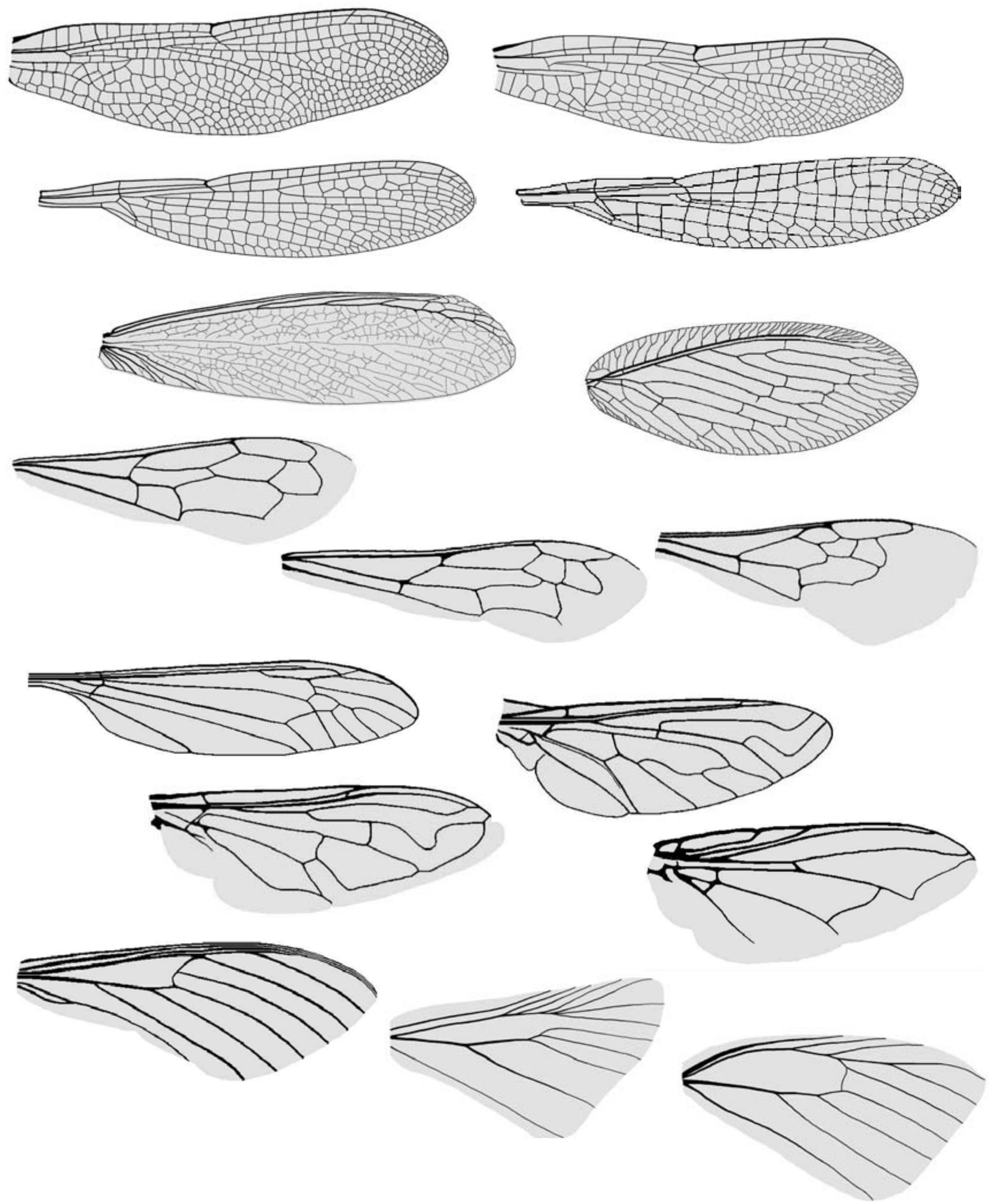
Is overall wing stiffness correlated with venation pattern? NO

	spanwise EI residual	chordwise EI residual
percent vein area	n.s.	n.s.
vein thick per span	n.s.	n.s.
log(vein intersections)	n.s.	n.s.
percent lead veins	n.s.	n.s.
lead edge vein density	n.s.	n.s.

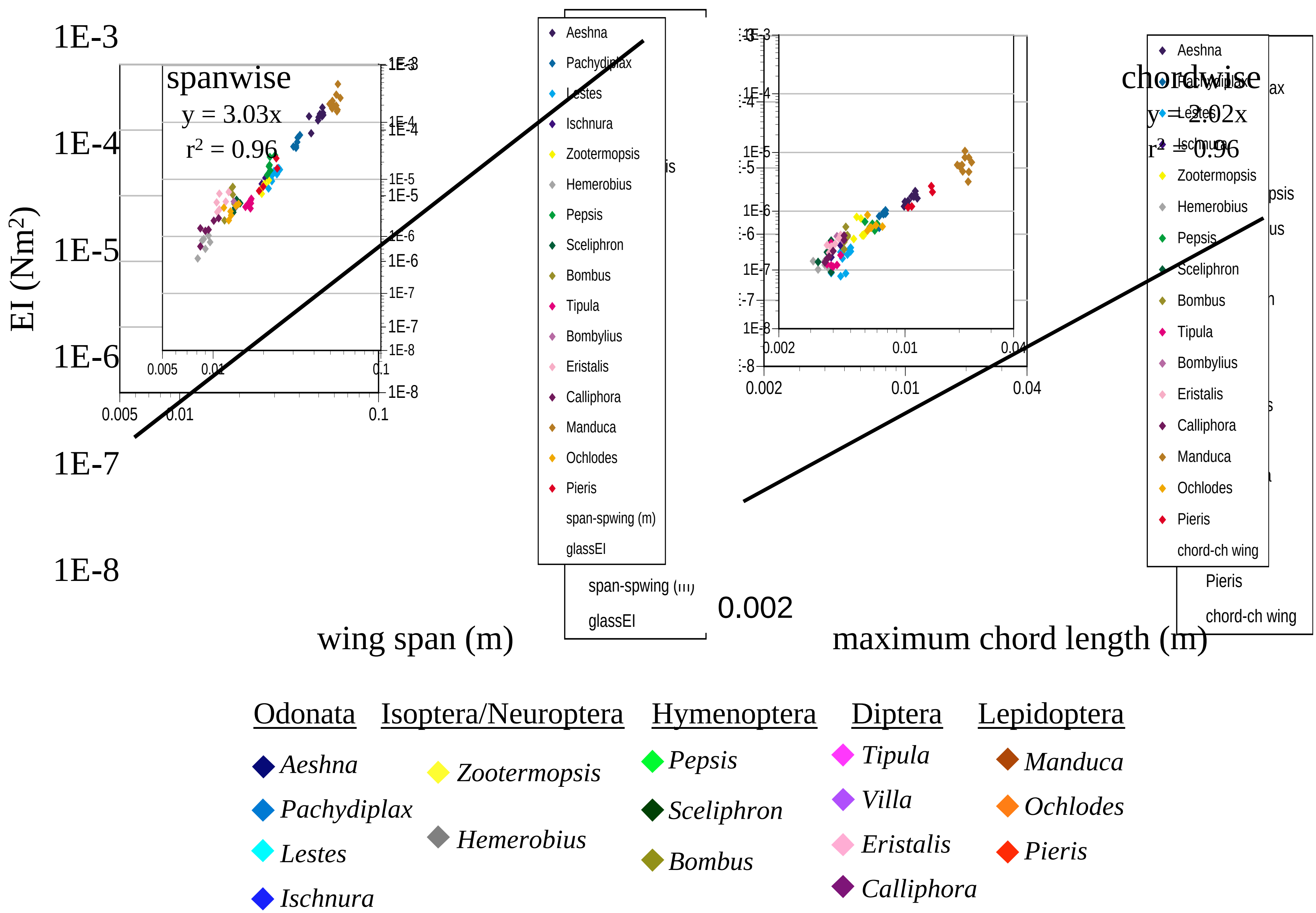


(*correlations between independent contrasts of traits)

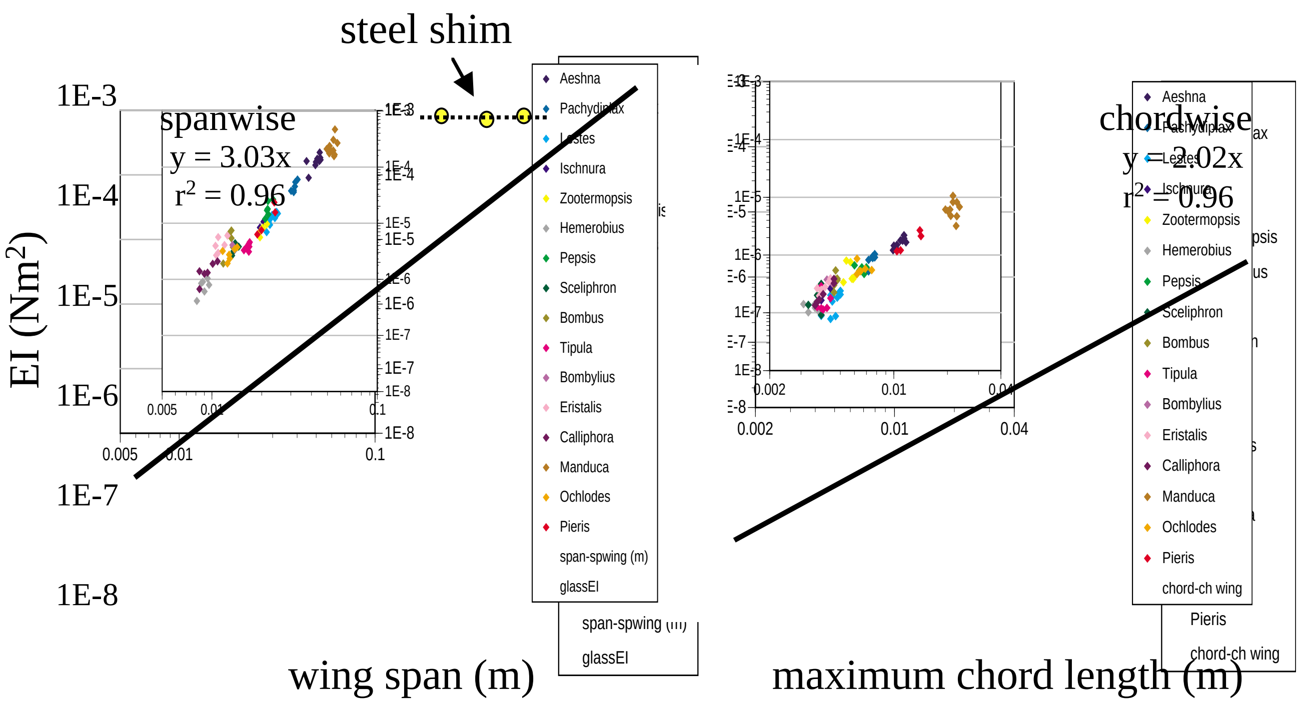
Despite enormous variation in the arrangement of wing veins, overall wing stiffness is independent of venation pattern!



Variation in wing stiffness is dominated by wing size



Why is insect wing stiffness so strongly related to wing size?



$$EI = \frac{FL^3}{3}$$

$$I = wt^3/12$$

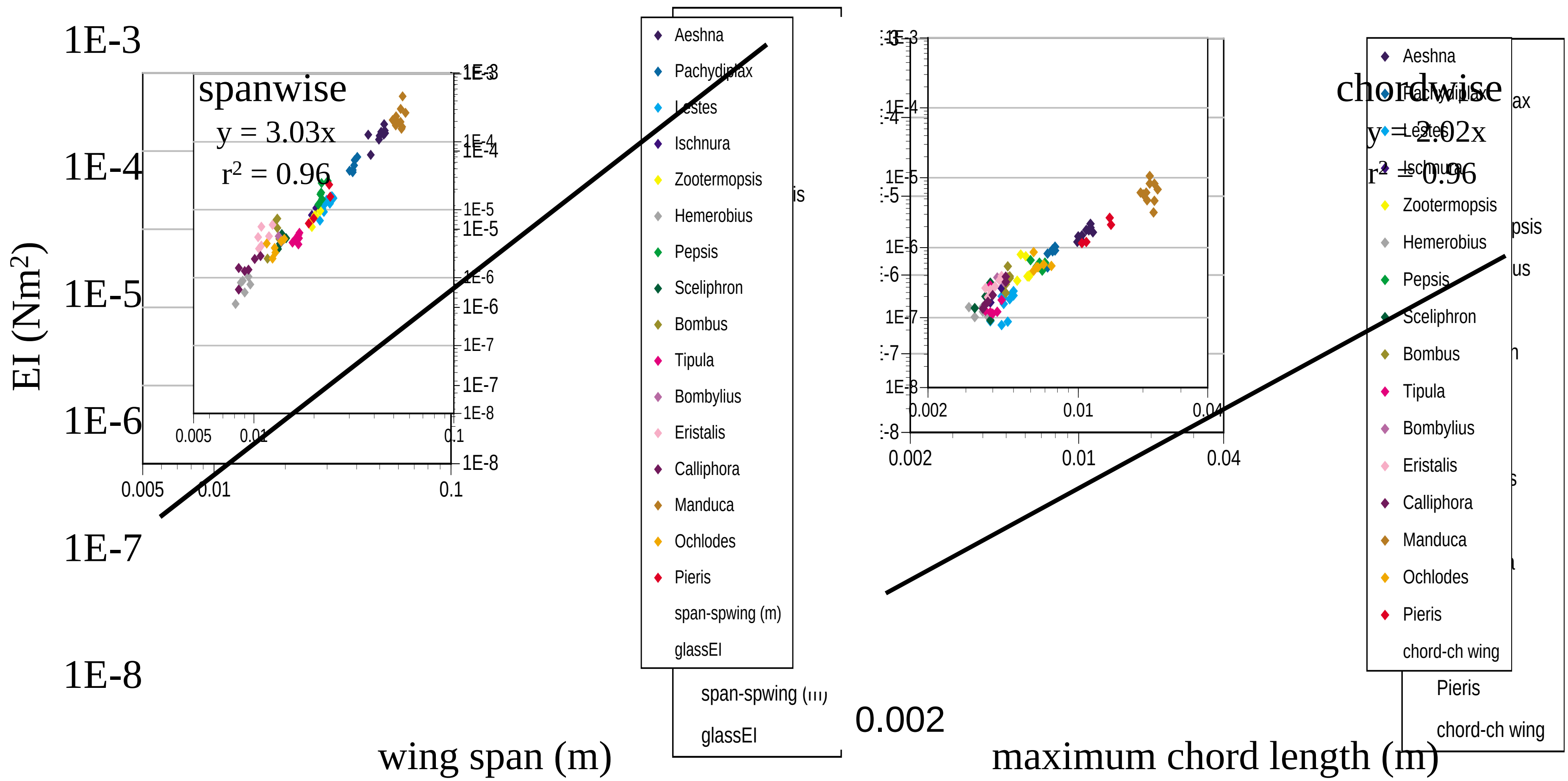
$$EI \; µ \; wt^3$$

• EI is a measure of cross-sectional shape ONLY (length independent)
But, insect wings get wider as they get longer.... w µ L

• How does thickness scale with wing length?
-- if thickness scales isometrically with length, EI µ L^4
-- if thickness is independent of length (single layer of cuticle), EI µ L

Perhaps scaling of wing stiffness provides functional, rather than geometric similarity (i.e. constant /L)?......

What is the source of spanwise-chordwise anisotropy in wings?



Odonata	Isoptera/Neuroptera	Hymenoptera	Diptera	Lepidoptera
Aeshna	*Zootermopsis*	*Pepsis*	*Tipula*	*Manduca*
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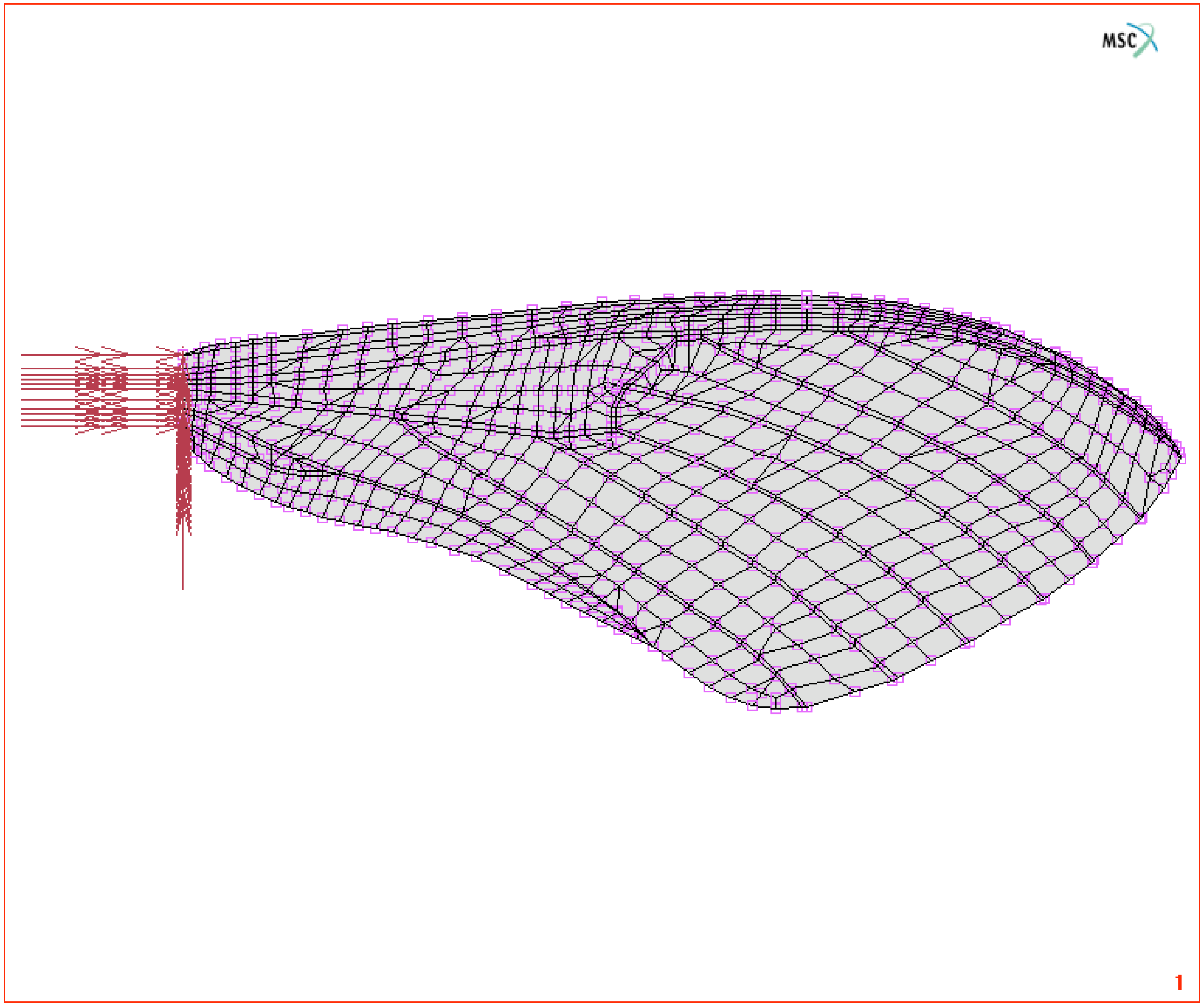
Lestes		*Bombus*	*Eristalis*	*Pieris*
Ischnura			*Calliphora*	

Finite element model of a moth wing

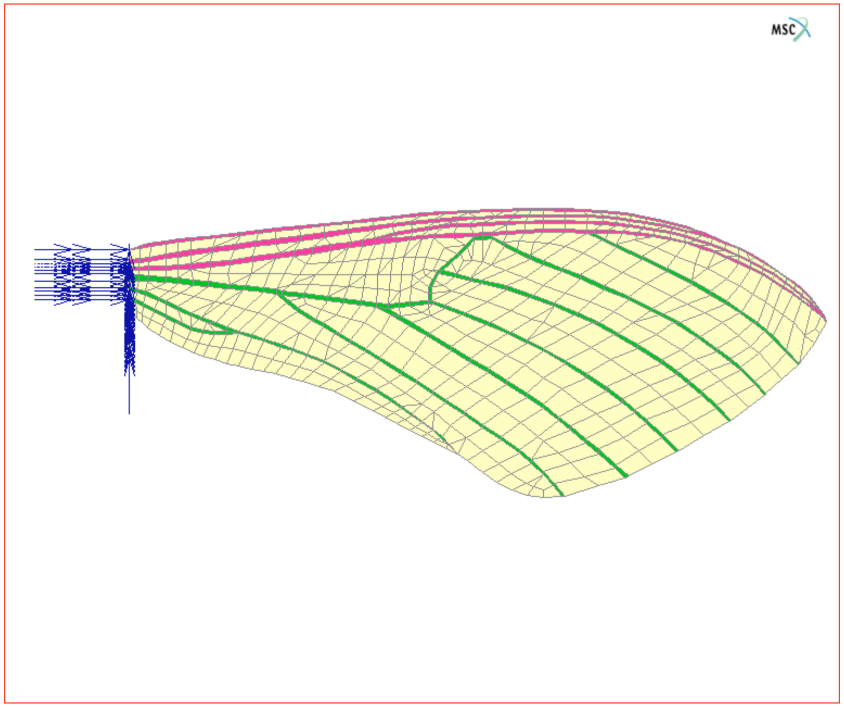


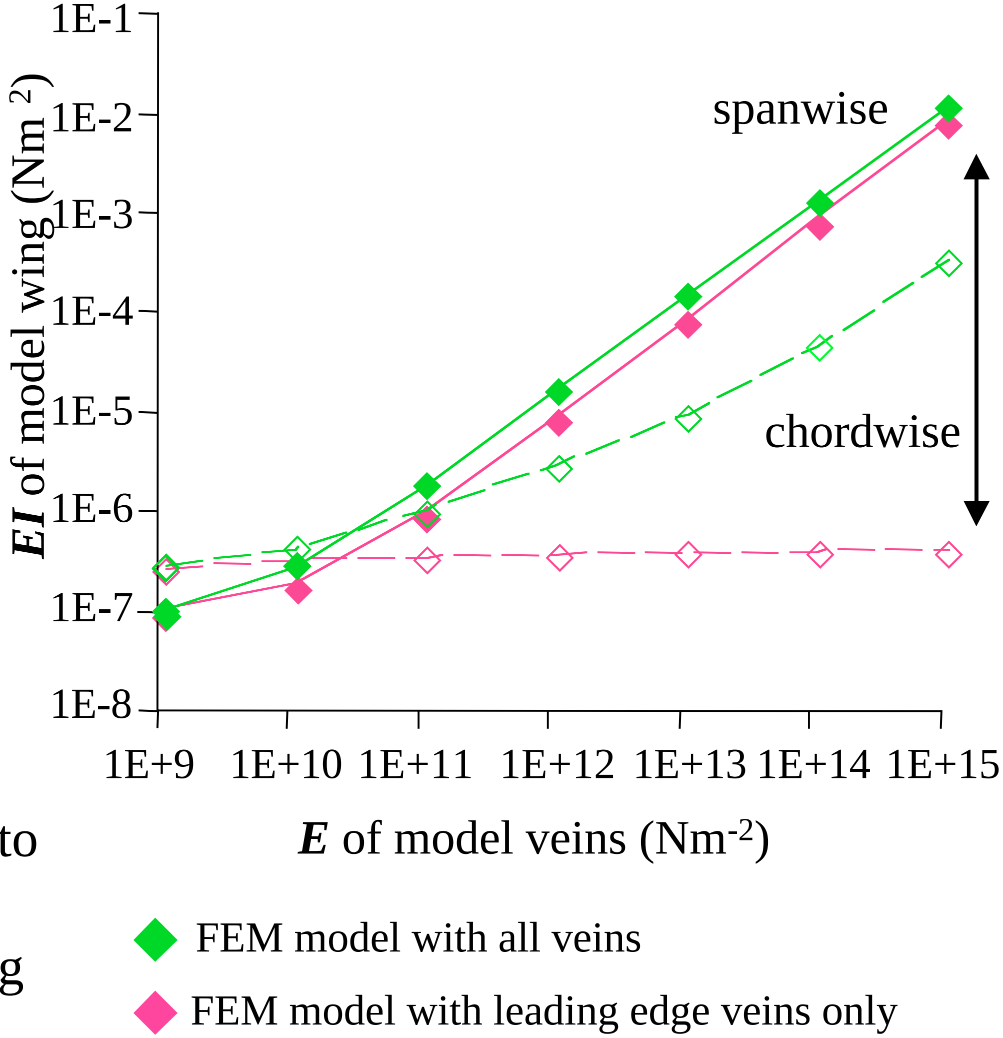
Divide wing into small, interconnected elements (flat plates)

- balance forces and moments around each simple plate and sum plates to find how whole complex structure behaves

Are wing veins the source of spanwise-chordwise anisotropy?

Leading edge veins generate spanwise-chordwise anisotropy



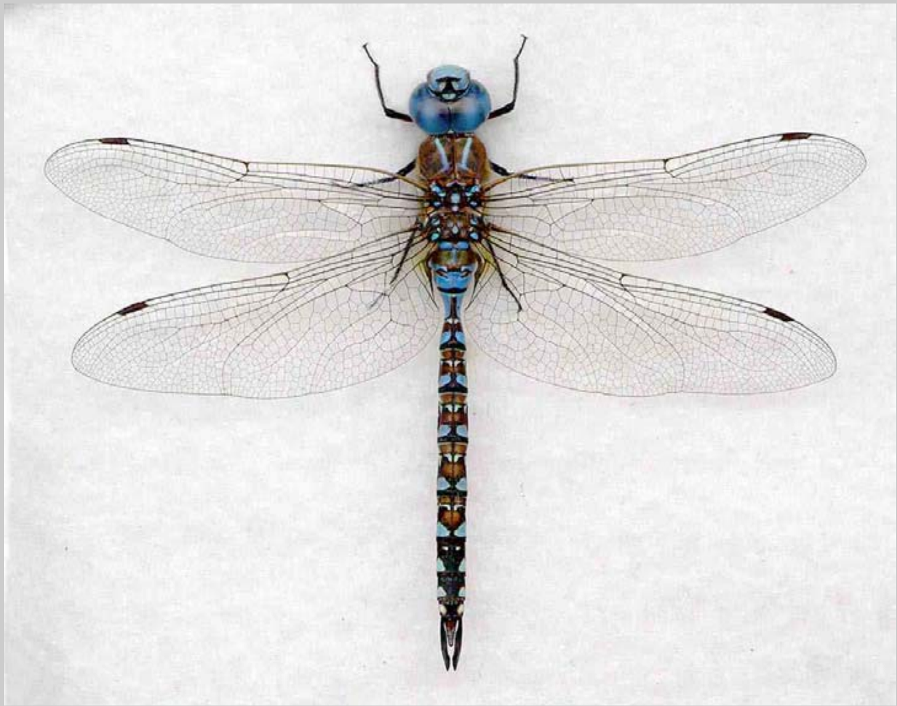


- Increase E of wing vein elements above that of surrounding membrane elements (to mimic increased I of tubular veins)
- Apply a point force to the tip of the wing
- Record computed tip displacement
- Calculate EI of whole model wing

E of model membrane = 1E+9 Nm-2

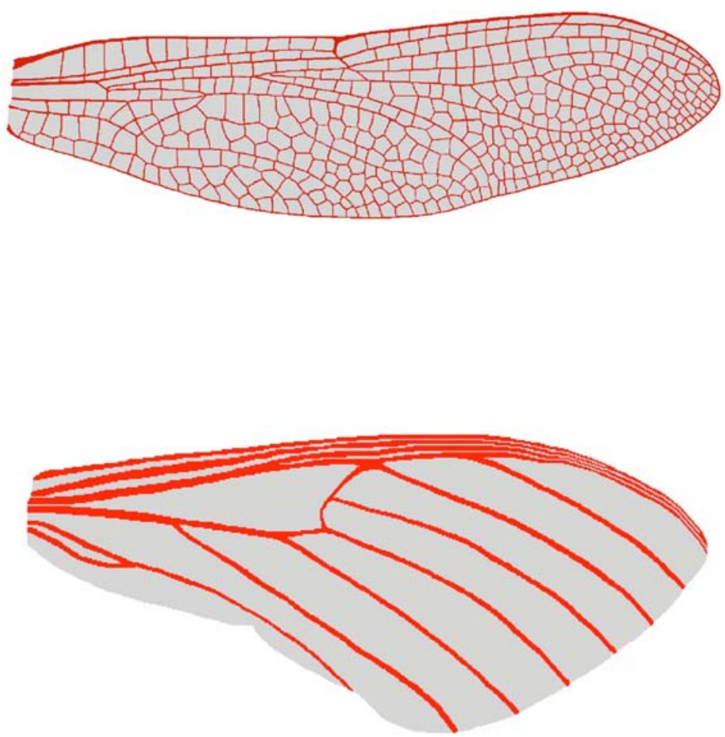
Are insect wings really homogeneous along their length?

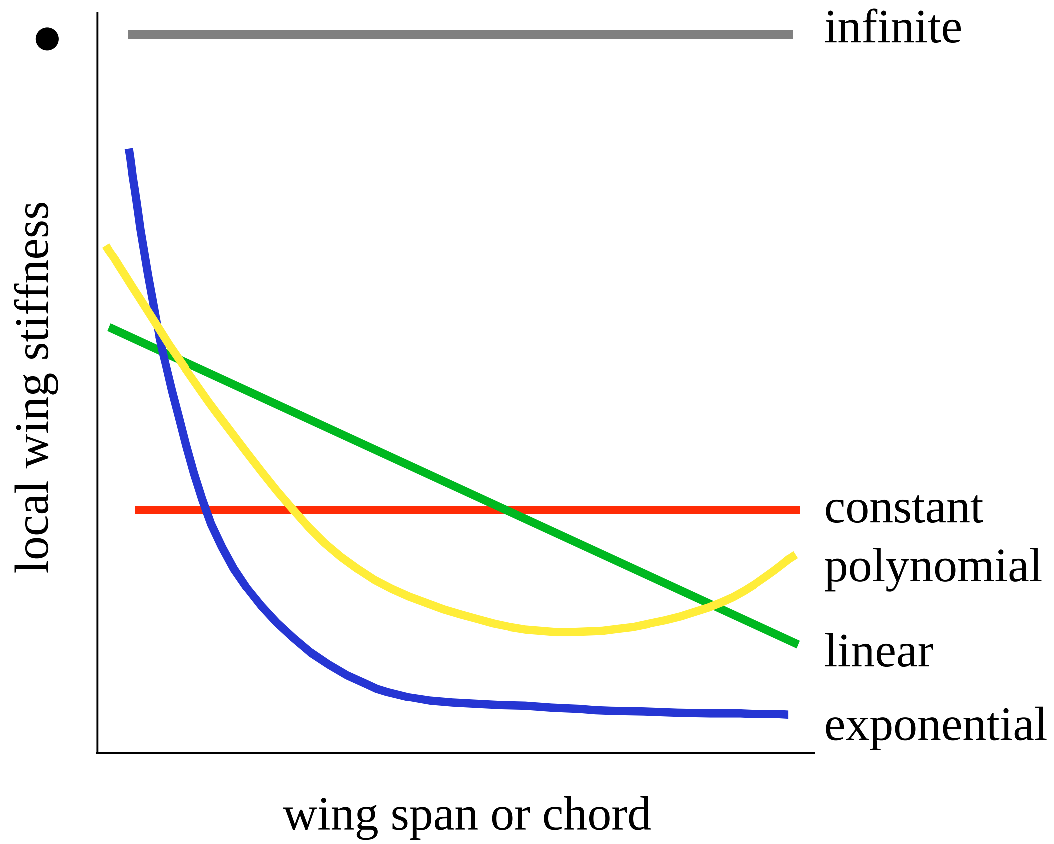
Manduca sexta



Aeshna multicolor

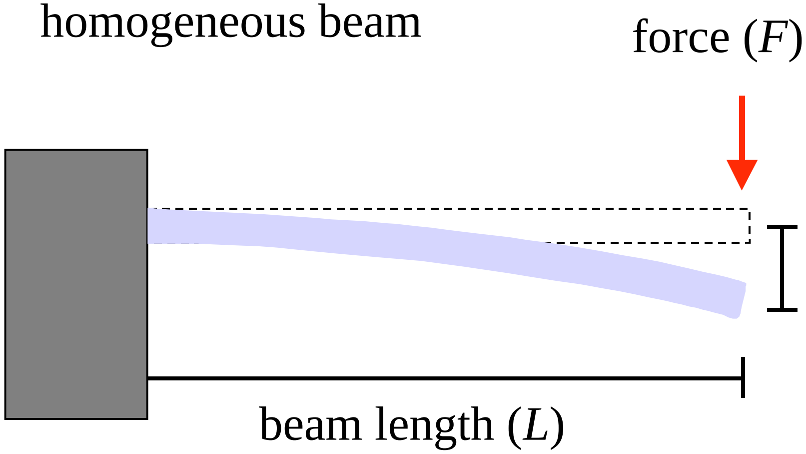
Possible patterns of spatial variation in wing stiffness



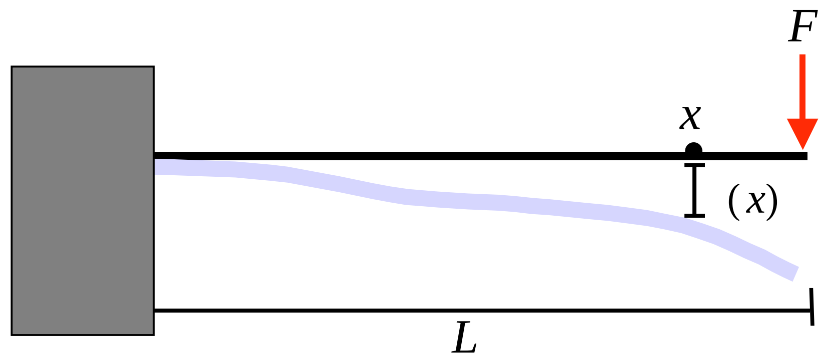
- 

Calculating regional variation in stiffness

homogeneous beam



heterogeneous beam

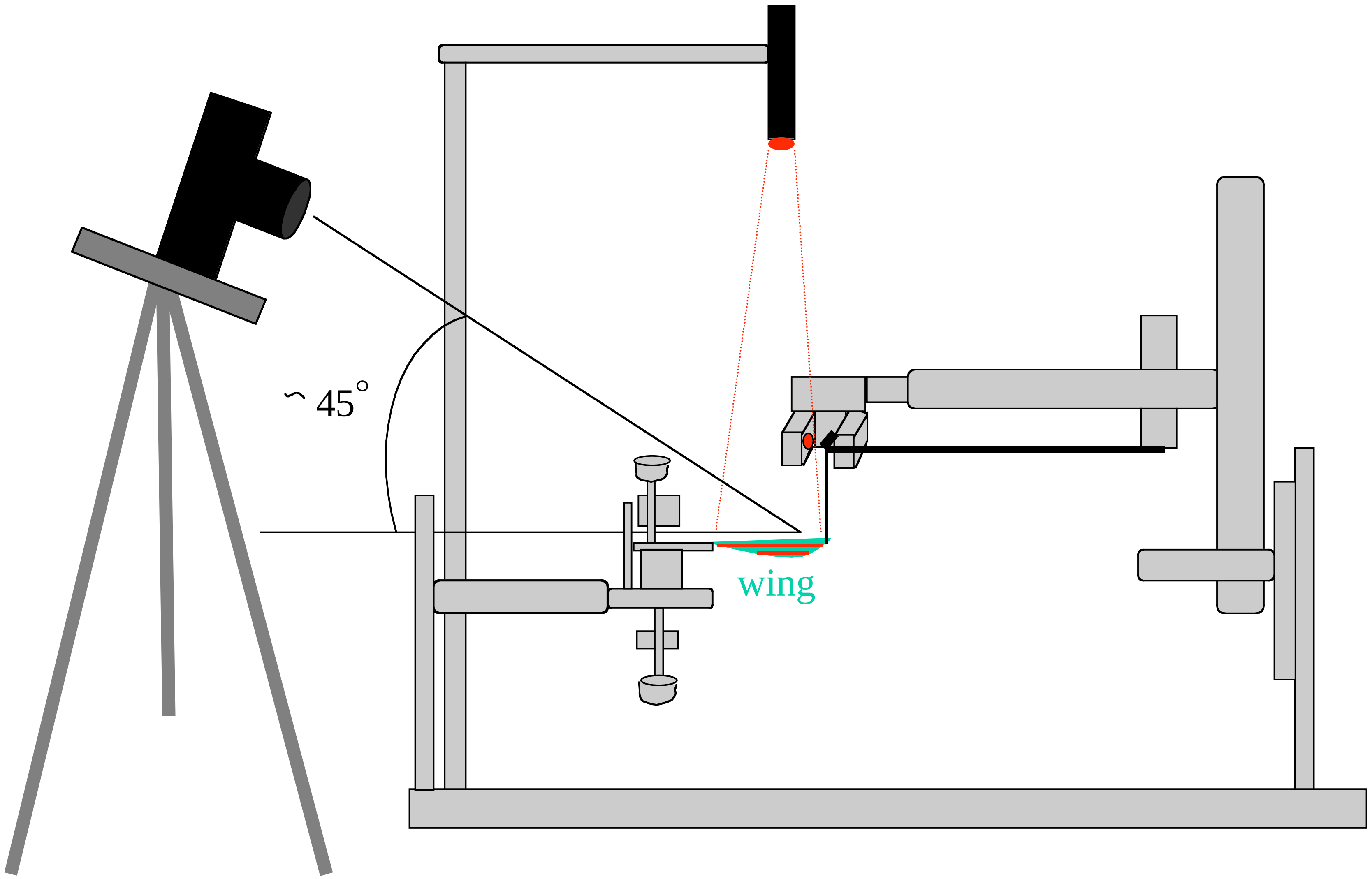


$$EI = \frac{FL^3}{3}$$

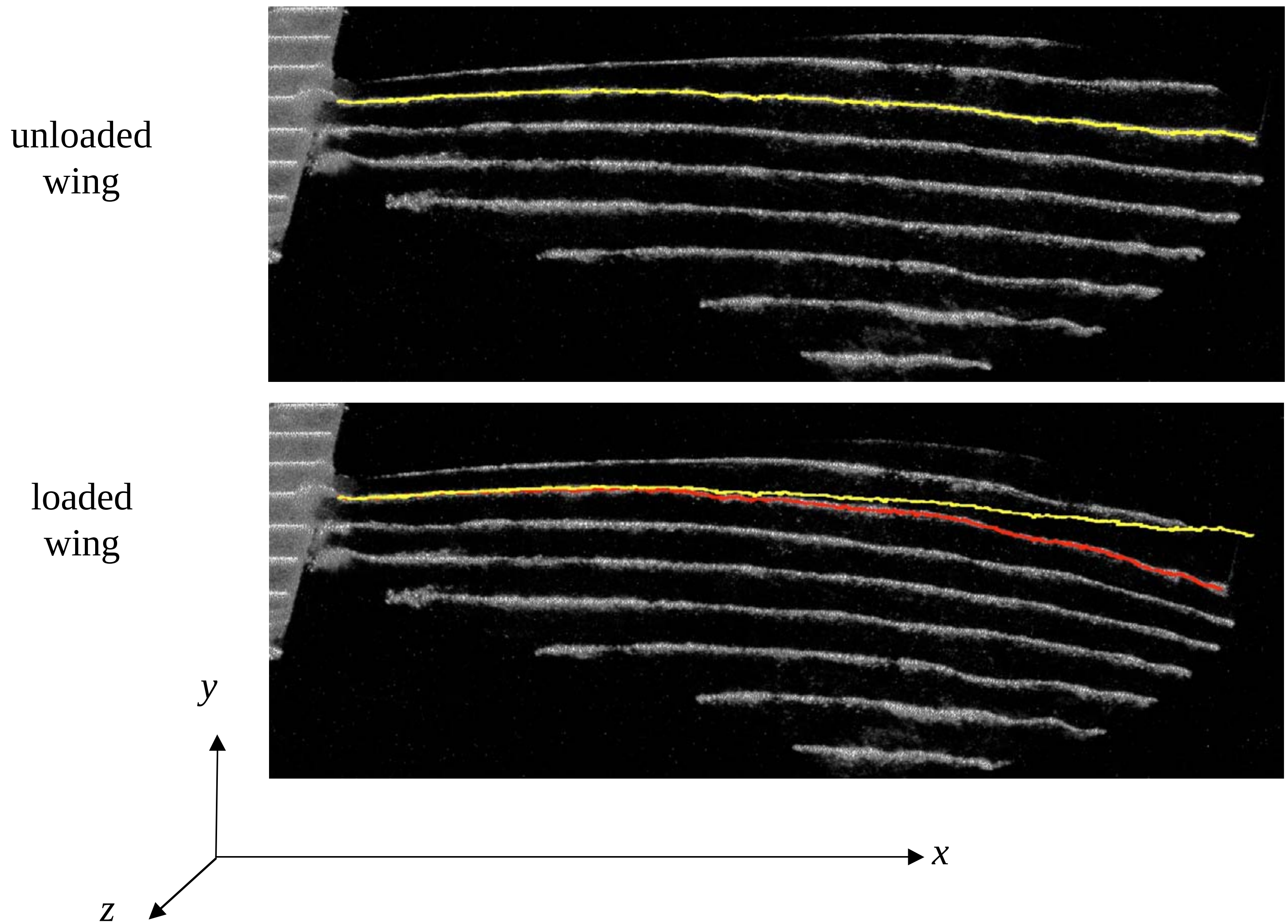
$$EI(x) = \frac{M(x)}{d^2 \quad / \, dx^2}$$

$$\boxed{EI(x) = \frac{F(L \quad x)}{d^2 \quad / \, dx^2}}$$

Measuring displacement continuously along a wing



Changes in the y-direction of the picture correspond to bending in the z-direction of the wing

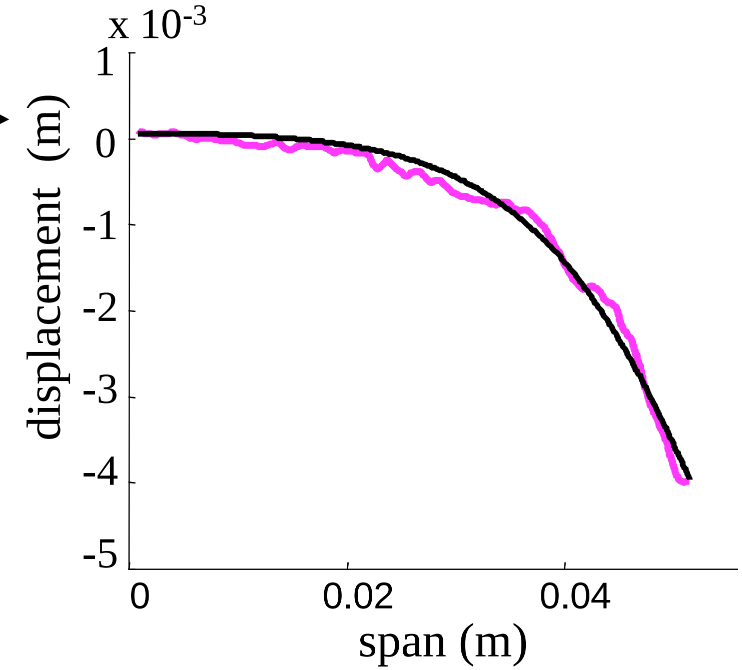


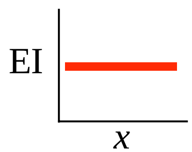
Solve for the EI distribution that best fits the measured wing displacement

$$EI(x) = \frac{F(L \quad x)}{d^2 \quad / dx^2}$$

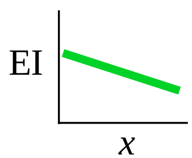
$$(x) = \int\int \frac{F(L \quad x)}{EI(x)} d^2 x$$

simplex minimization

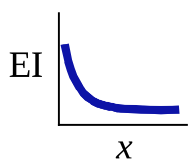




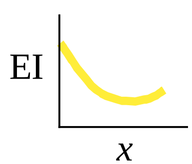
constant [$EI(x) = k$]



linear [$EI(x) = m\,x + b$]

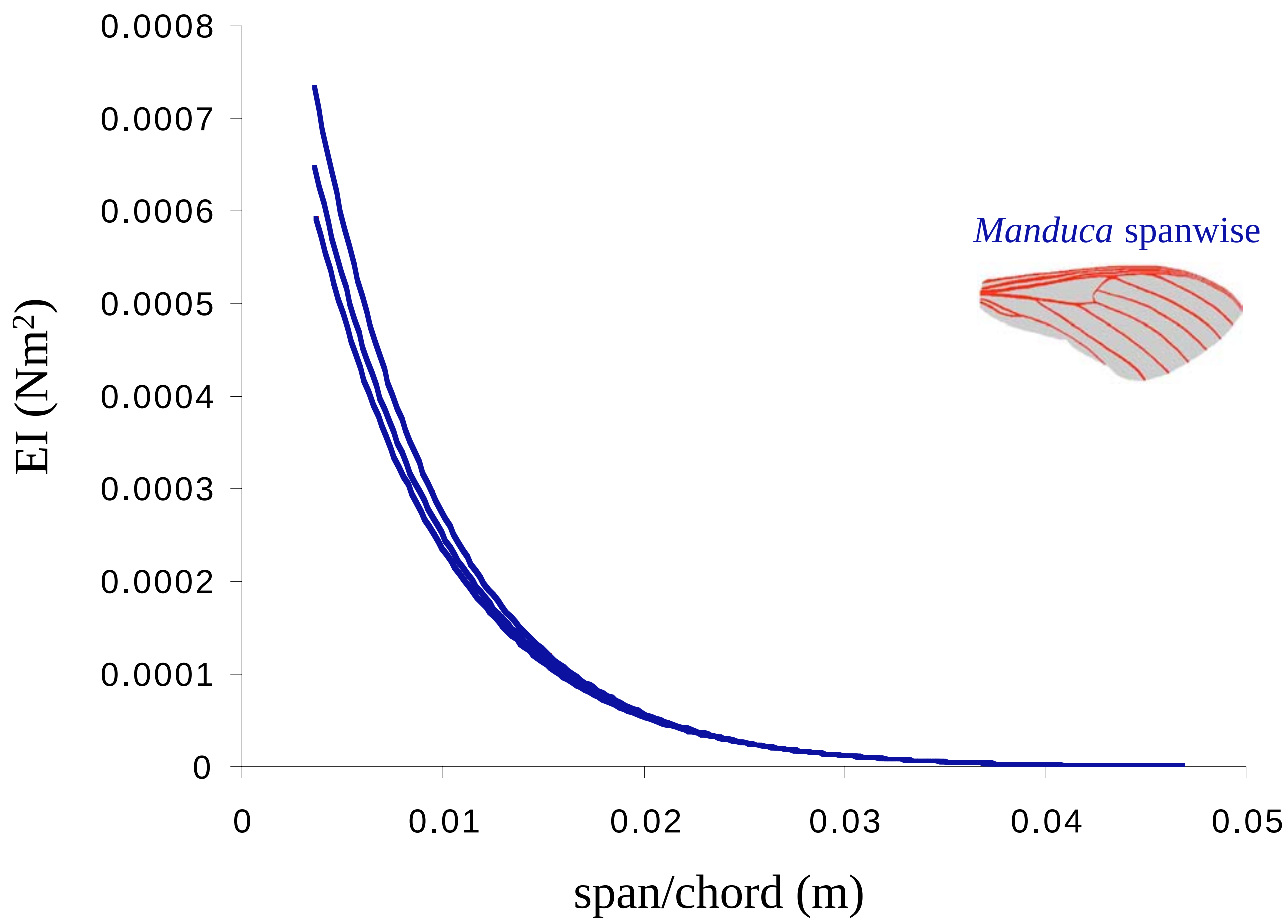


exponential [$EI(x) = c*\exp^{ax}$]

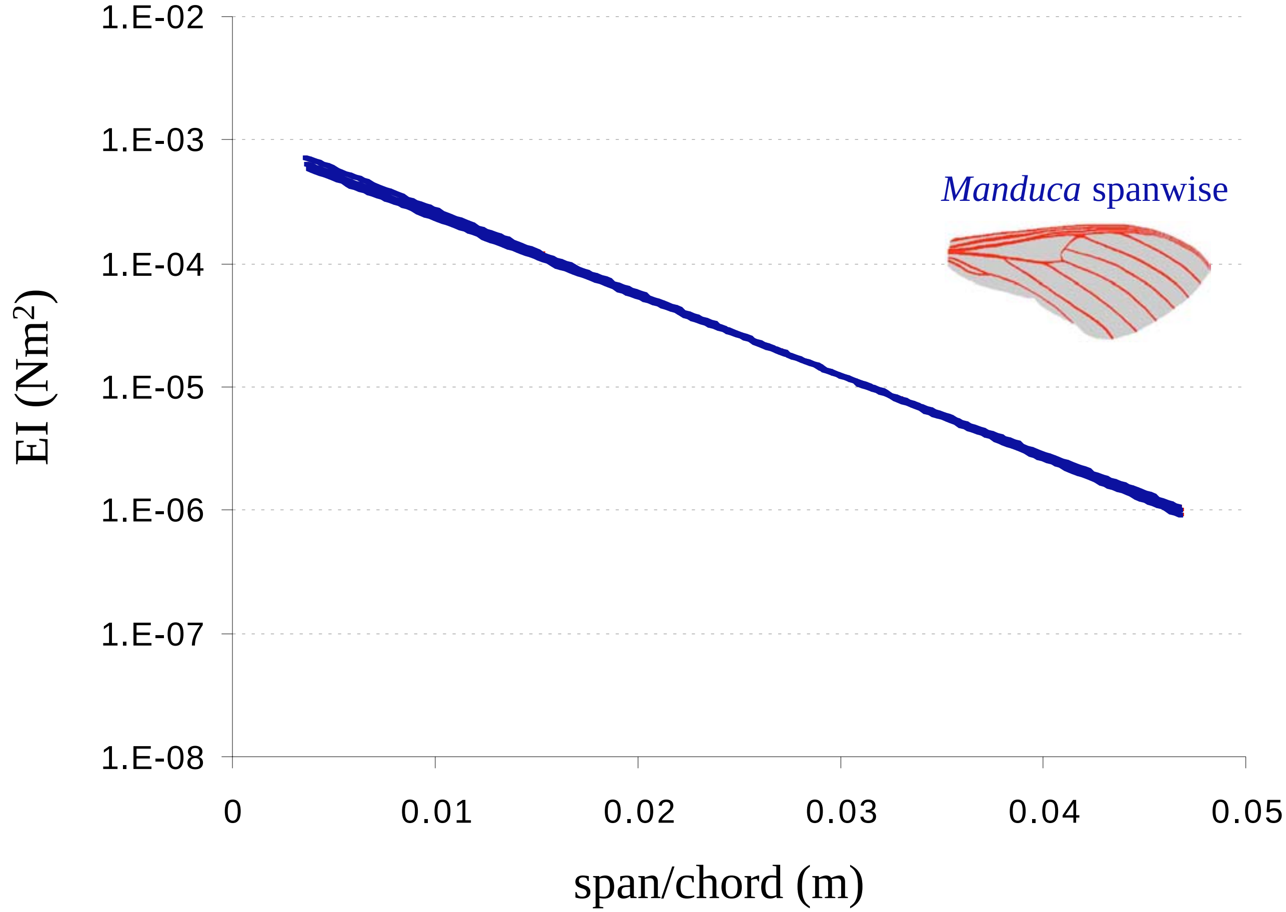


polynomial [$EI(x) = px^2+qx+r$]

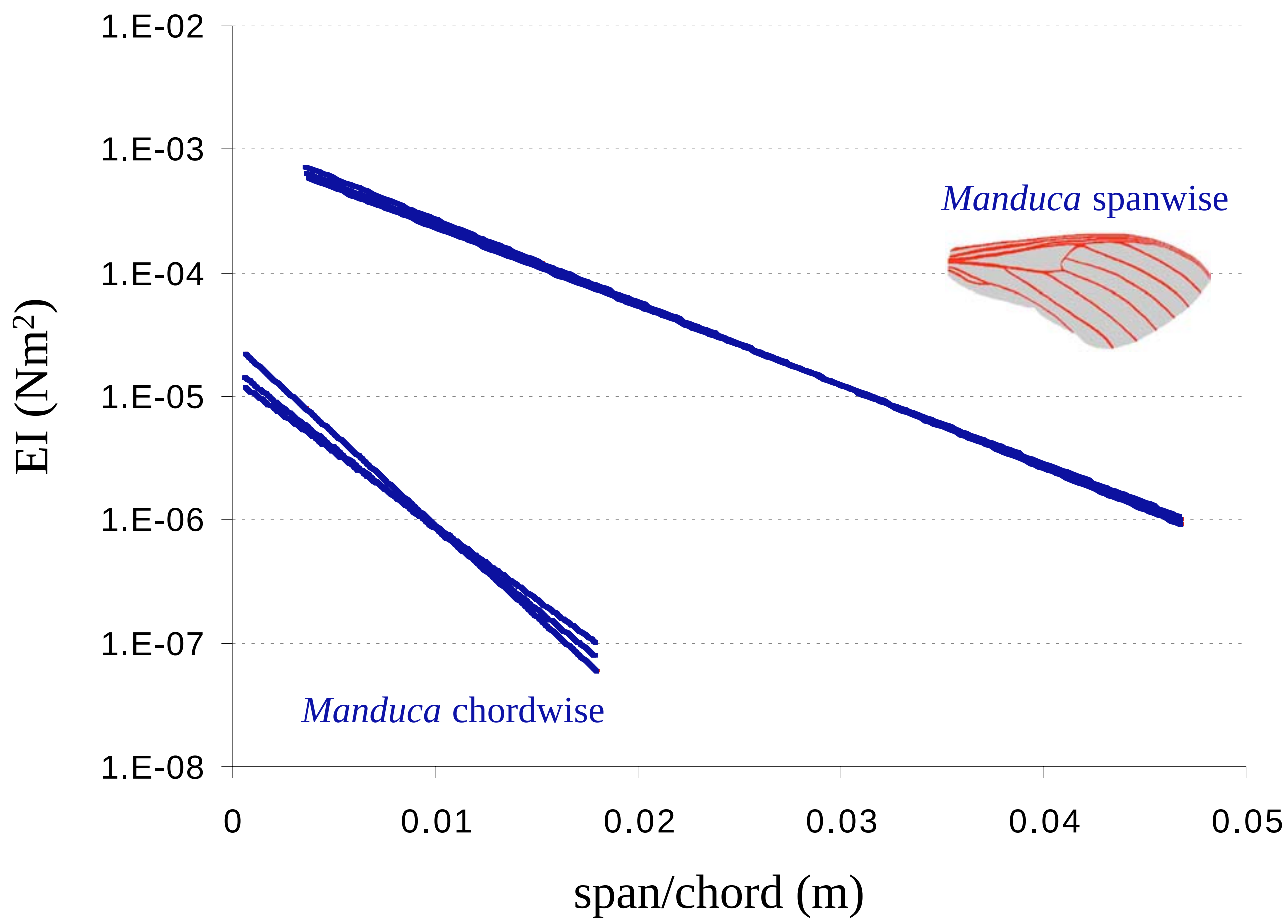
Spanwise wing stiffness declines exponentially in *Manduca*



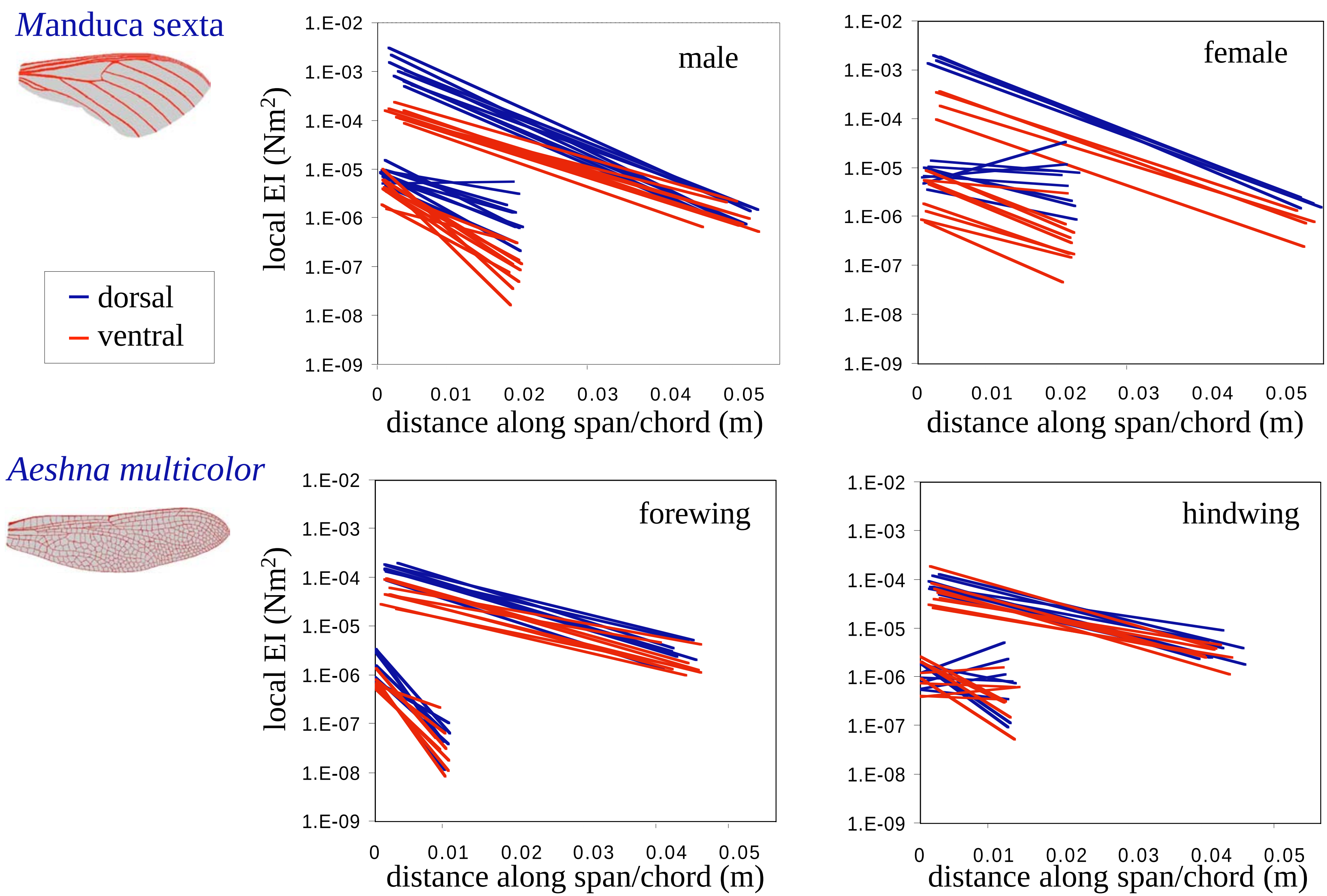
Spanwise wing stiffness declines exponentially in *Manduca*



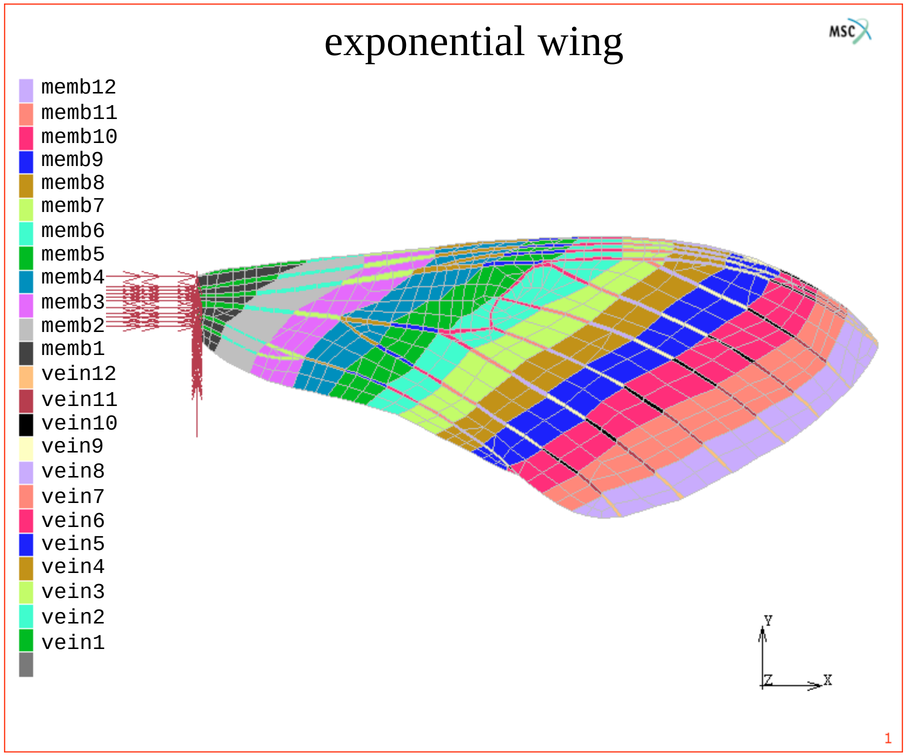
Chordwise wing stiffness decline exponentially in *Manduca*

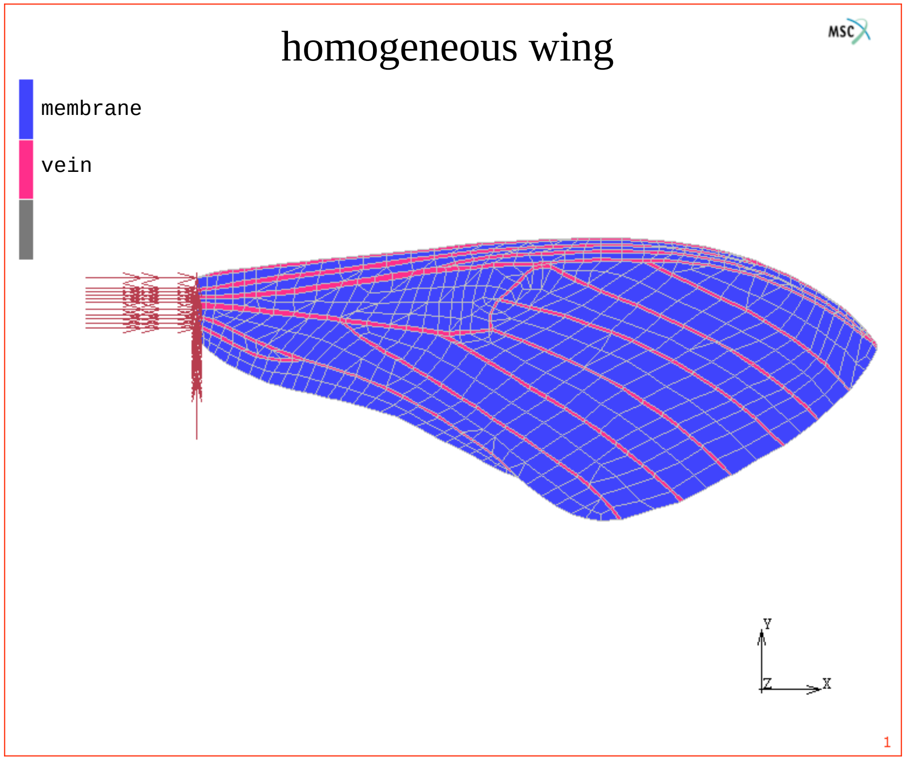


Spanwise and chordwise stiffness decline exponentially in both hawkmoths and dragonflies



How does an exponential decline in stiffness affect wing bending?

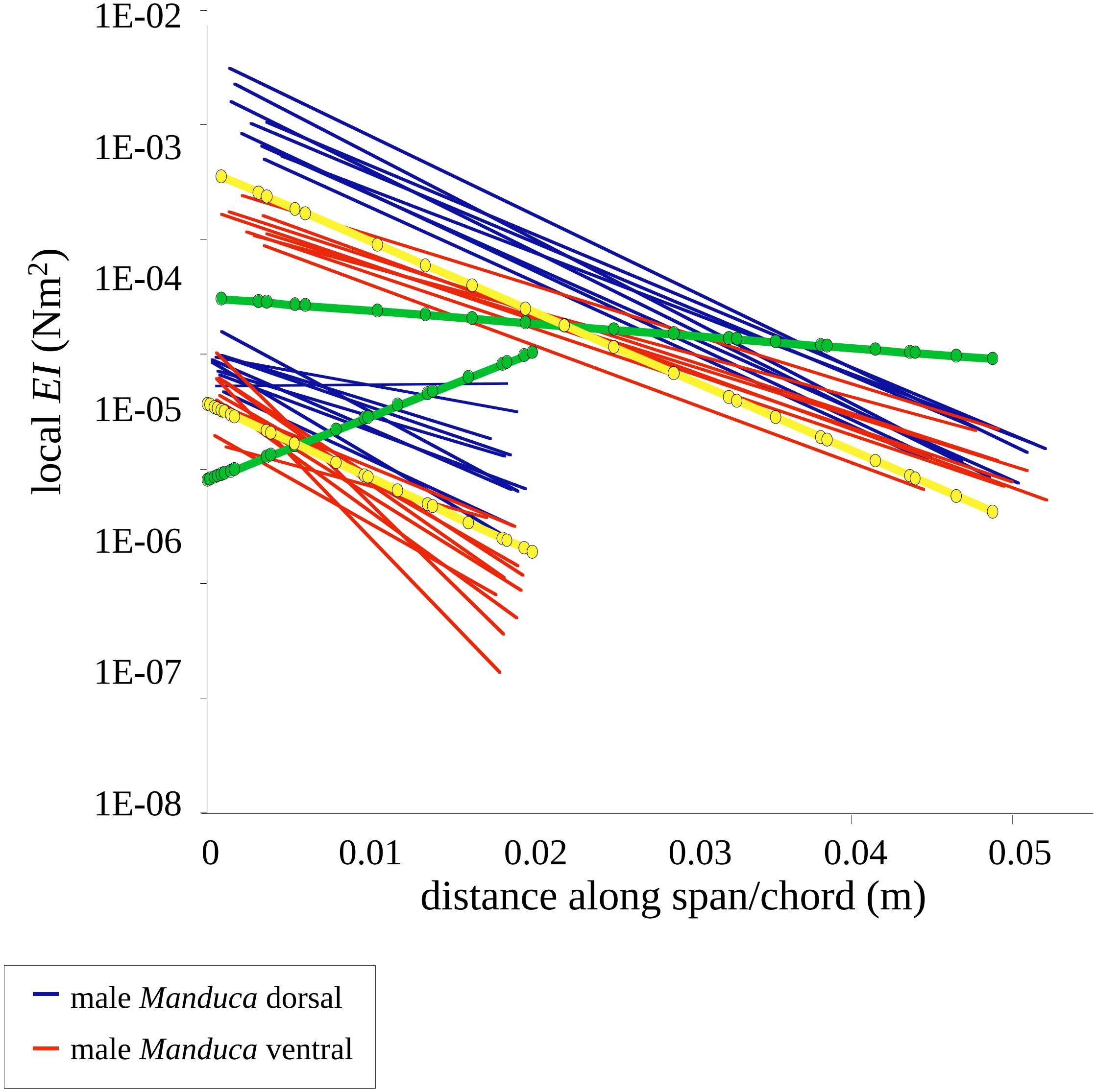


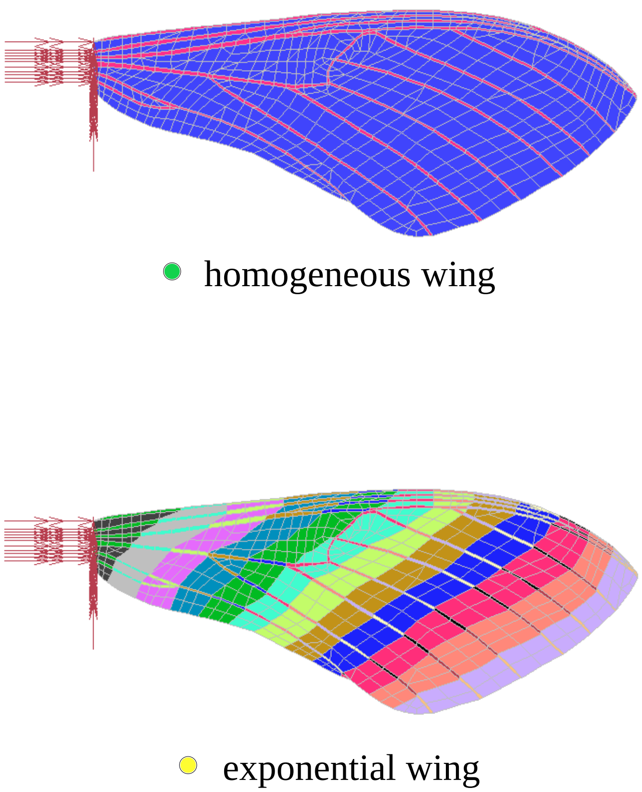


Create a FEM wing in which material properties decline exponentially, but veins are stiffer than membranes to provide anisotropy

*adjust material properties of both models so average tip and trailing edge displacement is the same as in a real *Manduca* wing

How does the spatial pattern of stiffness in model wings compare to real *Manduca* wings?



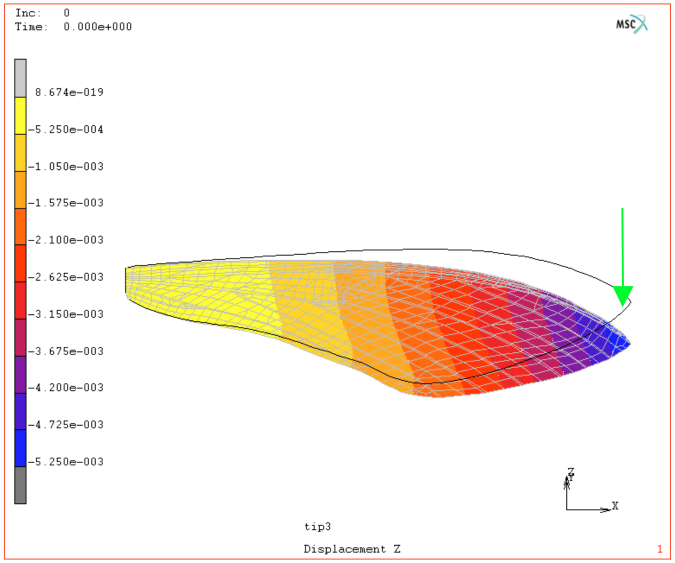


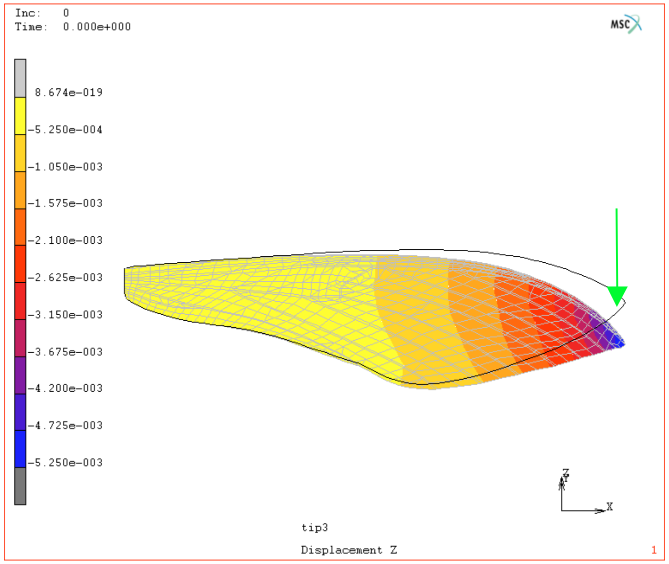
How do the model wings respond to a static load?

homogeneous wing

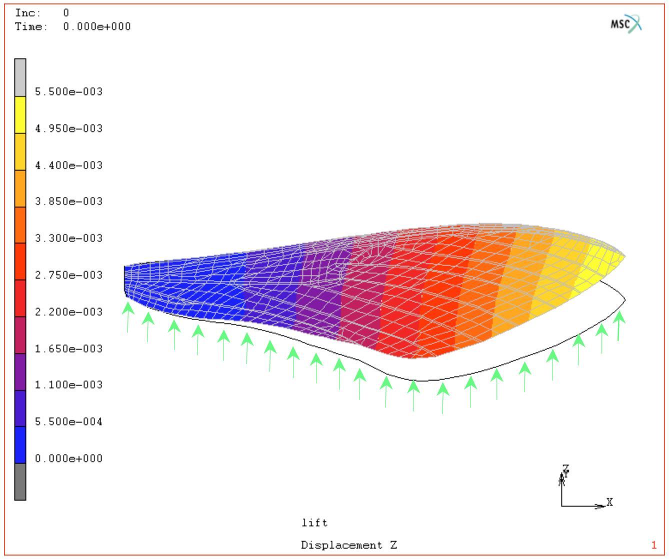
exponential wing

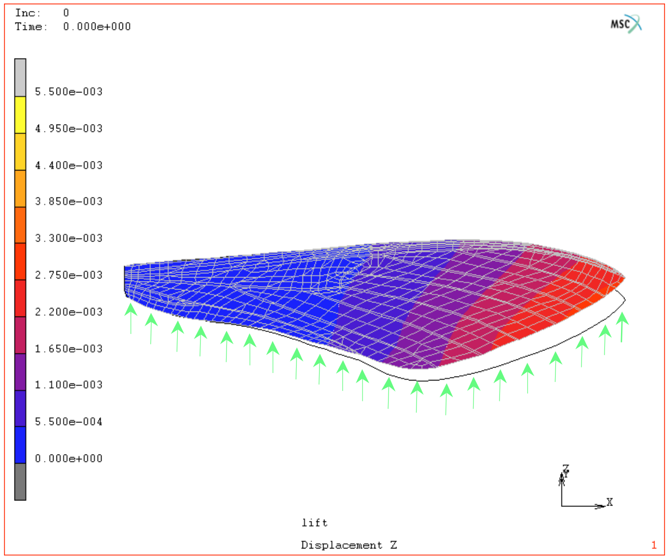
point force
at tip



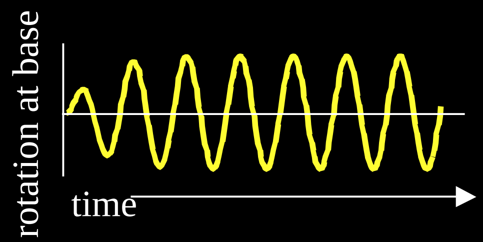


pressure force
on lower surface

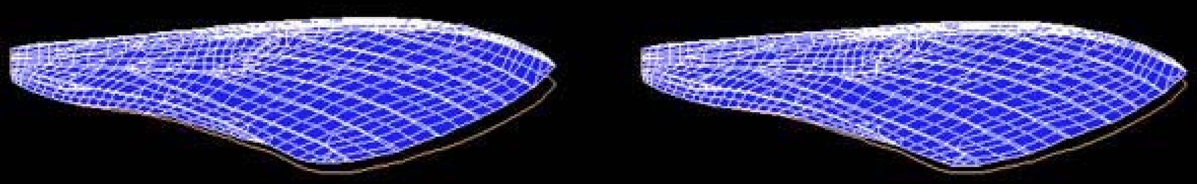




How do the model wings respond to a dynamic load?



An exponential decline in stiffness localizes wing bending to the tip and trailing edge of wings, where force production is most sensitive to changes in wing shape!



Biology 427 Biomechanics
Lecture 13. Finite elements, joints and skeletons

- *Recap flexural stiffness (EI), design for minimum weight, and stress distributions.*
- *Finite Element Analyses in Evolution*
- *Motion is permitted at joints with several degrees of freedom and low EI*
- *Mechanical advantage and speed ratio (those moment (or torque) balances)*
- *Rhinogrades: an enigmatic taxon*

Project proposals: due Friday, February 6

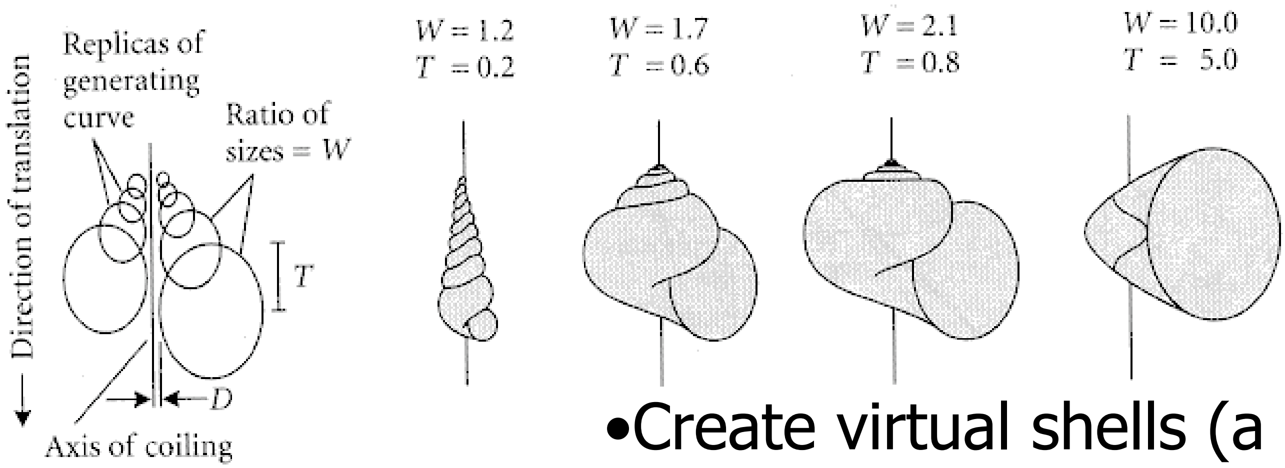
Proposals should be no more than 3 double-spaced pages, and should address the following:

- What is your question?
- Why is your question important/interesting?
- What is known about your question? (give background from literature*/web searches)
- How will you develop a quantitative analysis of your problem? (you do not need to provide any equations in the proposal, but should explain the quantitative approach/steps you will take)

- *Read "Advice for preparing projects" on the webpage!

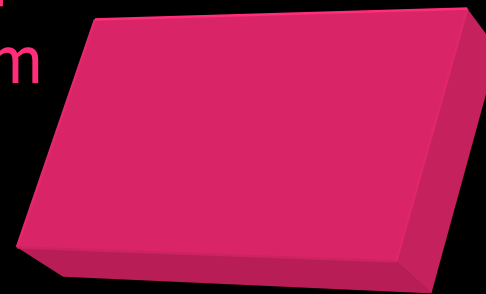
Does curvature reduce the stresses that result from predators?

Numerical experiments on shell shapes

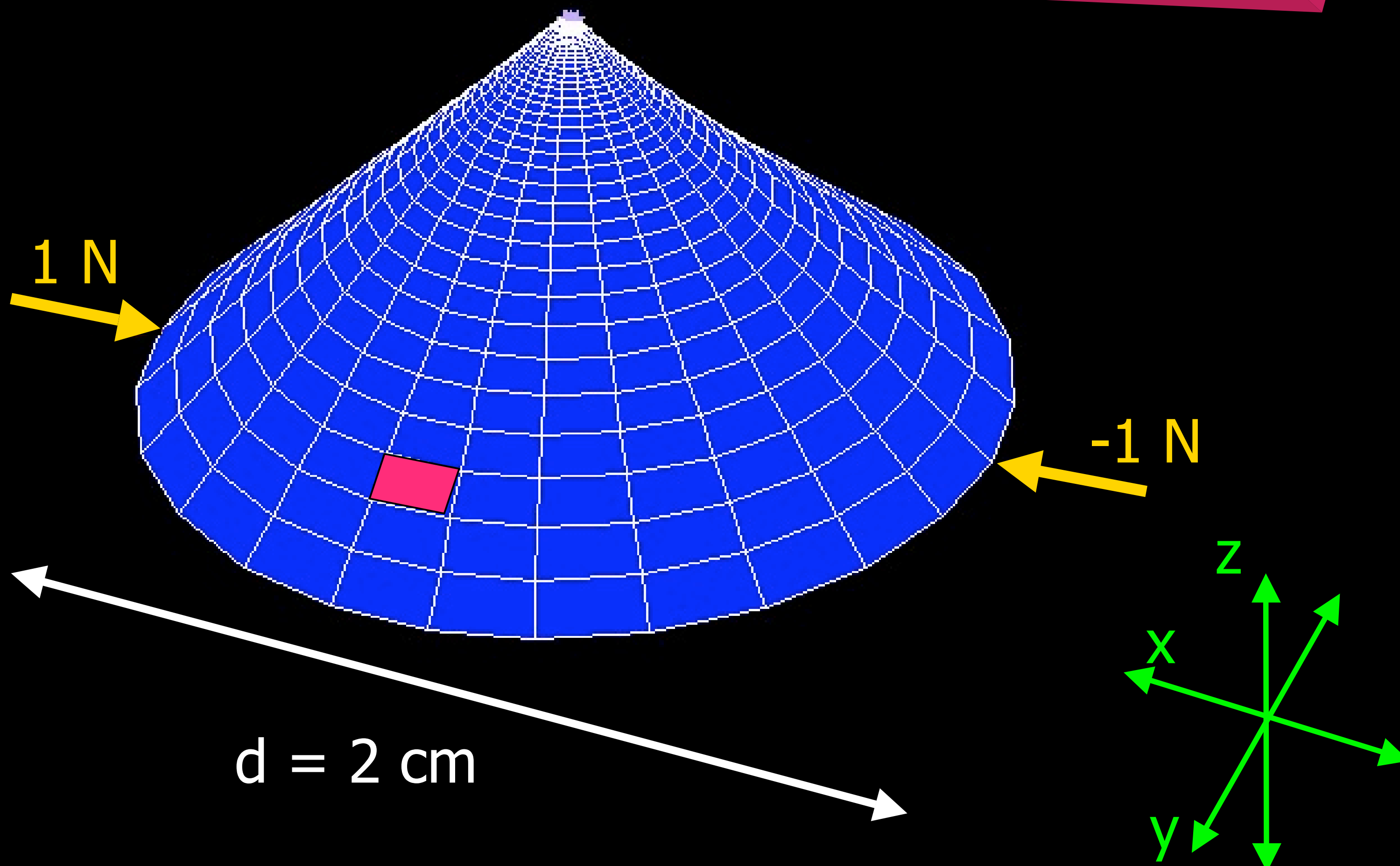


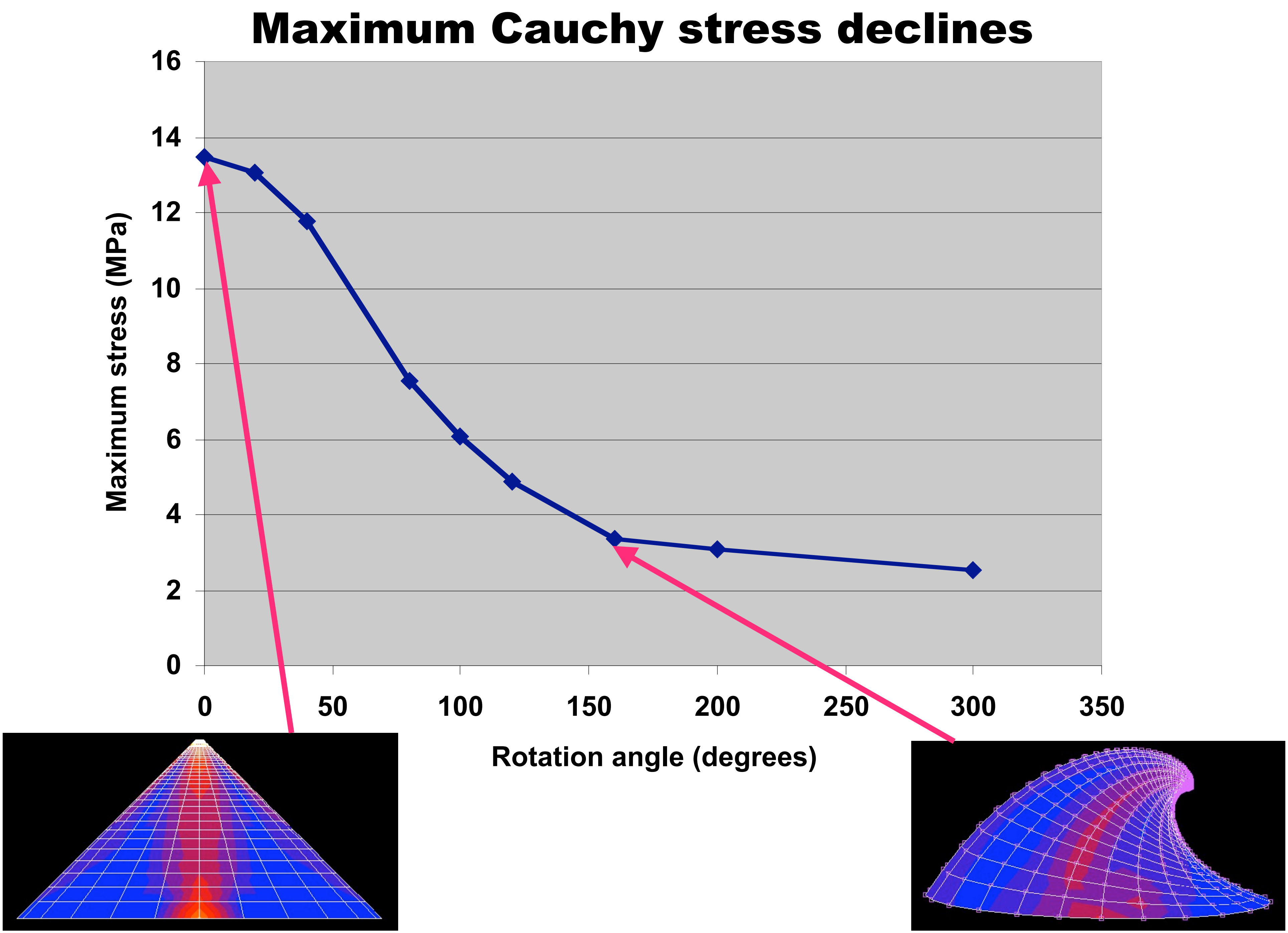
- Create virtual shells (a la Raup)
- Imbue them with mechanical characteristics that represent extant shells (Young’s modulus = 1 GPa)
- Place point forces that mimic crushing predators.
- Examine the stress distribution.
- Explore how coiling affects stress distribution.

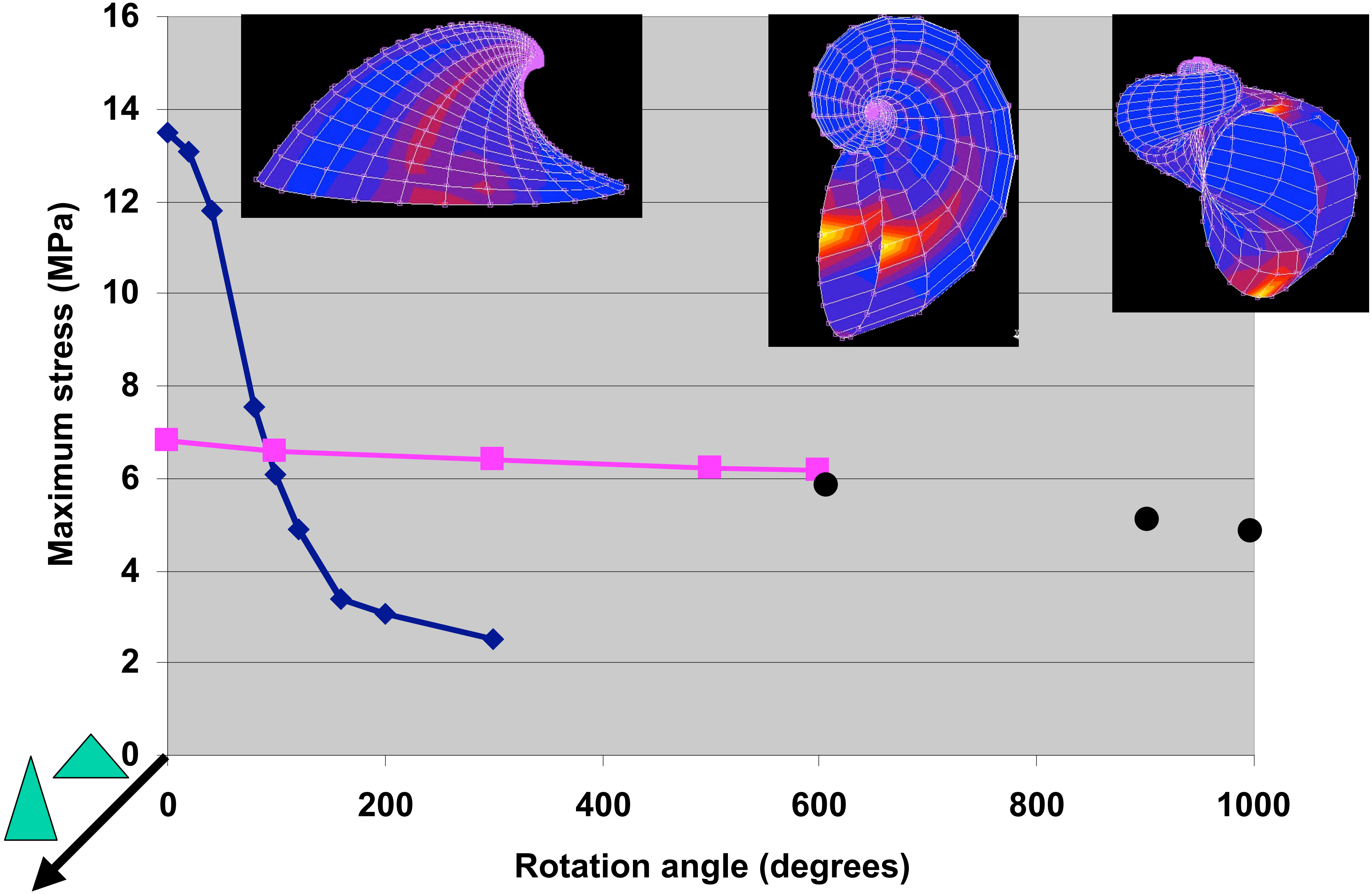
Finite Element Analysis



How do size and shape affect the magnitude and distribution of stress?




Maximum Cauchy stress declines
Maximum stress (MPa)
0
2
4
6
8
10
12
14
16
0
50
100
150
200
250
300
350
Rotation angle (degrees)



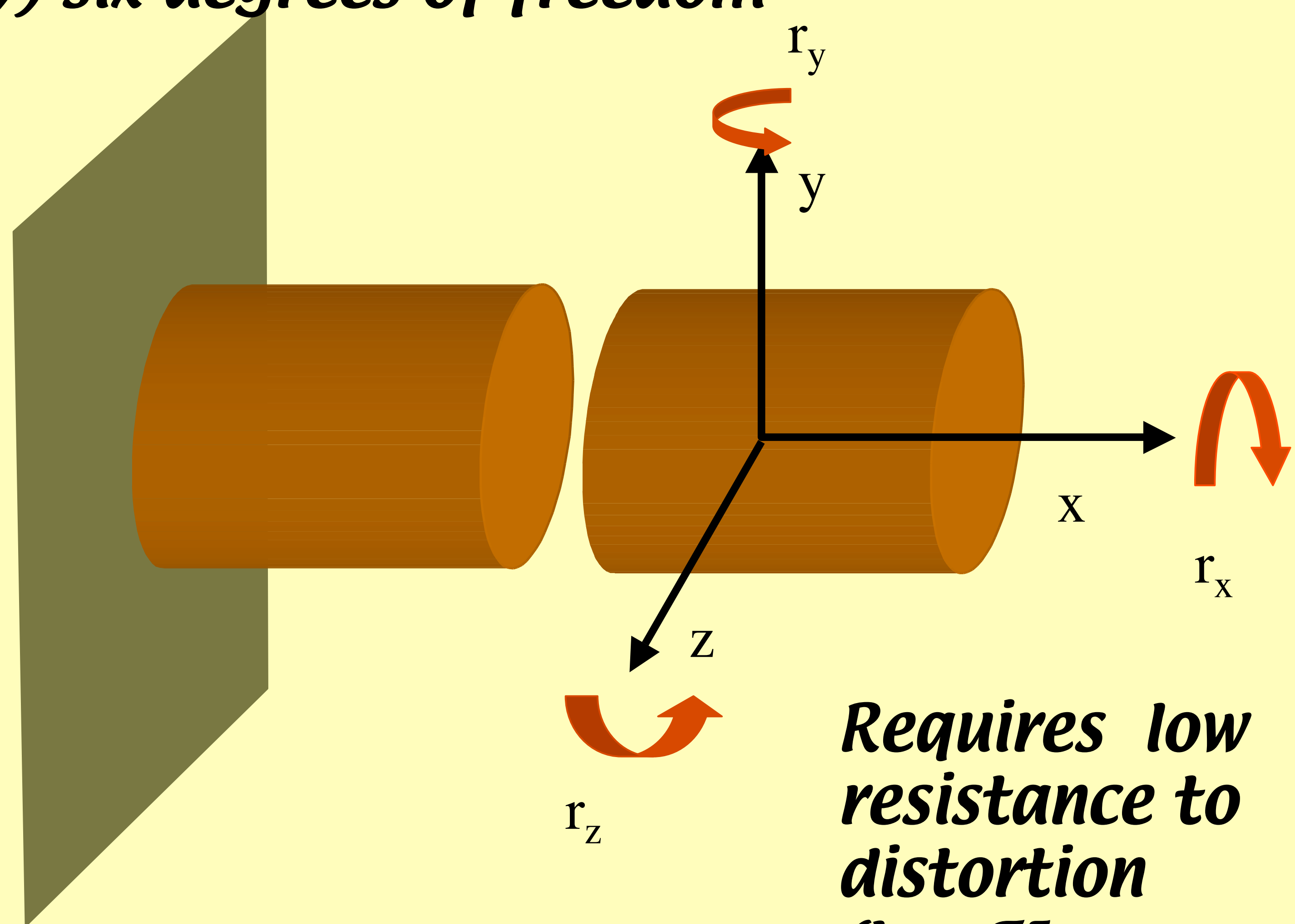
Maximum stress (MPa)
16
14
12
10
8
6
4
2
0
0
200
400
600
800
1000
Rotation angle (degrees)

Brief review of skeletons

- *Mechanical support*
- *Protection*
- *Force transmission*
- *Energy storage*

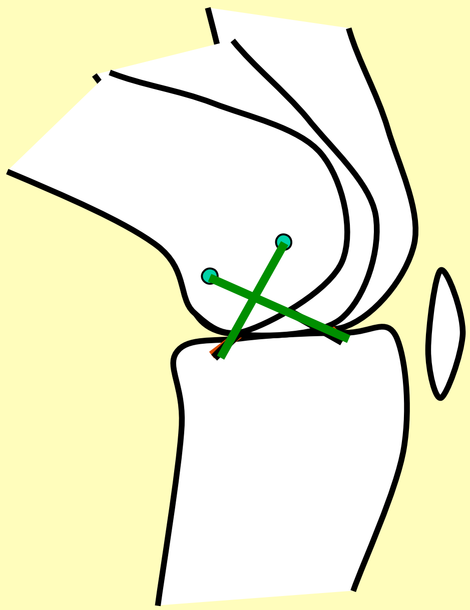
- *Rigid and flexible elements: Endo- and exoskeletons*
- *Fluid filled cavities: hydraulic skeletons*
- *Solid muscle: muscular hydrostats*
- *Protein filaments: cytoskeleton*

Rigid elements connected by joints with (potentially) six degrees of freedom



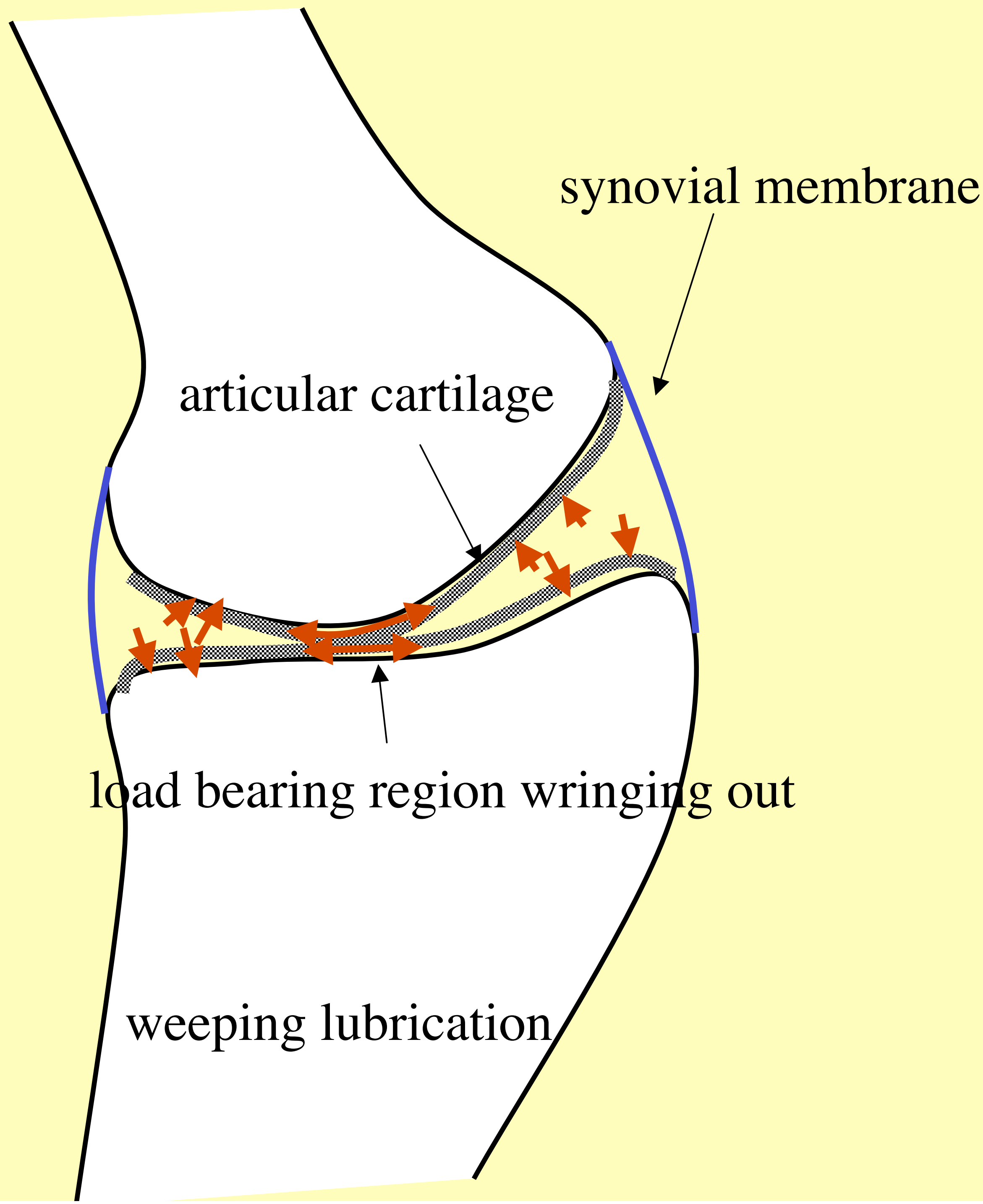
Requires low resistance to distortion (low EI, lubrication)

Commonly 1 - 3 degrees/joint.
Multiple single degree joints give



cruciate ligaments
constrain knee motions

coefficient of friction is
about 0.003
good ball bearings
~0.02



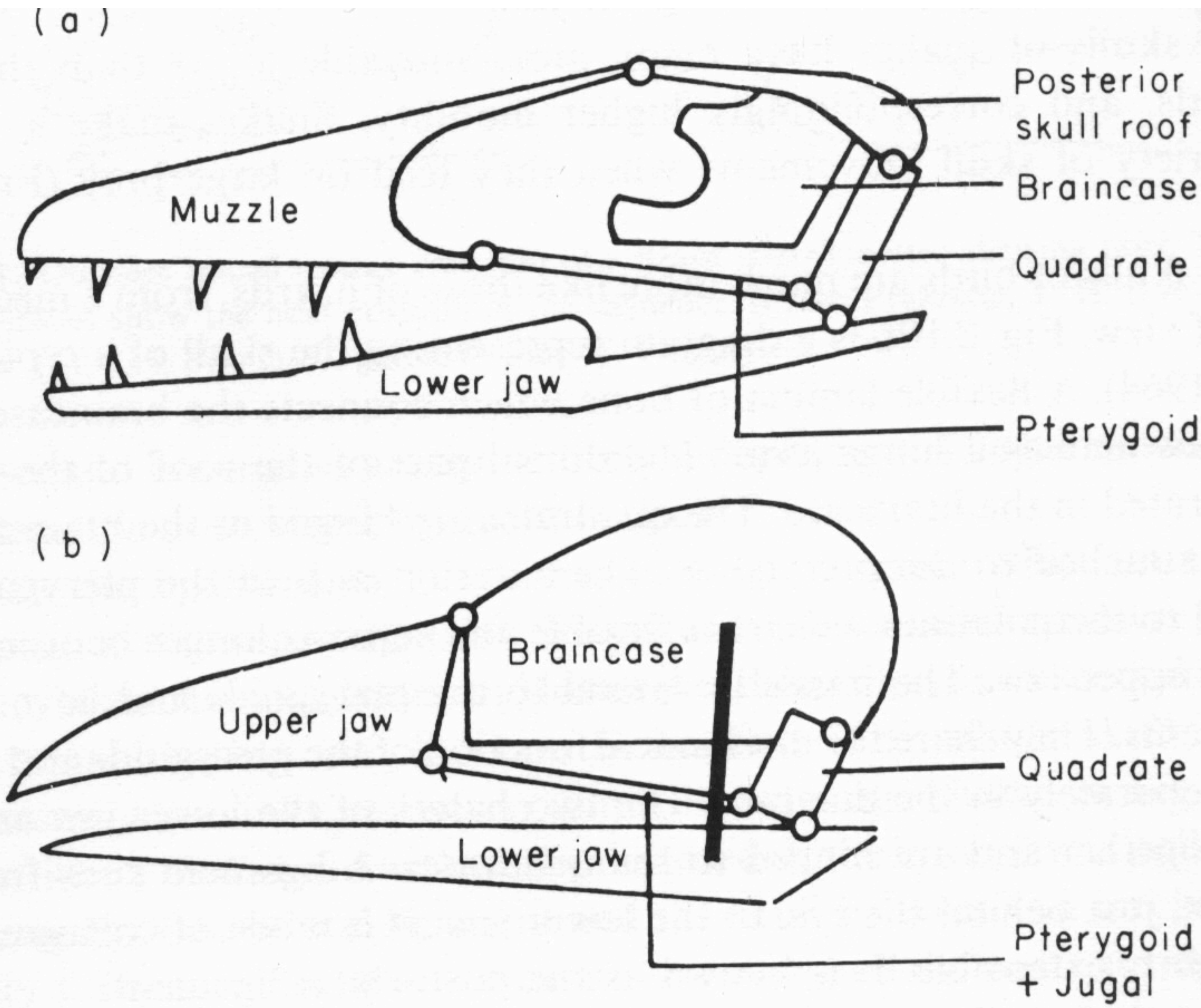
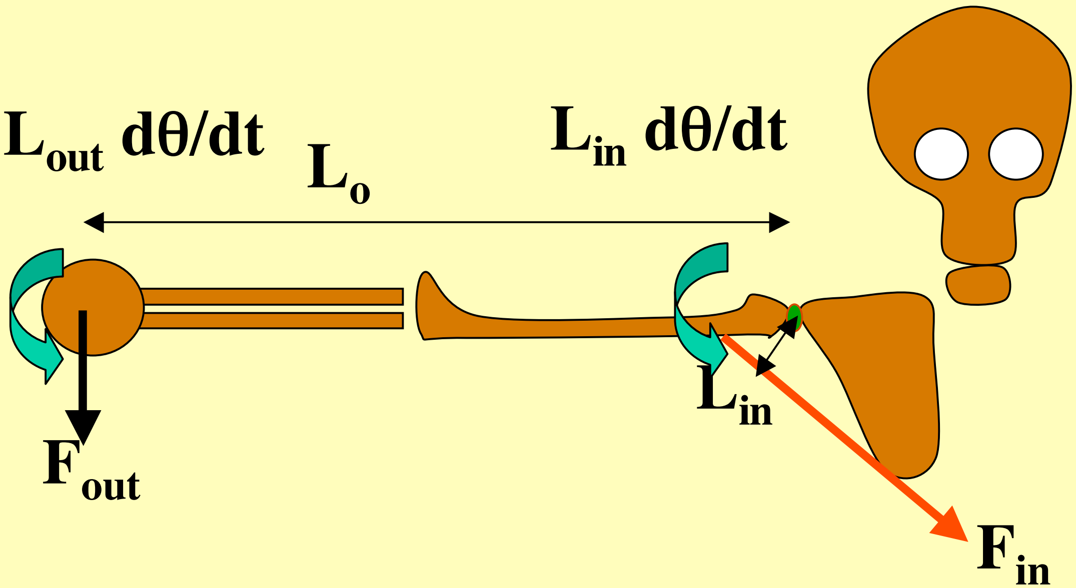


Fig. 2.10. The mechanisms of the kinetic skulls of (a) a monitor lizard (*Varanus*) and (b) a bird. The postorbital ligament of the bird is shown black.



At static equilibrium

$M_{in} = M_{out}$

$F_{in}\ L_{in} = F_{out}\ L_{out}$

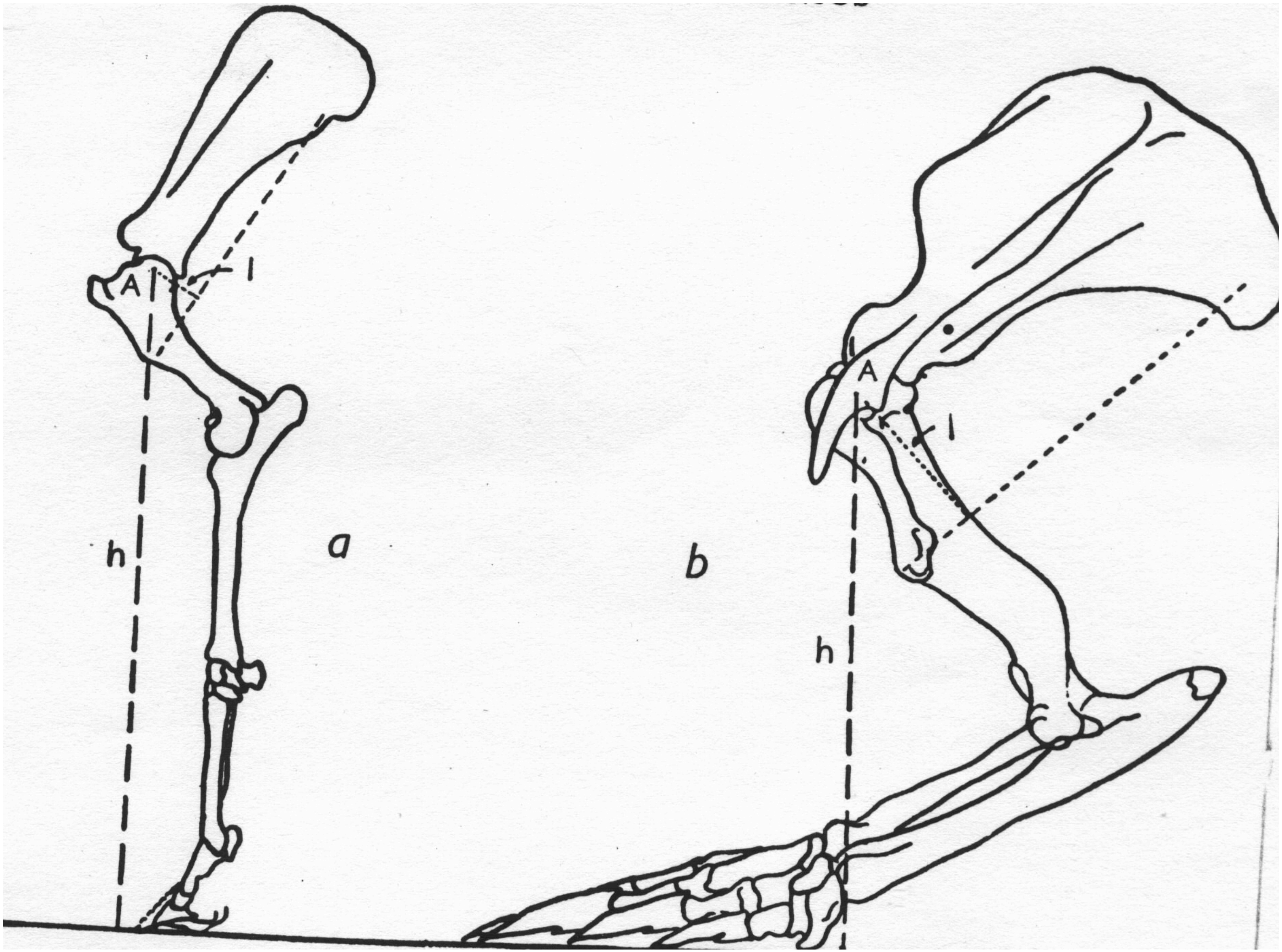
$F_{out}/F_{in} = L_{in}/L_{out}$

= ***Mechanical Advantage***

The speed ratio

$$\frac{L_{out}\ d\theta/dt}{L_{in}\ d\theta/dt}$$

$= L_{out}/L_{in}$



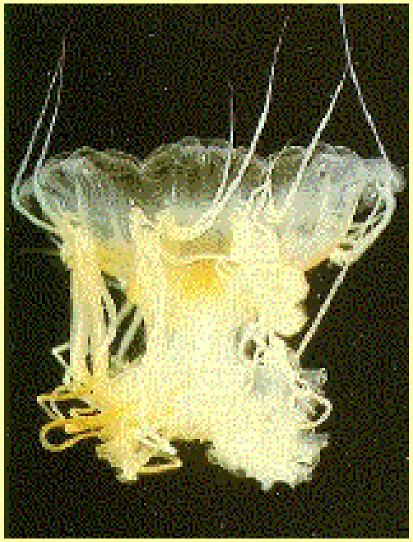
A
l
h
a
b
A
l
h

Biology 427 Biomechanics
Lecture 14. Structural systems and adhesives

- Recap skeletal systems, joints, mechanical advantage, speed ratio
- Worms, tongues and tentacles: Hydrostats and muscular hydrostats for support and movement
- Staying put: Mechanisms of adhesion

Brief review of skeletons
- Mechanical support
- Protection
- Force transmission
- Energy storage

- Rigid and flexible elements: Endo- and exoskeletons
- Fluid filled cavities: hydraulic skeletons
- Solid muscle: muscular hydrostats



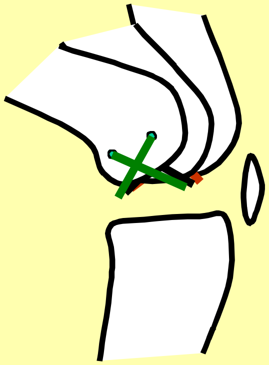
Functions of skeletal systems

- Mechanical support
- Protection
- Force transmission
- Energy storage

antagonistic muscles/materials are needed to return elements to original position after a motion

Elements of skeletal systems resist compression, tension or both

struts - can take both tension and compression (i.e. bones)



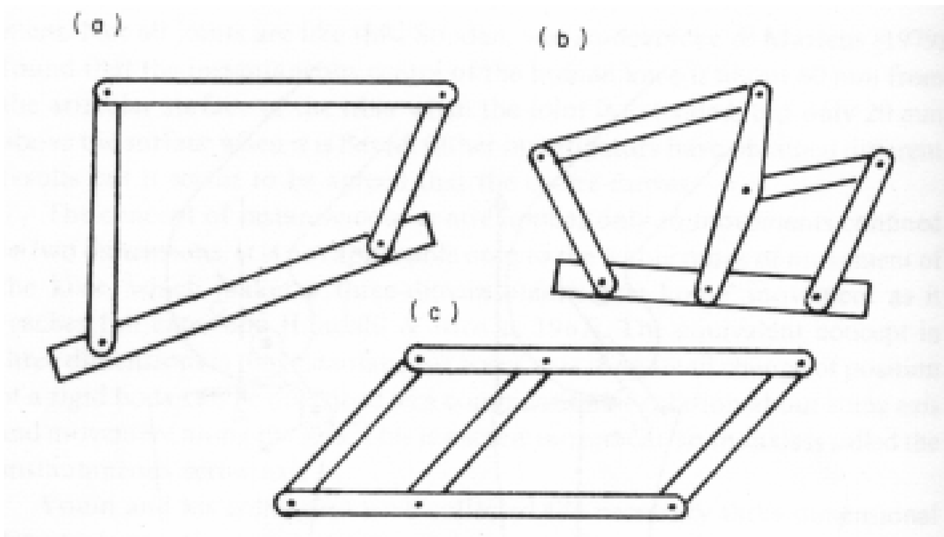
ties - resist tension only (i.e. tendons)

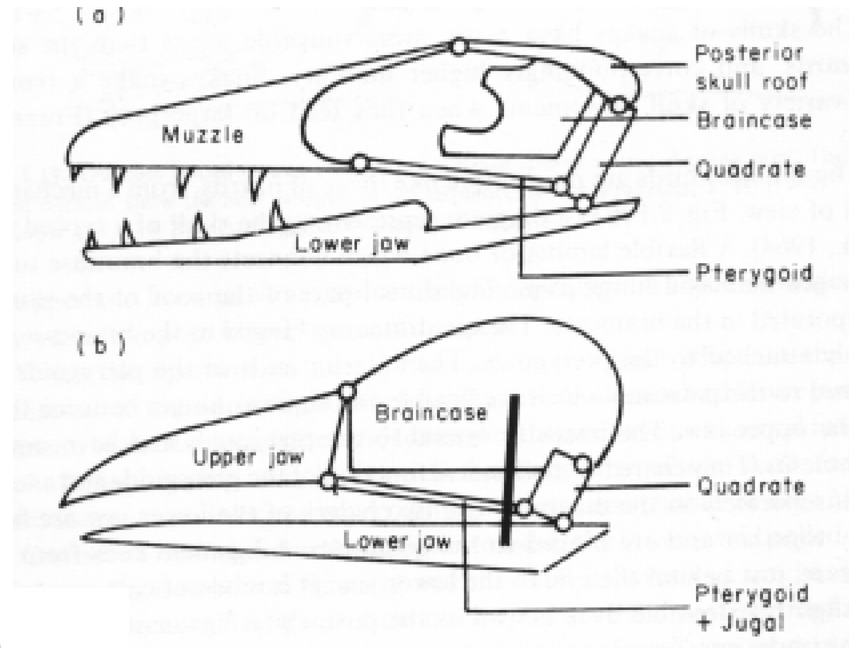
incompressible elements (i.e. water-filled cavities or muscle)

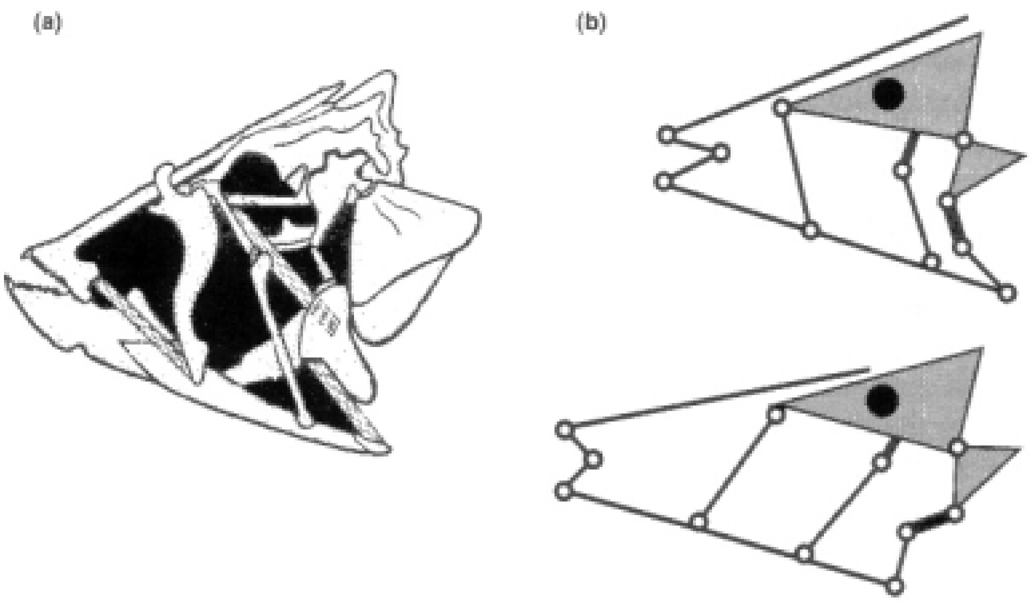
Categorizing supportive systems

1. *Tensile systems - built to resist tension only (i.e. algal stipes, fruit stems, toe-pad setae)*
2. *Strutted systems*
   - *single or branched struts (i.e. tree branches, coral)*
   - *articulated struts (i.e. vertebrate skeletons, insect exoskeletons)*

Articulated struts can have multiple linkages



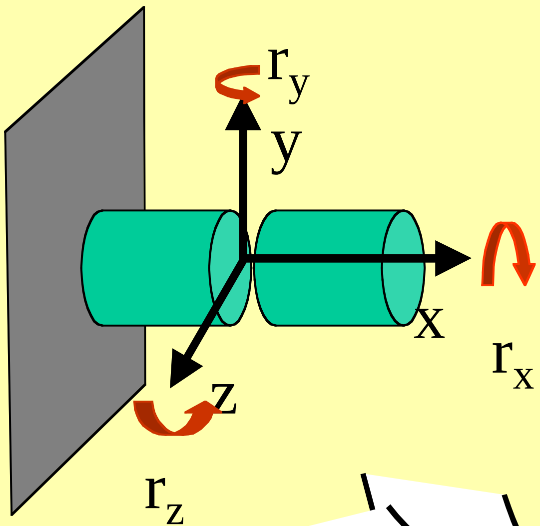




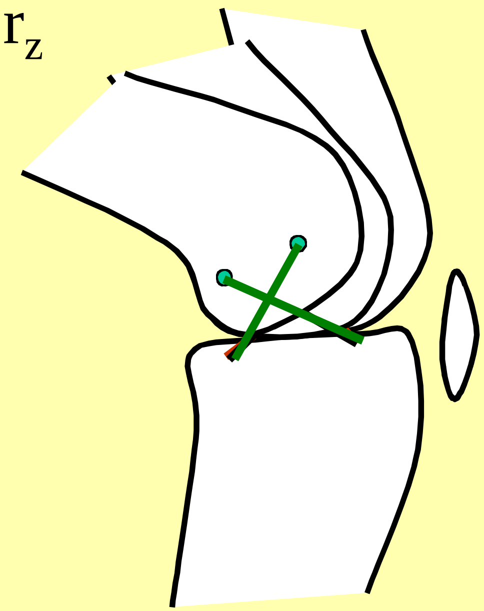
<u>*slingjaw wrasse*</u>

- 11 jaw linkages
- protrusion to 65% head length in 1/30 s
- acceleration = 100 m/s^2
- snout speed = 5 mph

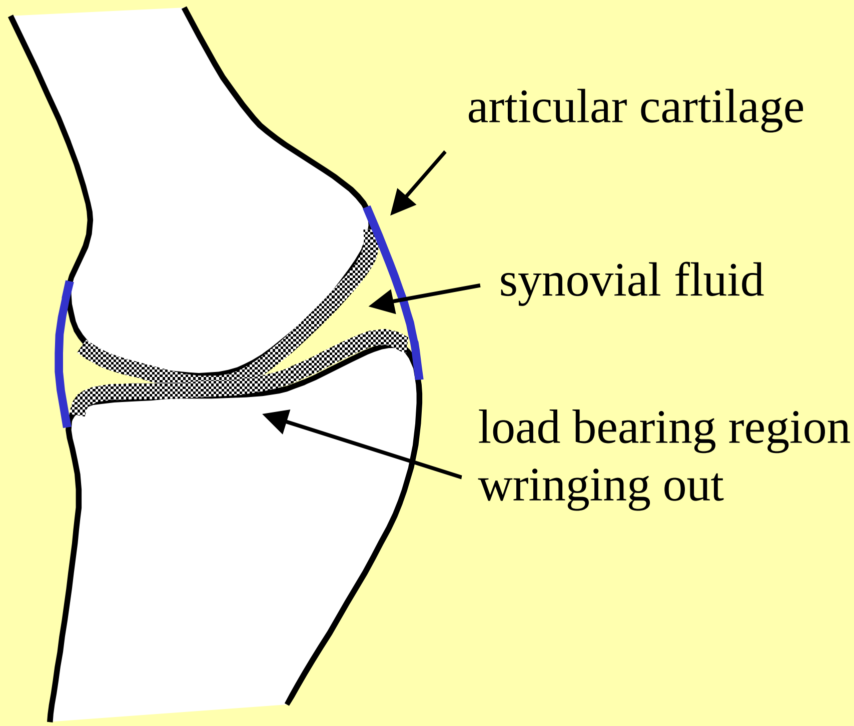
Most biological joints contain 1-3 rotational degrees of freedom



Movement at joints requires low resistance to distortion (lubrication, low EI)

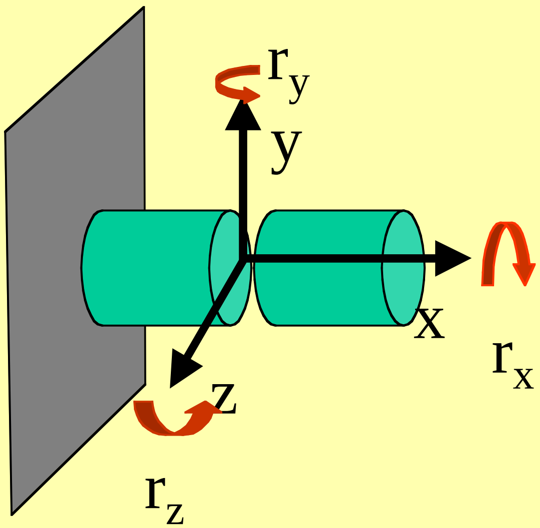


Human knee: cruciate ligaments constrain knee motions



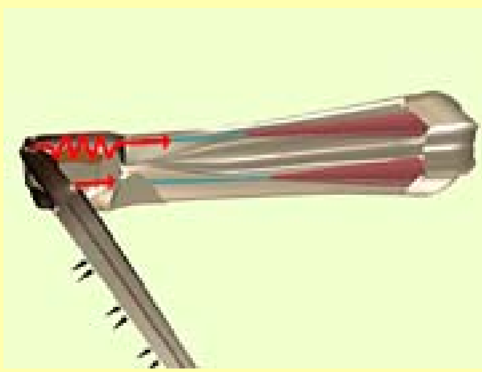
WEEPING LUBRICATION

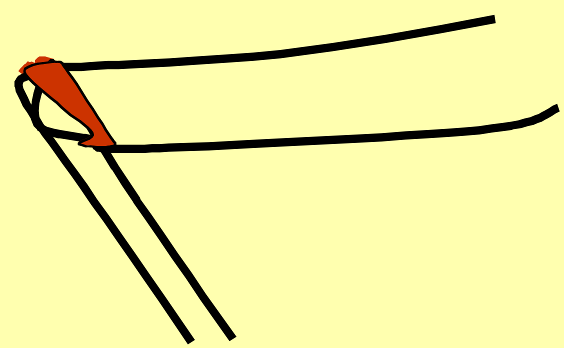
Most biological joints contain 1-3 rotational degrees of freedom



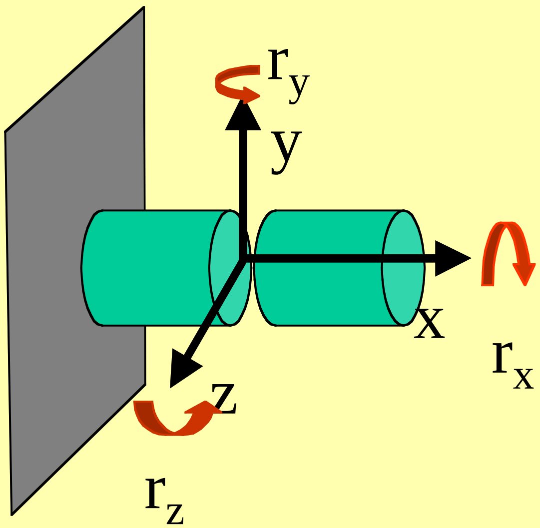
Movement at joints requires low resistance to distortion (lubrication, low *EI*)

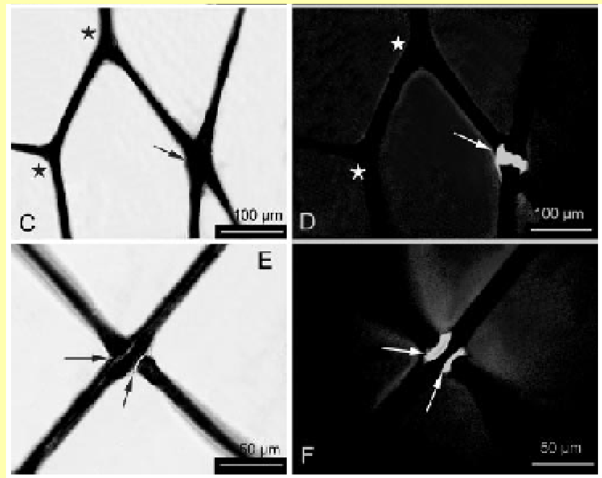
Arthroidal membrane (untanned insect cuticle - low *EI*)

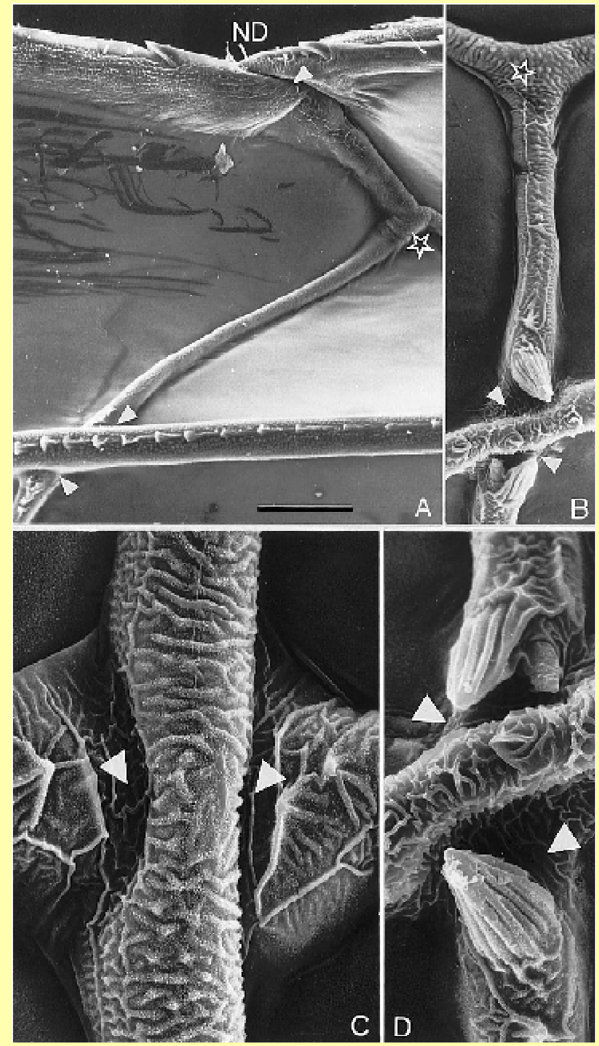




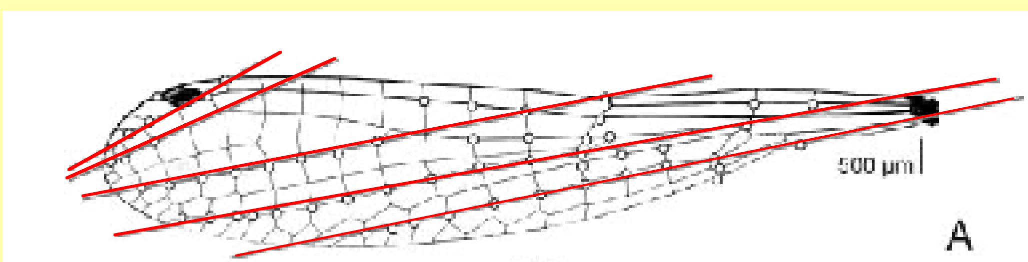
Most biological joints contain 1-3 rotational degrees of freedom



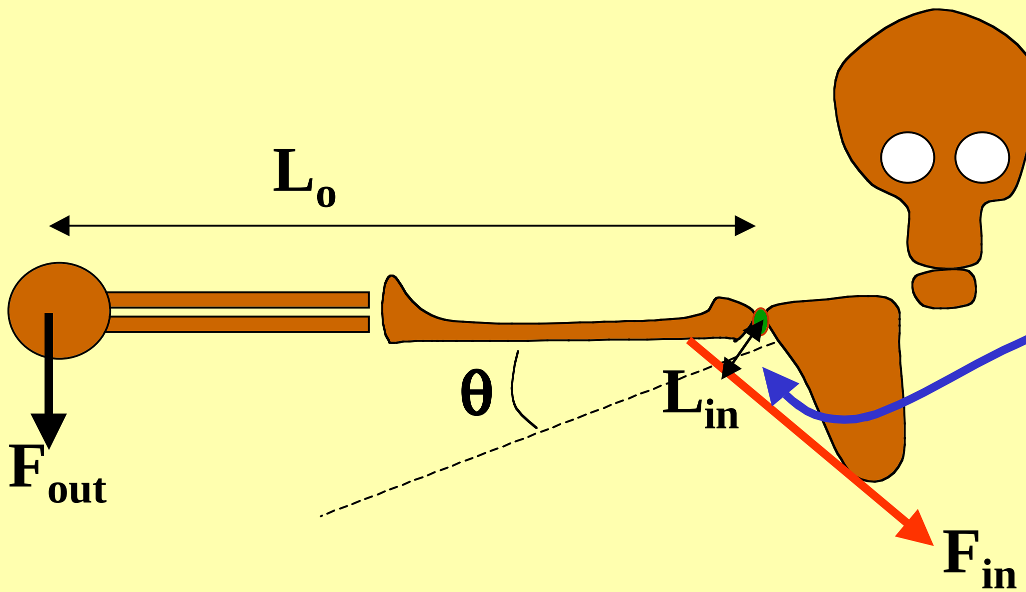




*damselflies have mobile ***resilin*** wing vein joints (Gorb, 1999)



Speed vs. Strength in articulated support systems



***** L_{in} is perpendicular line from F_{in} to joint***

Mechanical Advantage

$M_{in} = M_{out}$

$F_{in} L_{in} = F_{out} L_{out}$

$F_{out}/F_{in} = L_{in}/L_{out}$

Speed Ratio

$$= \frac{L_{out}\, d\theta/dt}{L_{in}\, d\theta/dt}$$

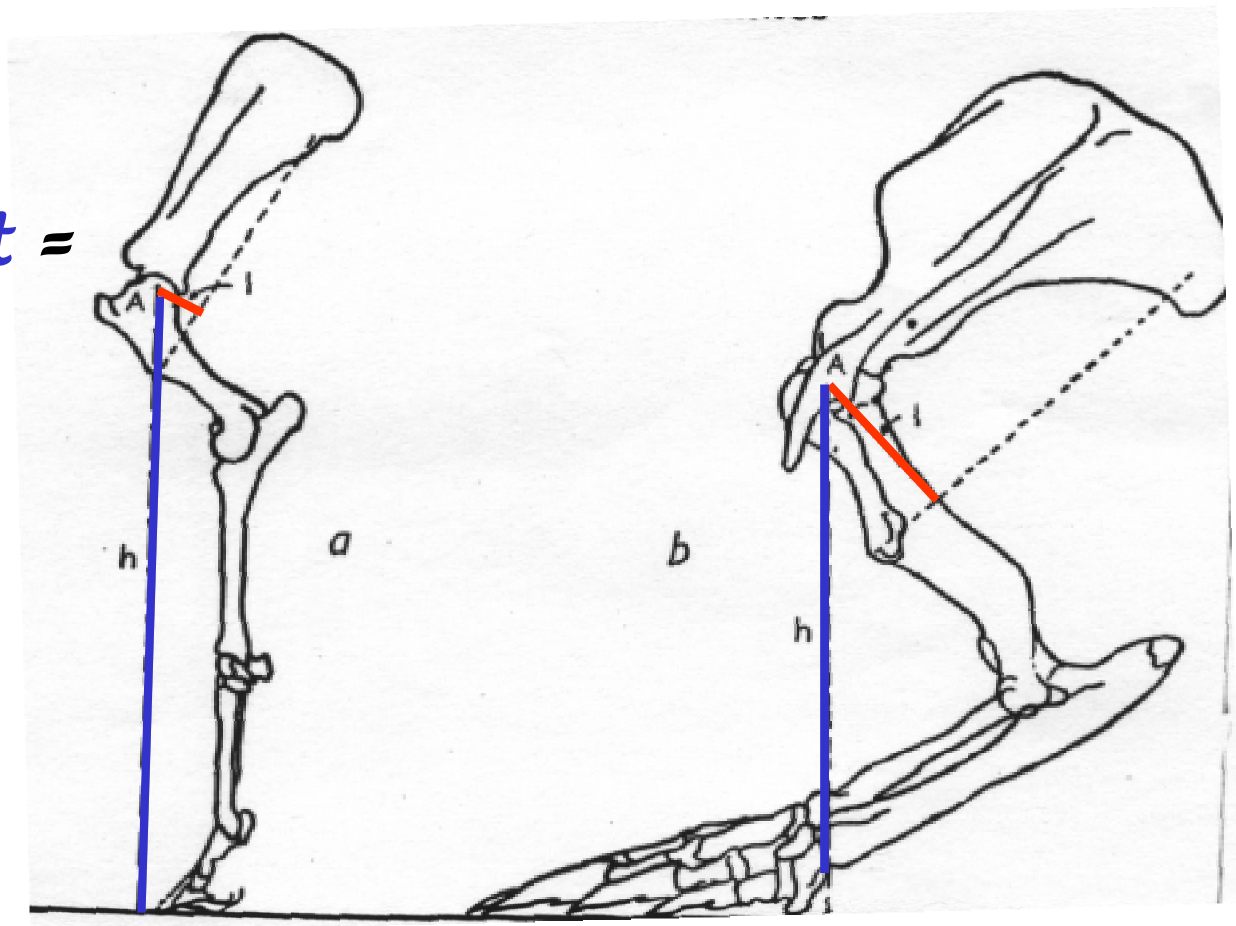
$$= L_{out}/L_{in}$$

Speed vs. Strength in articulated support systems

Horse:
low mechanical advantage
high speed ratio
FAST

Armadillo:
high mechanical advantage
low speed ratio
STRONG

Lin/Lout = small



Lin/Lout = big

Categorizing supportive systems

1. Tensile systems - built to resist tension only (i.e. algal stipes, fruit stems, toe-pad setae)
2. Strutted systems
   - single or branched struts (i.e. tree branches, coral)
   - articulated struts (i.e. vertebrate skeletons, insect exoskeletons)

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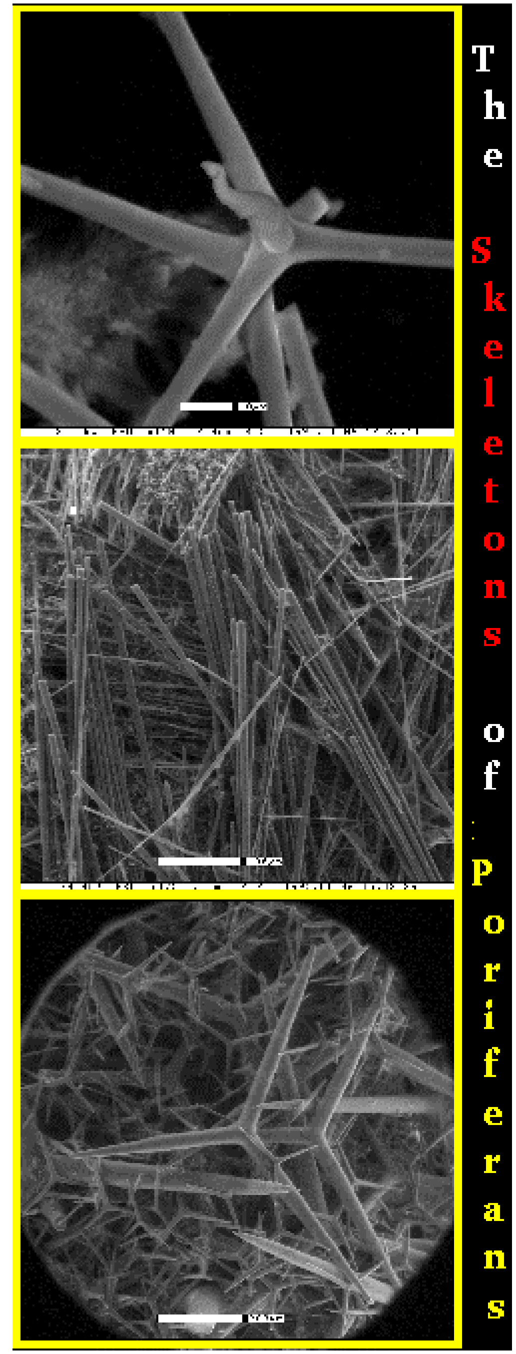
   - *dispersed struts (i.e. dispersed spicules in sponges)*



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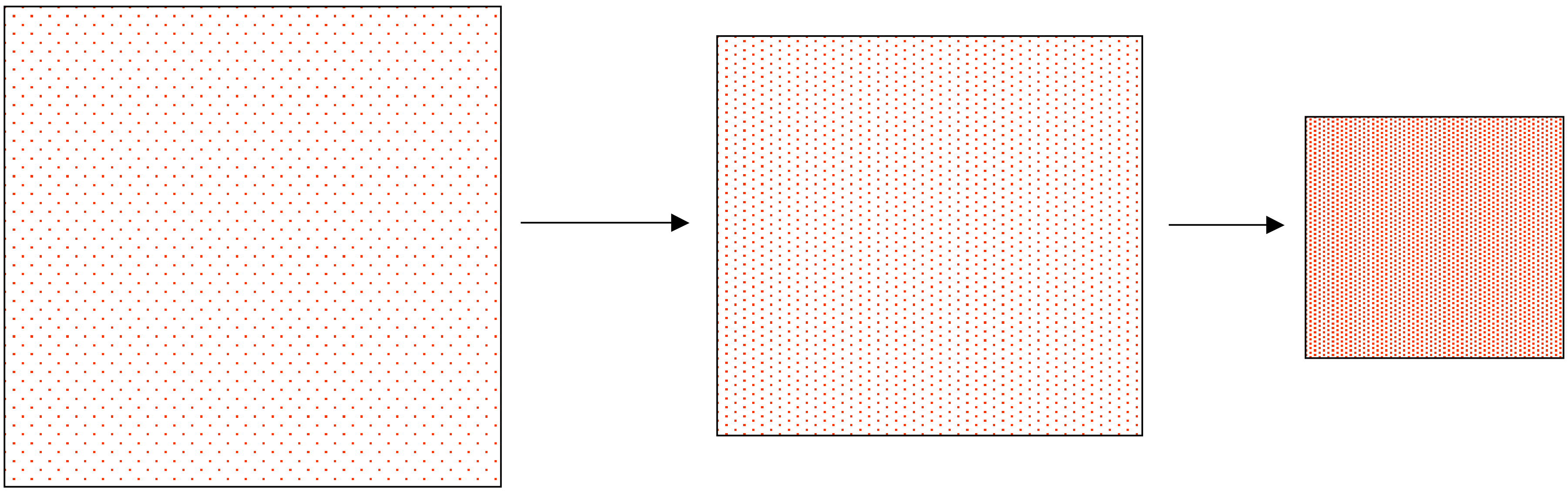
   - *dispersed struts (i.e. dispersed spicules in sponges)*

3. *Internally pressurized systems*
   - *hydrostats - watery-filled cavities under internal pressure (i.e. worms, plant stems)*

A very quick introduction to fluids and pressure.....



chamber filled with gas molecules

lower volume = higher pressure

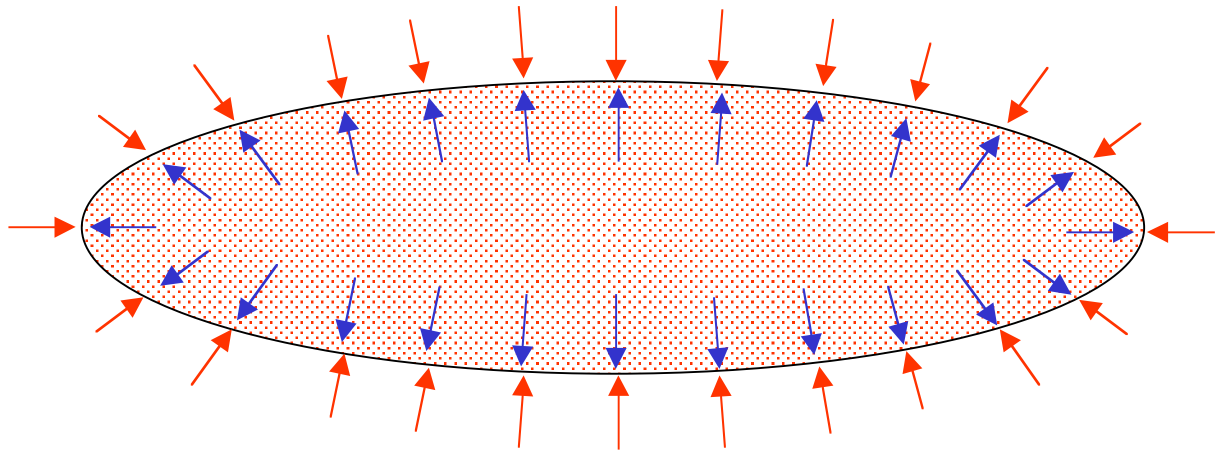
$$PV = nRT$$

•pressure and volume inversely proportional in gases

$$\boxed{P \approx 1/V}$$

assume constant if no change in temperature

Hydrostats are fluid-filled structures under internal pressure

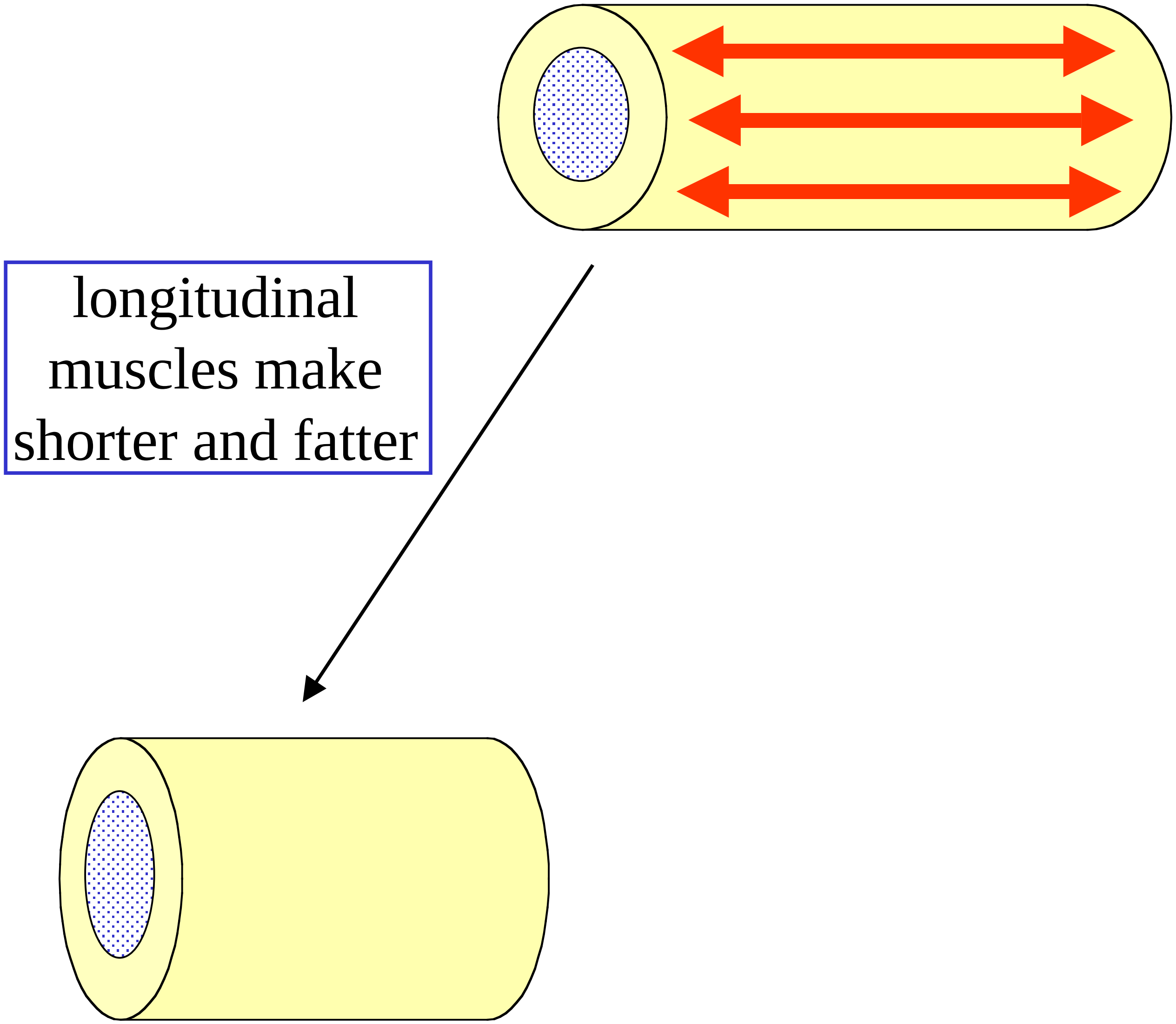


• pressurized fluid exerts an outward force on the membrane (membrane is in tension)

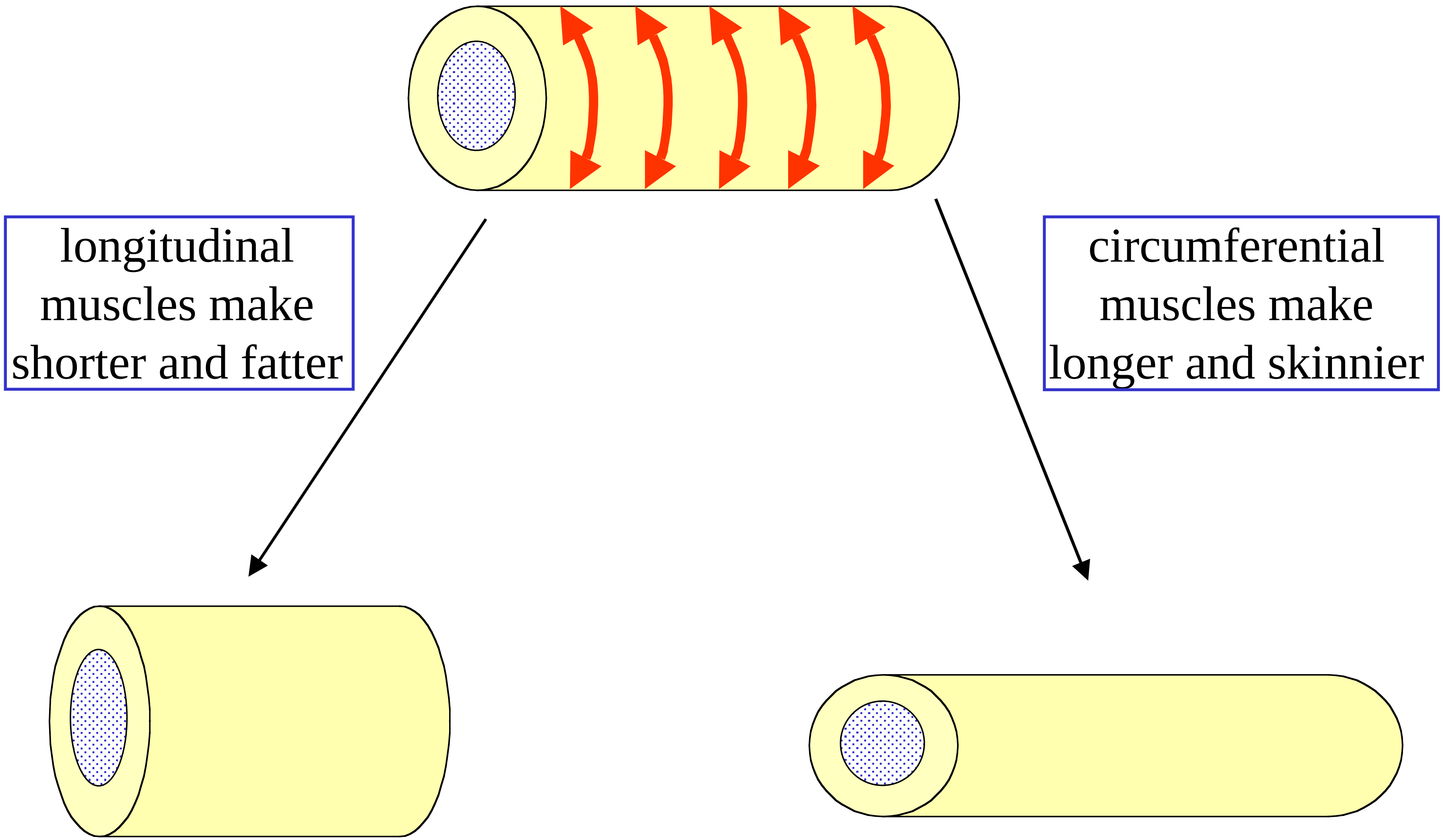
• membrane exerts an inward force on the fluid

• fluid is essentially incompressible, so hydrostats can behave like "solid" structures that muscles can act upon

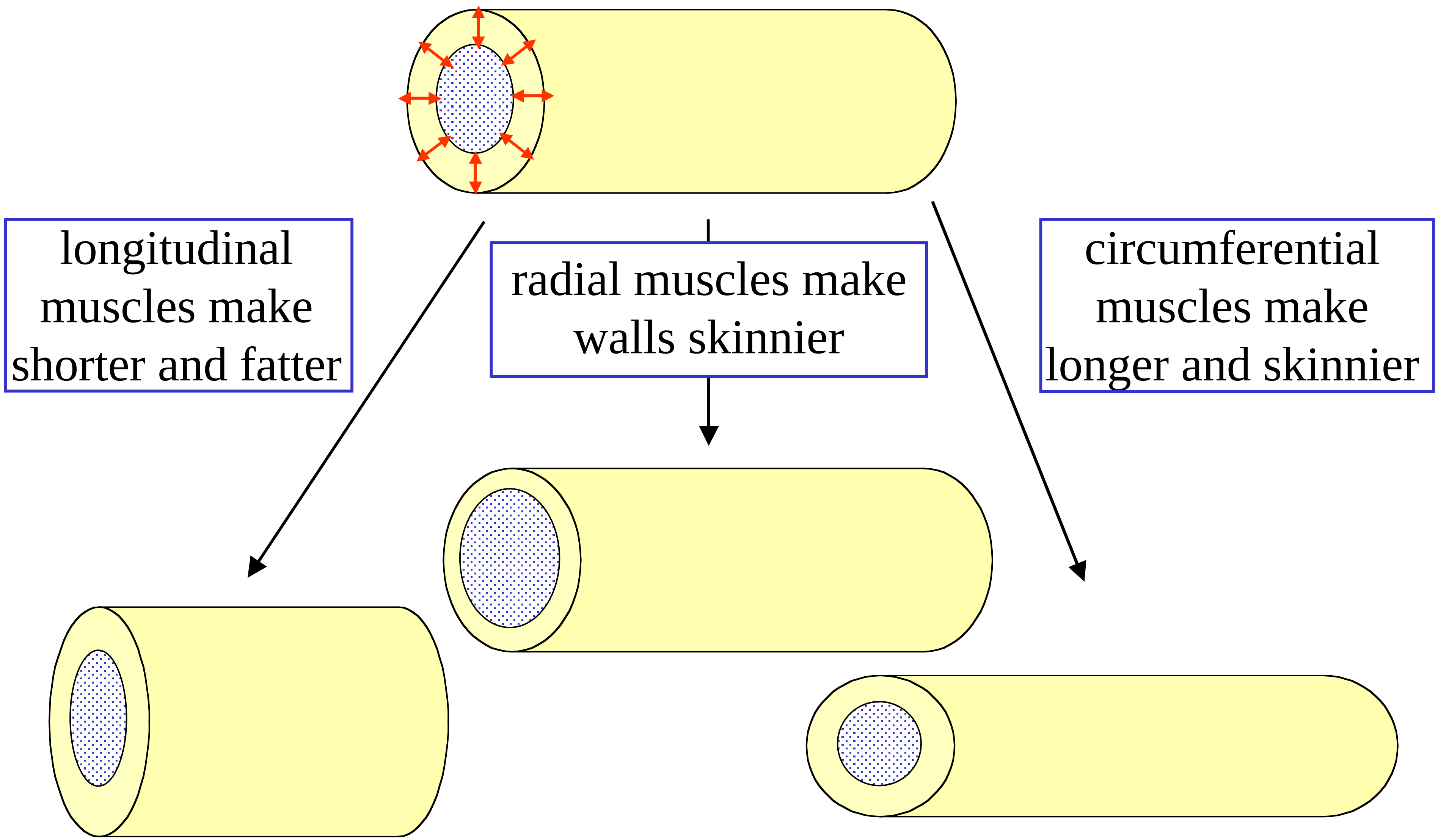
Biological hydrostats are often cylindrical



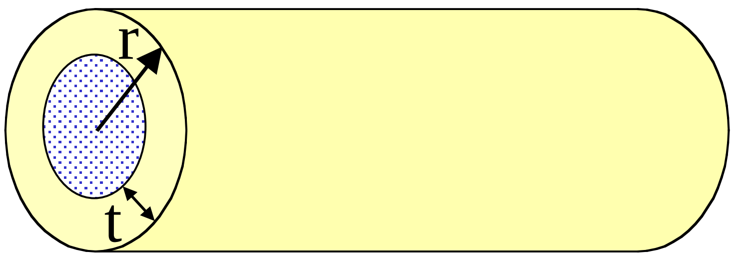
Biological hydrostats are often cylindrical



Biological hydrostats are often cylindrical



Shape changes in hydrostats can drive locomotion or provide support



ΔP = pressure difference

Circumferential stress:

$\sigma_C = \Delta Pr/t$

Longitudinal stress:

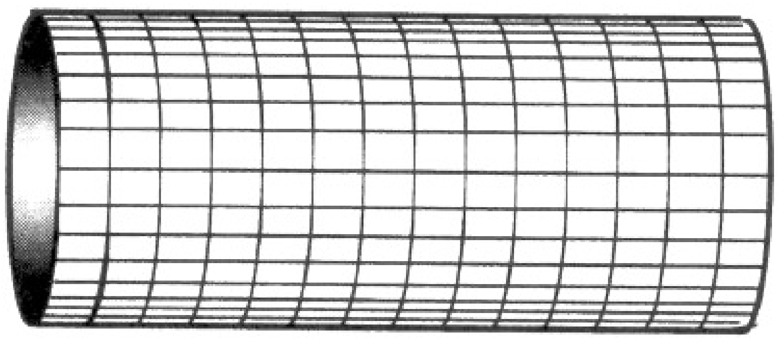
$\sigma_L = \Delta Pr/2t$

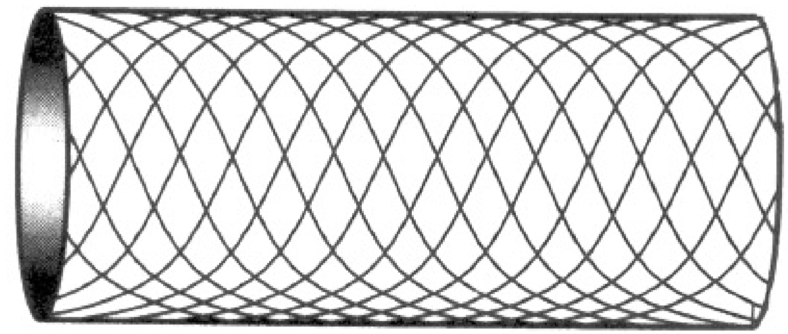
*smaller cylinders can withstand relatively larger pressure differences

*cylinders with thicker walls can withstand larger pressure differences

But...
this means cylinders will bulge outwards twice as fast as they will lengthen

Hydrostat behavior can be controlled by tension-resisting fibers in the outer membrane





longitudinal and circumferential fibers

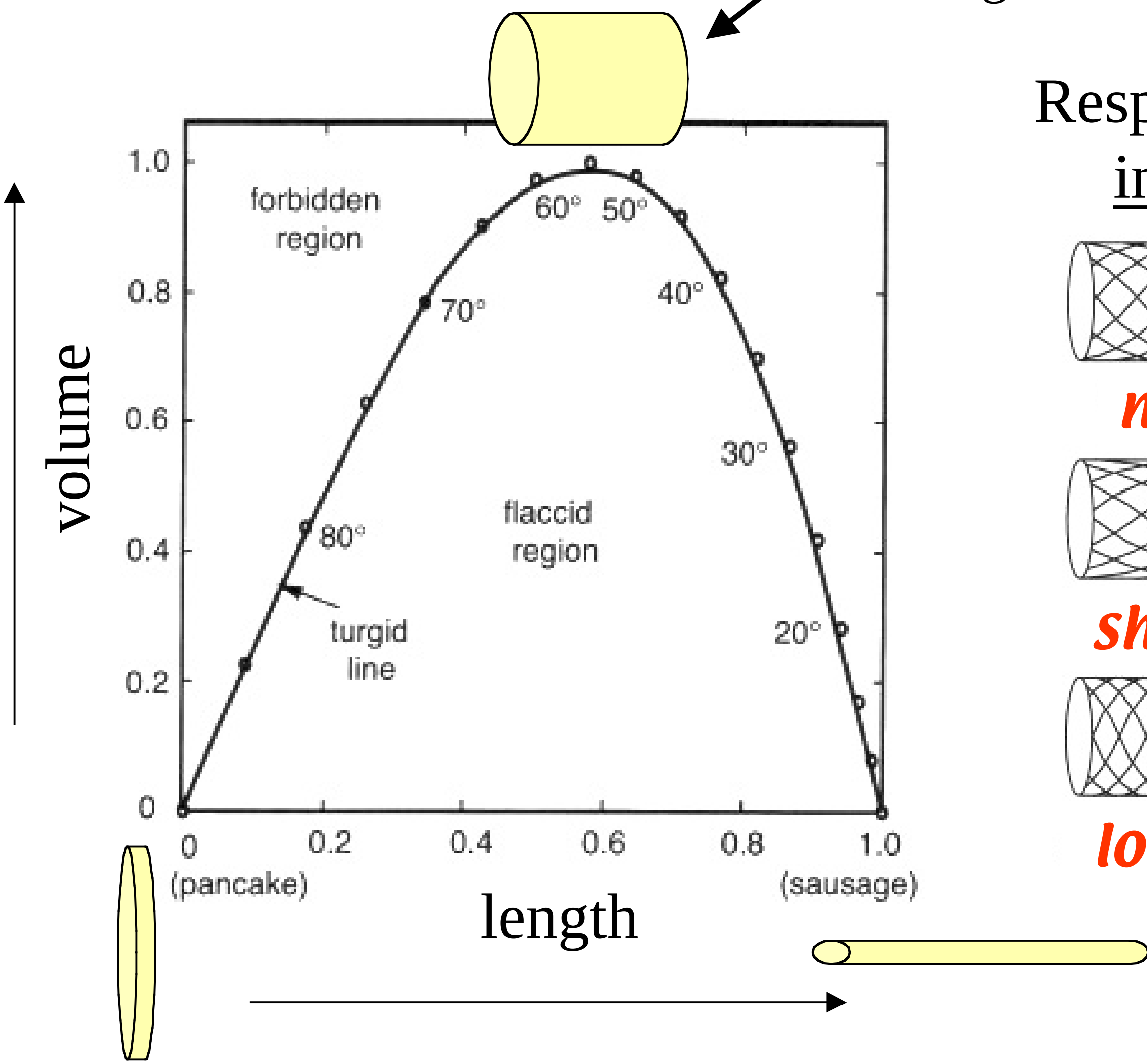
- cylinder resists compression and tension, high flexural stiffness
- local buckling with large compressive loads
- low resistance to torsion

helical fibers

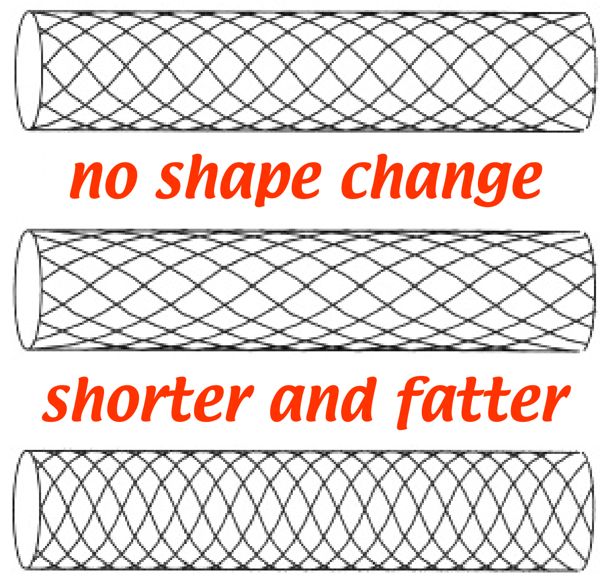
- lengthens and shortens smoothly, bends more easily
- less prone to local buckling
- high resistance to torsion
- more common in biological systems

The angle of helical fibers determines the behavior of fiber-reinforced hydrostats

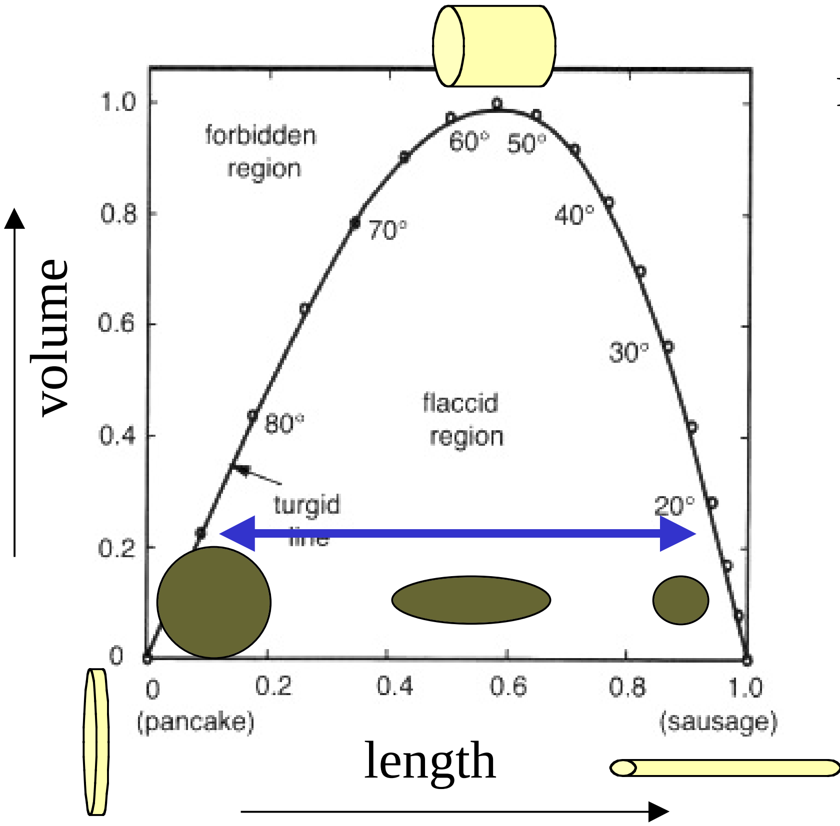
maximum volume at fiber angle of 55°



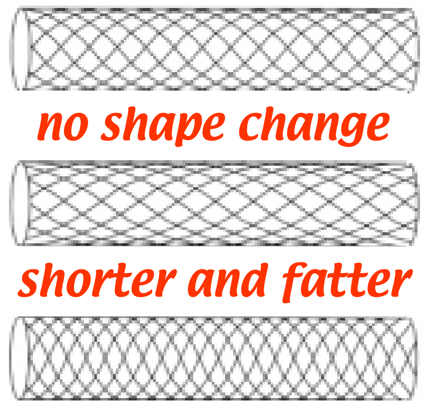
Response to increased internal pressure	fiber angle
	55°
no shape change	
	<55°
shorter and fatter	
	>55°
longer and thinner	



The angle of helical fibers determines the behavior of fiber-reinforced hydrostats



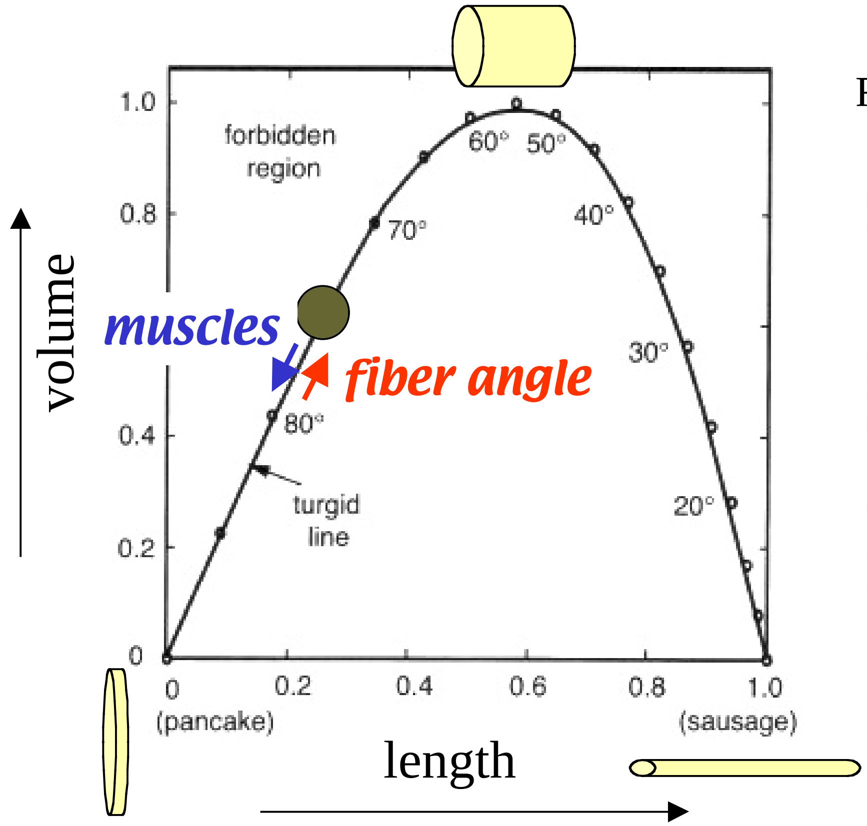
Response to increased internal pressure	fiber angle
no shape change	***55°***
shorter and fatter	***<55°***
longer and thinner	***>55°***



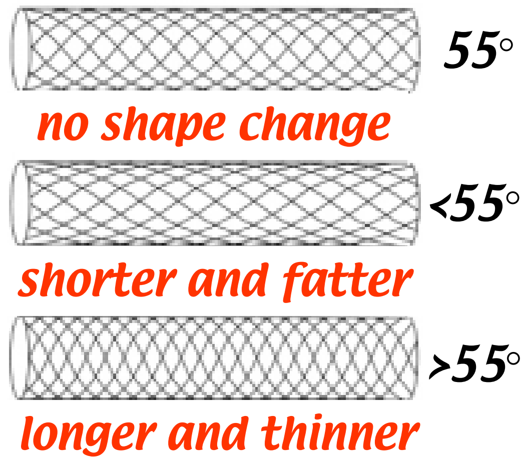
Limp worms - long, flat, unsegmented, i.e. **nemertean worms**

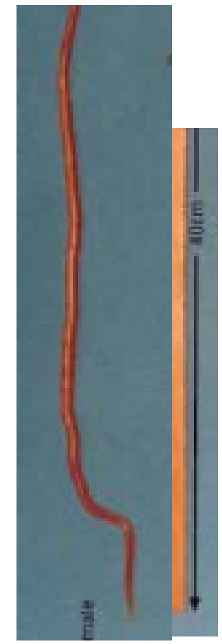
- Lie in flaccid region because not circular cylinders (not full volume)
- Contraction of longitudinal muscles makes shorter and more round
- Contraction of circumferential muscles makes longer and more round

The angle of helical fibers determines the behavior of fiber-reinforced hydrostats



Response to increased internal pressure | fiber angle

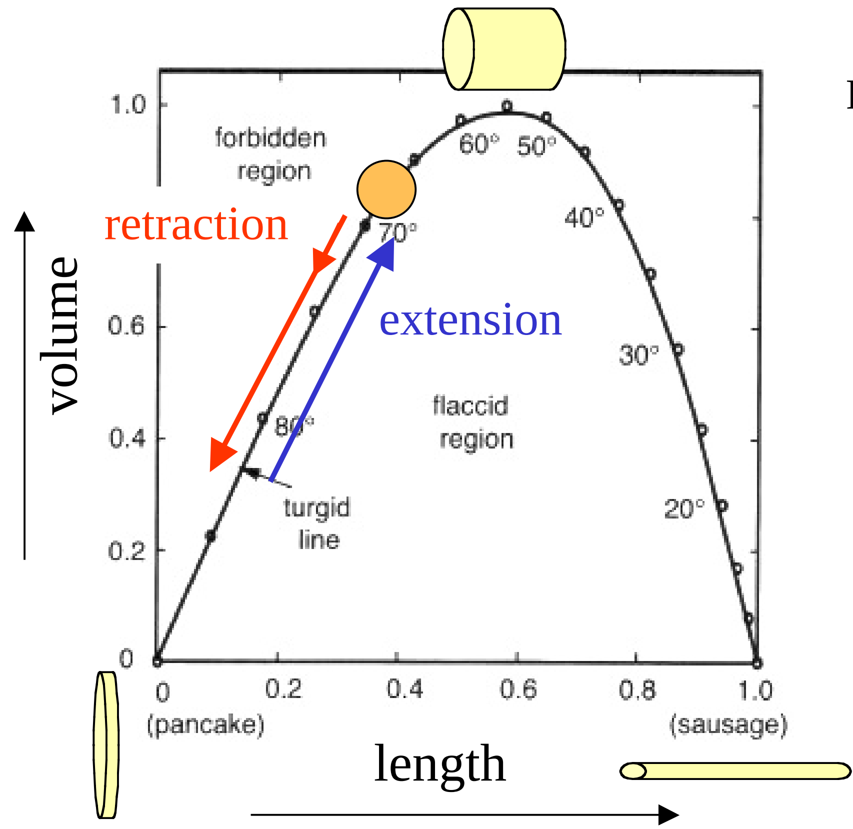


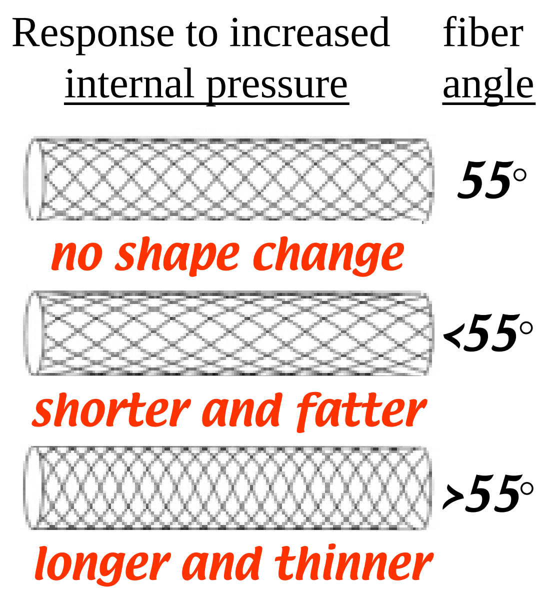


Stiff worms - unsegmented **nematodes, or roundworms**

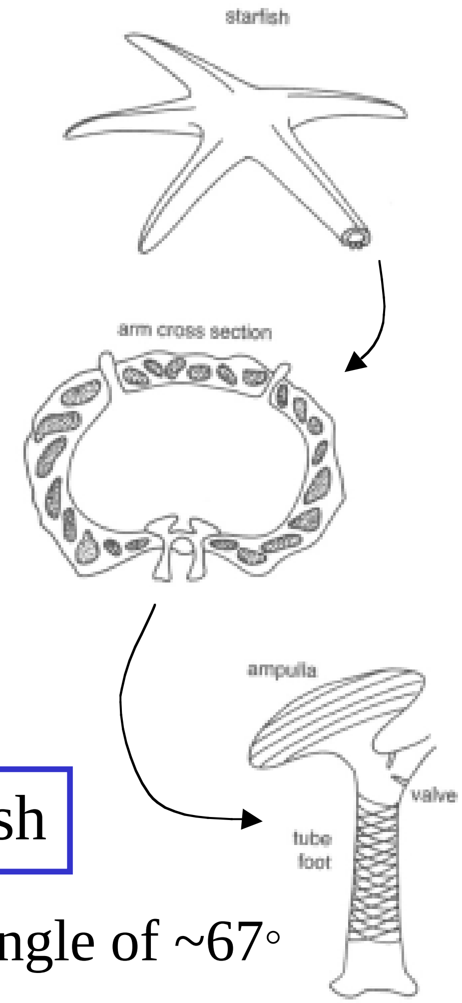
- Strong cuticle, round cross-section, LONGITUDINAL MUSCLES ONLY
- *Ascaris* (intestinal parasite) has a fiber angle of 75°
- Because volume cannot change, contraction generates pressures of up to 30 kPa - worms also get a little shorter
- Recoil of fibers to lower angle antagonizes the action to restore shape

What if hydrostat volume can change?



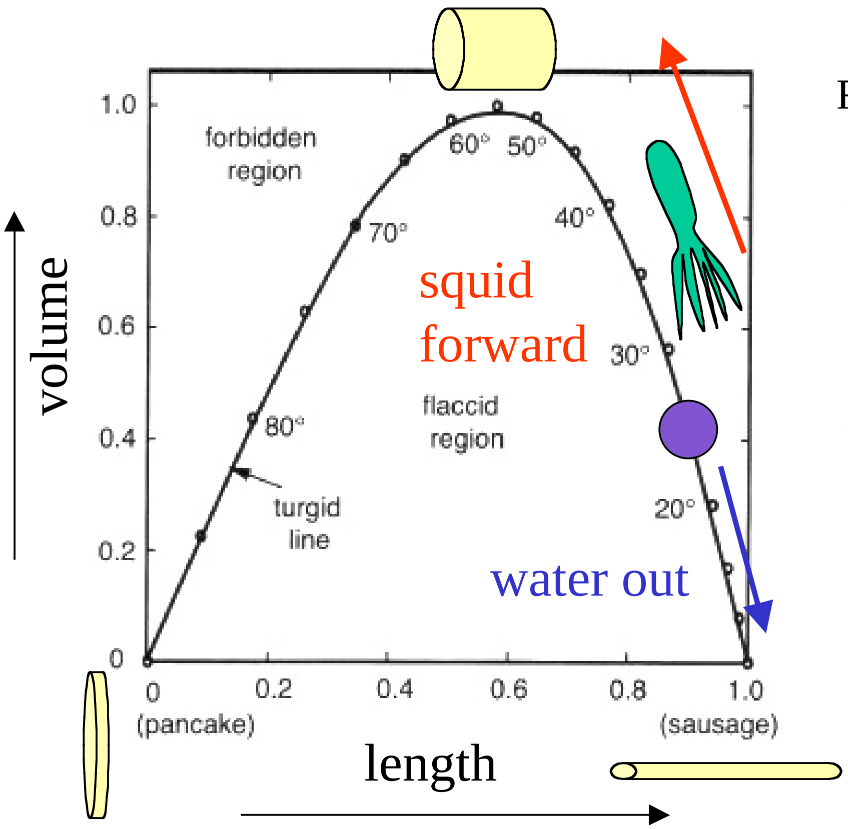


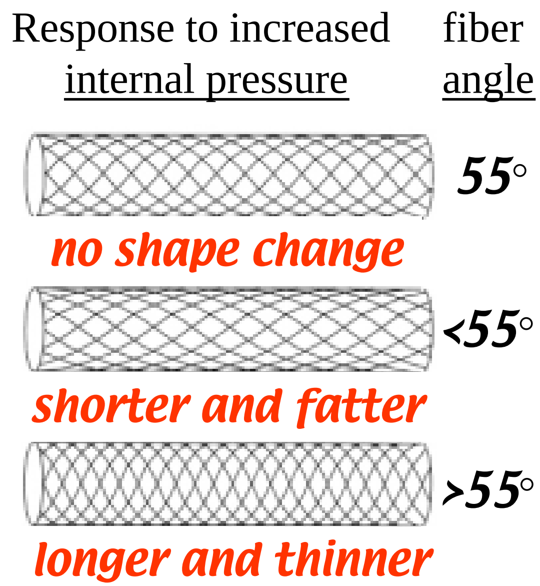
Tube *feet* of starfish

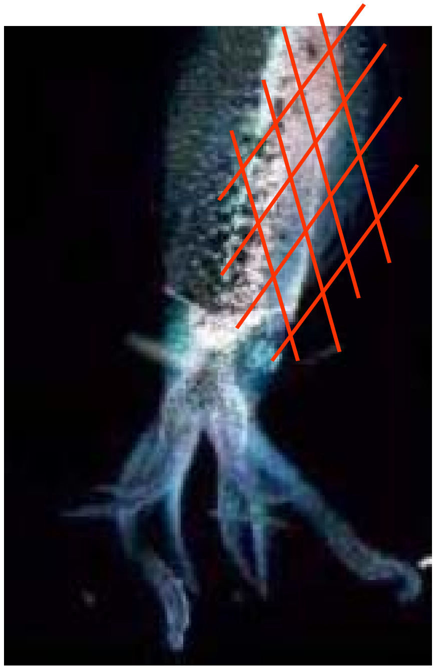


- Tube foot has *longitudinal muscles only* and fibers at an angle of ~67°
- Contraction of longitudinal muscles makes shorter, but fiber angle resists shortening
- Pressure rises and fluid is expelled into ampulla --> foot retracts
- When ampulla contracts, fluid is expelled back into tube foot and foot extends

What if hydrostat volume can change?



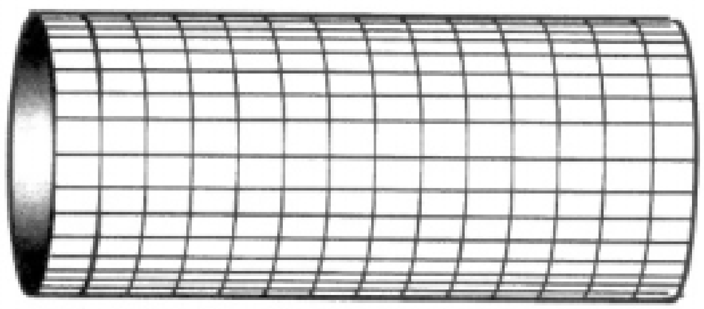


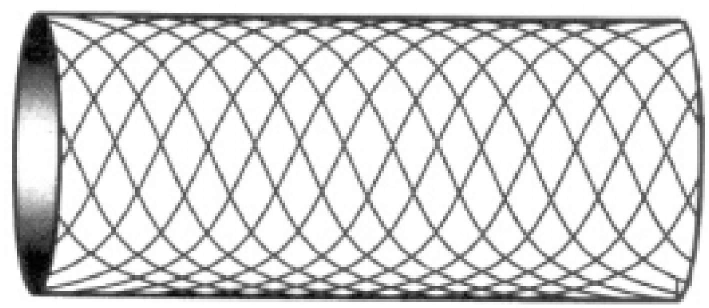


Squid mantle

- Squid mantle has *circumferential muscles only* and fibers at an angle of ~25°
- Contraction of circumferential muscles tends to extend mantle
- Extension at low fiber angle implies a reduction in volume
- Water squirts from mantle and squid accelerates in opposite direction

Do any biological structures use longitudinal and circumferential fibers instead of helical?





longitudinal and circumferential fibers
- cylinder resists compression and tension, high flexural stiffness

helical fibers
- lengthens and shortens smoothly, bends more easily
- more common in biological systems

Mammalian penises depend on hydrostatics for functioning, and fibers run longitudinally and circumferentially, not helically

* Fiber orientation provides high stiffness and minimizes shape changes and compression

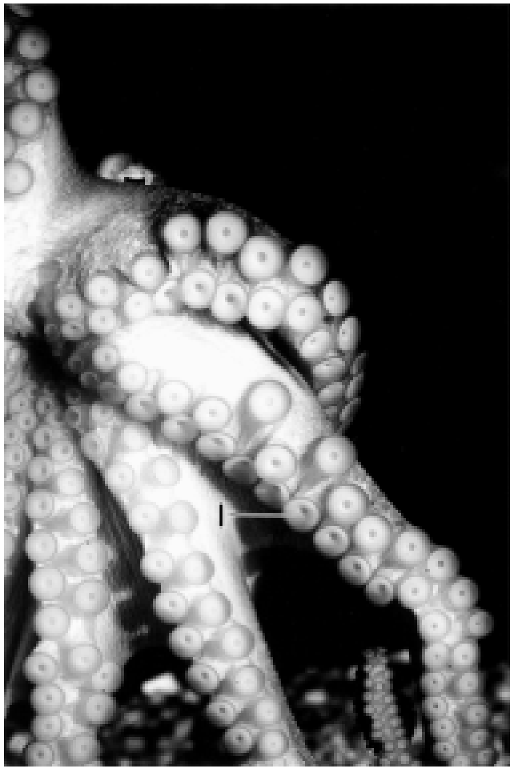
Categorizing supportive systems

1. Tensile systems - built to resist tension only (i.e. algal stipes, fruit stems, toe-pad setae)

2. Strutted systems
   - single or branched struts (i.e. tree branches, coral)
   - articulated struts (i.e. vertebrate skeletons, insect exoskeletons)
   - dispersed struts (i.e. dispersed spicules in sponges)

3. Internally pressurized systems
   - hydrostats - watery-filled cavities under internal pressure (i.e. worms, plant stems)
   - muscular hydrostats - contraction of one group of muscles causes extension of another group (i.e. trunks, tentacles and tongues)

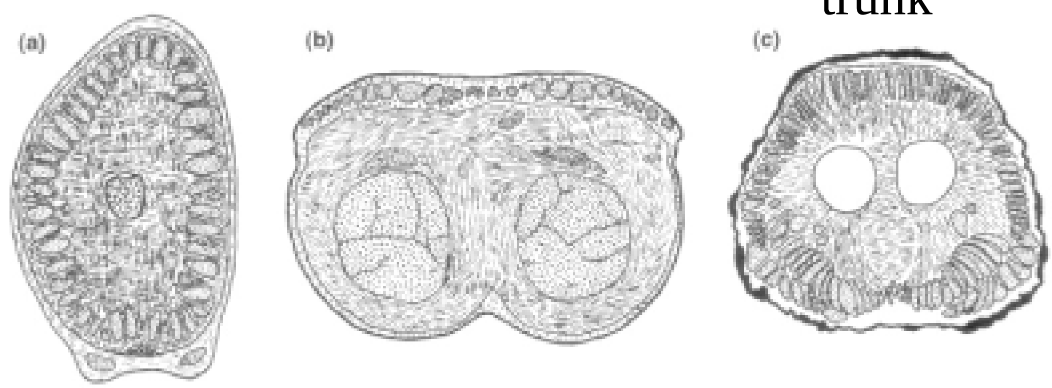
Muscular hydrostats rely on the fact that muscles themselves are incompressible



<u>Muscles wrap around each other</u>

- Contraction of one group causes extension of the other because volume cannot change

squid tentacle lizard tongue elephant trunk



What if you don't want to go anywhere? How can you stay put just where you are?

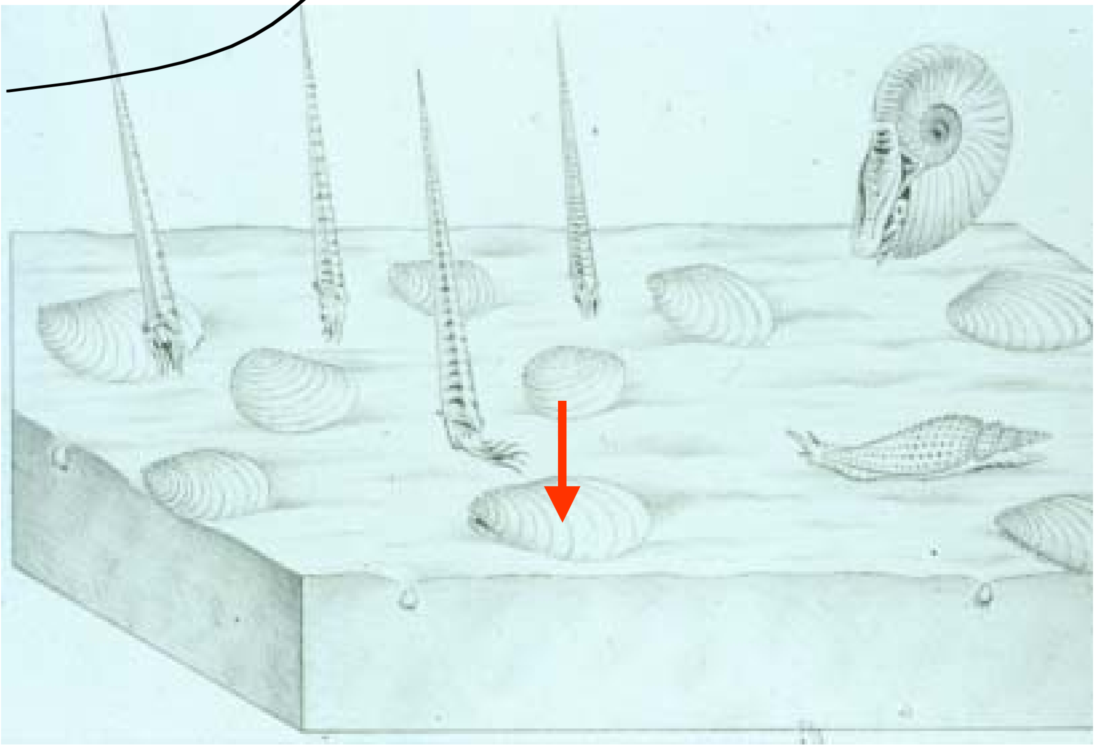
Attachments may be permanent or temporary, and must resist forces that are often quite large (i.e. gravity, wind and wave currents)

usually only fails if reaches breaking stress

<u>Attachments may fail in:</u>

(less of a problem)

compression



(more of a problem)

What if you don't want to go anywhere? How can you stay put just where you are?

Attachments may be permanent or temporary, and must resist forces that are often quite large (i.e. gravity, wind and wave currents)

<u>Attachments may fail in:</u>

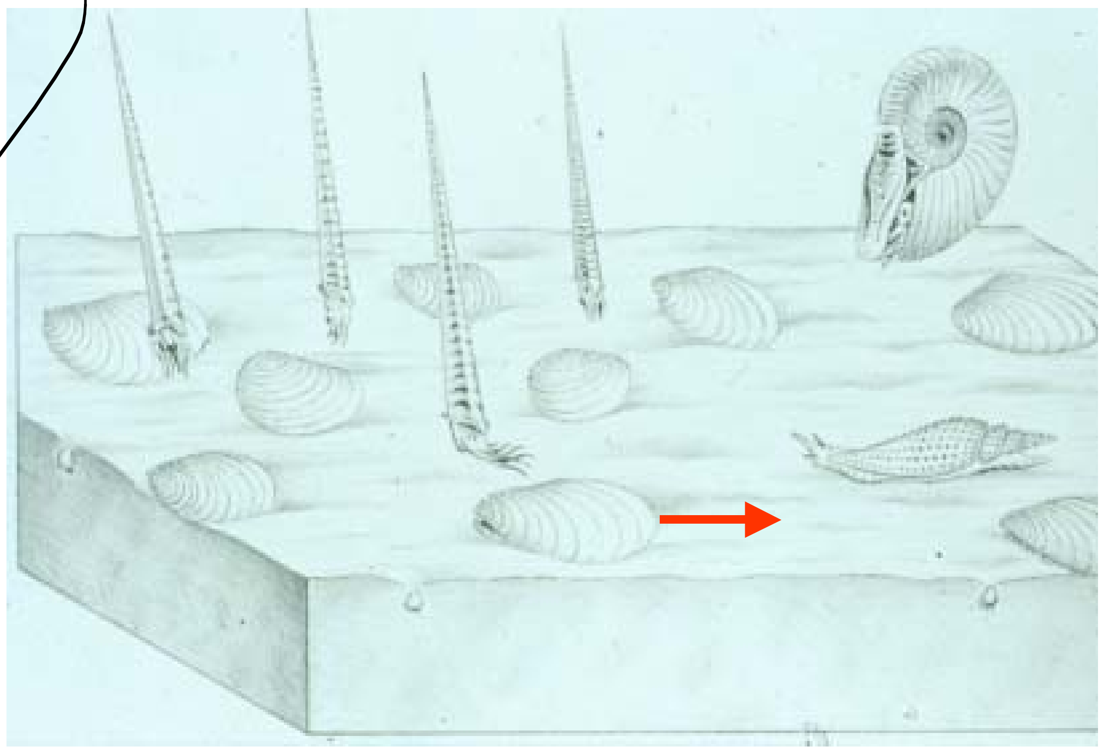
(less of a problem)

compression

shear

(more of a problem)

surface irregularities help resist shearing



What if you don't want to go anywhere? How can you stay put just where you are?

Attachments may be permanent or temporary, and must resist forces that are often quite large (i.e. gravity, wind and wave currents)

more difficult to resist

• interdigitating attachments can convert tension to shear - i.e. tendon and bone

<u>Attachments may fail in:</u>

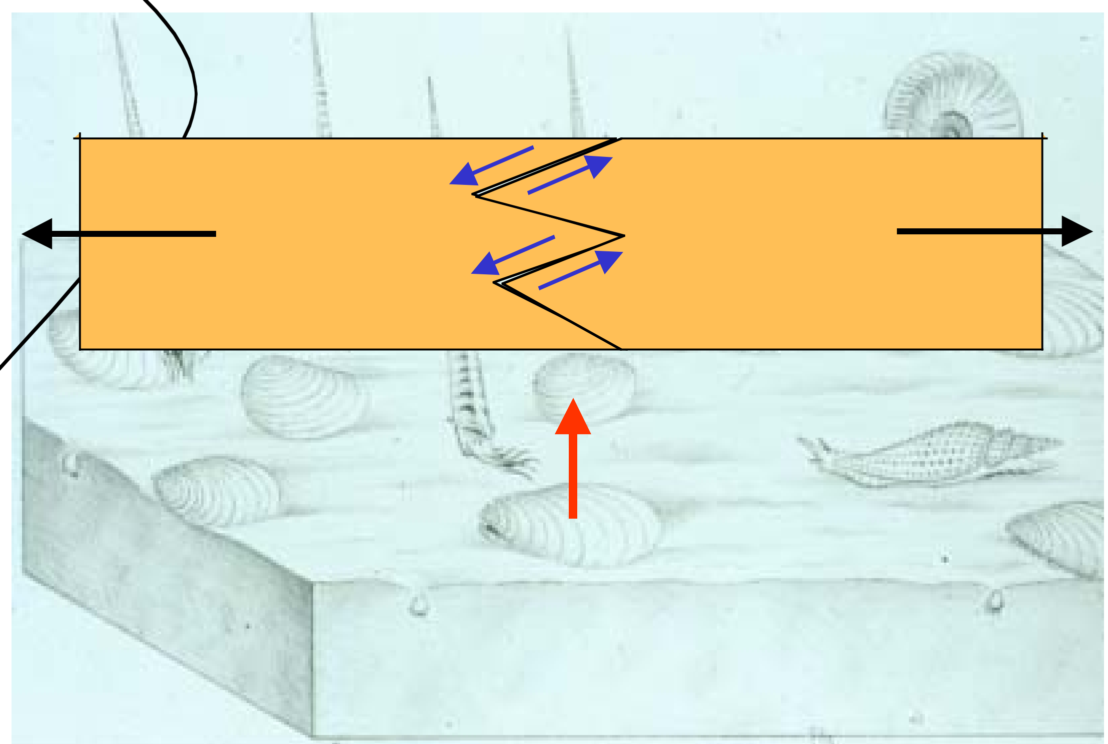
(less of a problem)

compression

shear

tension

(more of a problem)



What if you don't want to go anywhere? How can you stay put just where you are?

Attachments may be permanent or temporary, and must resist forces that are often quite large (i.e. gravity, wind and wave currents)

<u>Attachments may fail in:</u>

(less of a problem)

compression

shear

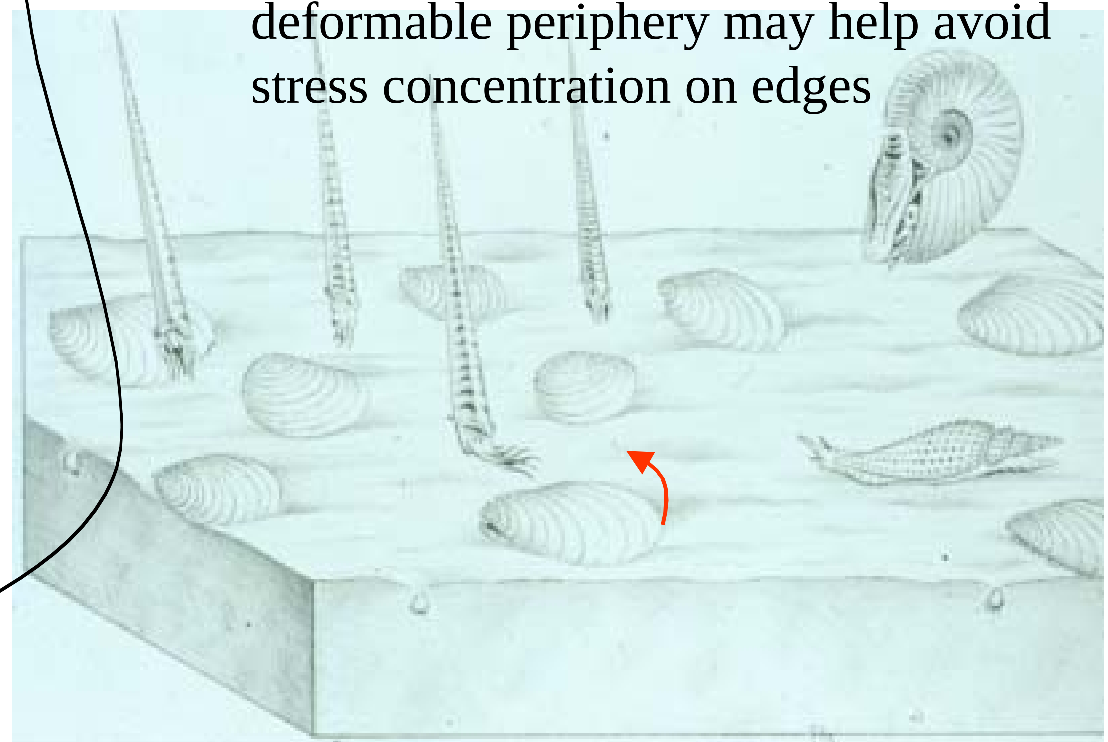
tension

peel

(more of a problem)

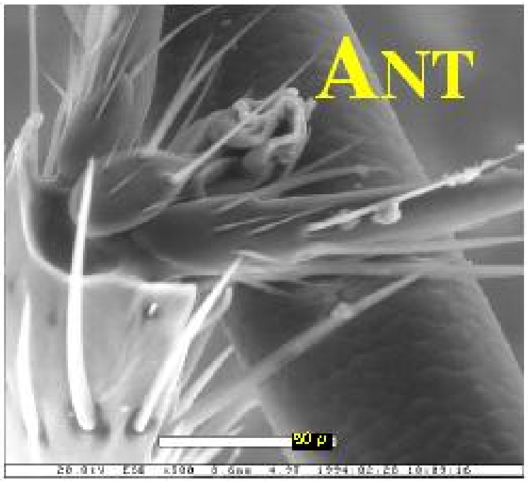
difficult to resist because concentrates force into a small area ("peel line")

• tapering disk of attachment with deformable periphery may help avoid stress concentration on edges

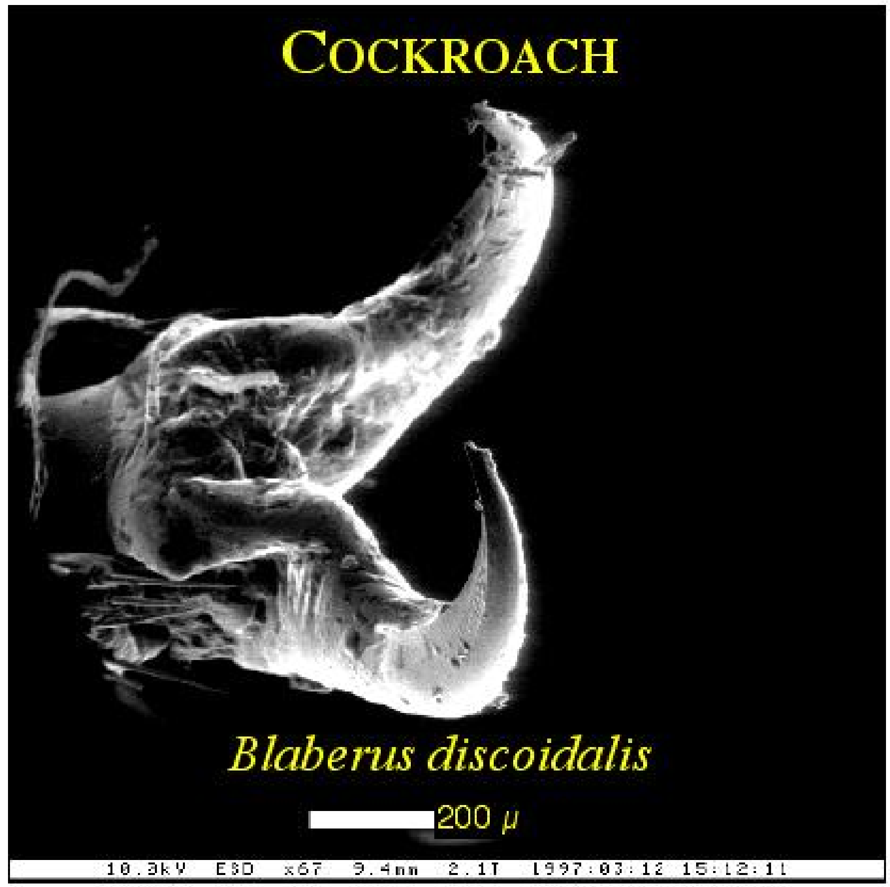


Methods of Adhesion

- ***Interlocking devices* - *hooks, spines or claws***



Passive spines and claws very effective on rough surfaces.

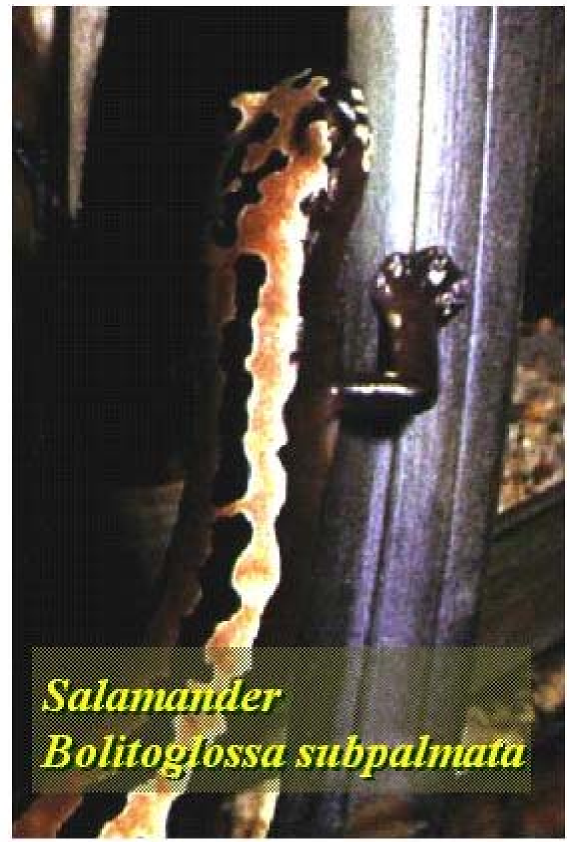


Methods of Adhesion

- *Interlocking devices*
- *Suction - adhesion depends on pressure difference between fluid inside suction cup and atmosphere*

Slow locomotion, high duty factor, move only one foot at a time.

Effective only on smooth surface.



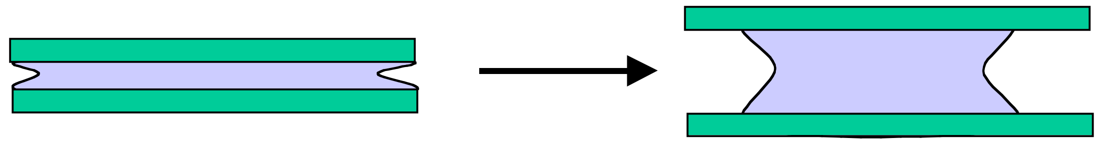
Methods of Adhesion

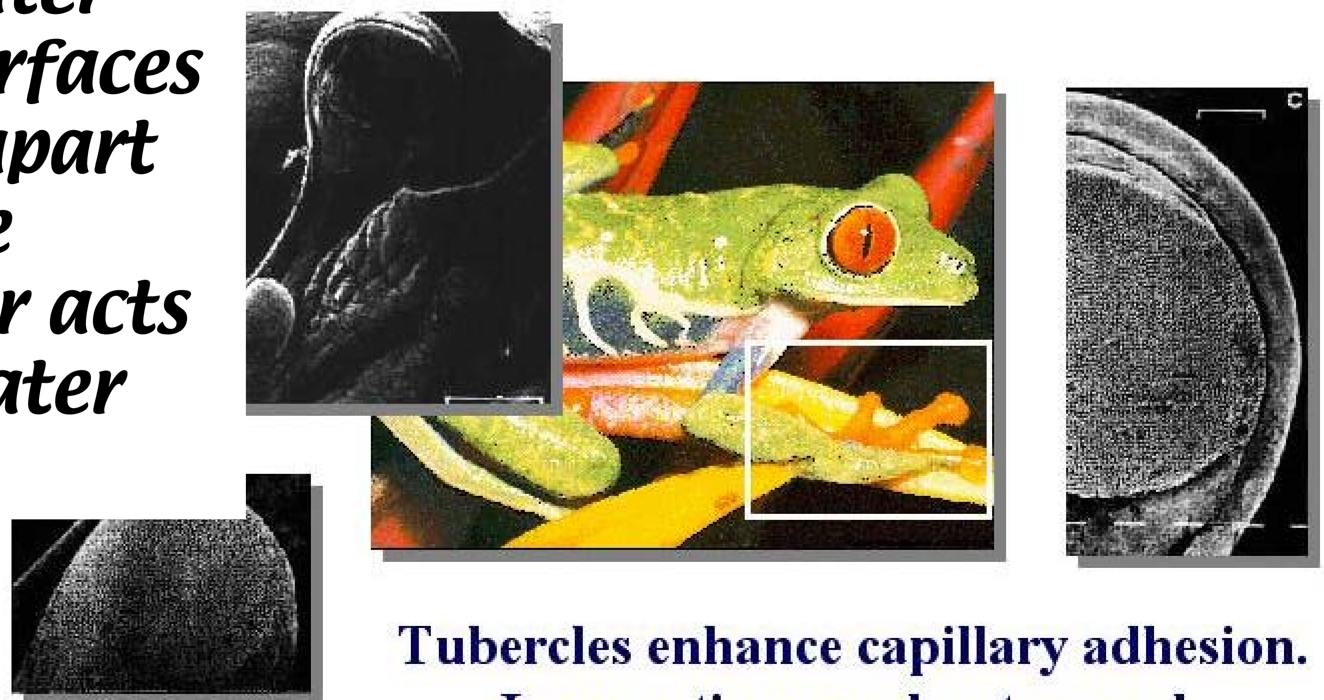
- Interlocking devices
- Suction
- Extruded goo - glue, mucus etc. that fixes animal to substrate

  - Stefan adhesion - thin layer of viscous fluid resists shear

Methods of Adhesion

- *Interlocking devices*
- *Suction*
- *Extruded goo*
- *Capillary action* - ***thin layer of water between two surfaces resists pulling apart because surface tension of water acts to reduce air-water interface***





Tubercles enhance capillary adhesion. Locomotion: moderate speed, walking gait.

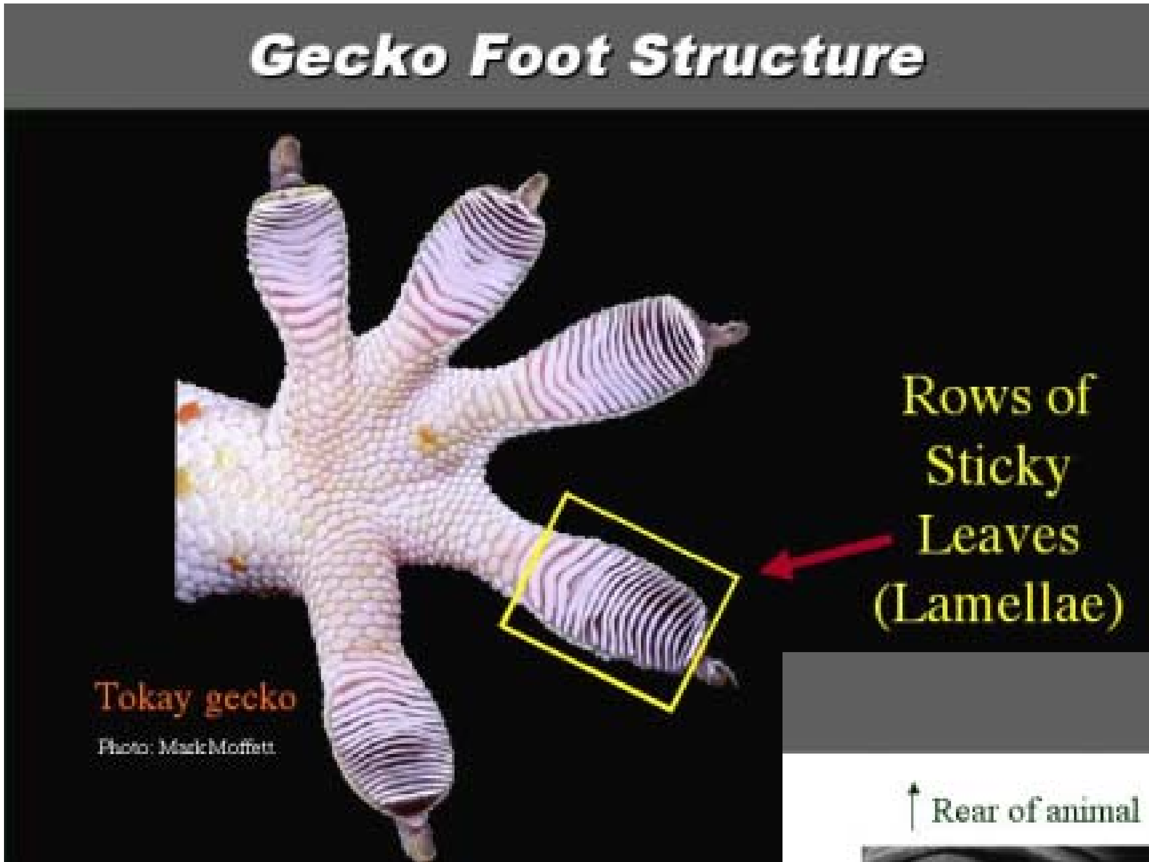
Methods of Adhesion

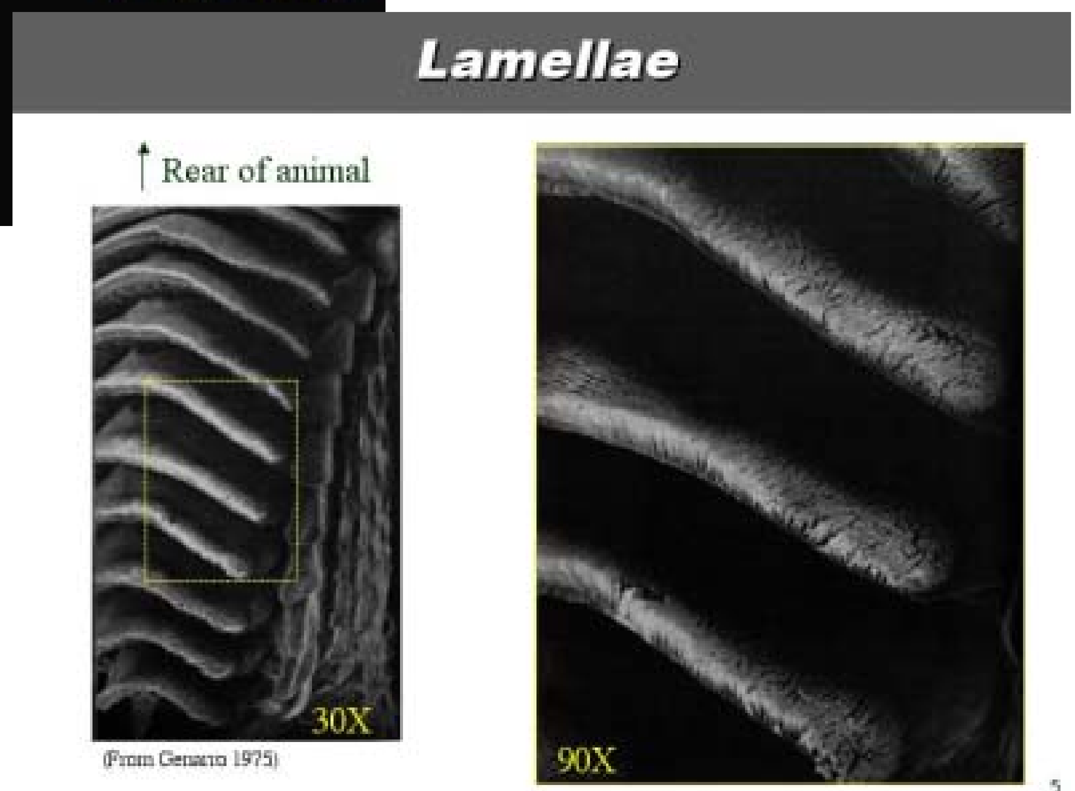
- Interlocking devices
- Suction
- Extruded goo
- Capillary action
- Intermolecular forces
  - electrostatic attraction - interaction between charged ions
  - polar interactions - attraction between molecules with a charge separation (i.e. hydrogen bonds in H_2O)
  - van der Waals forces - transient interactions between positive and negative portions of molecules as electrons rotate to opposite sides of orbits

Geckos can run upside down, accelerate on polished glass, and hang from one toe…….

How do gecko feet adhere to surfaces so well?

Huge diversity of gecko feet.....


Gecko Foot Structure
Rows of Sticky Leaves (Lamellae)
Tokay gecko
Photo: Mark Moffett


Lamellae
Rear of animal
30X
(From Genaro 1975)
90X

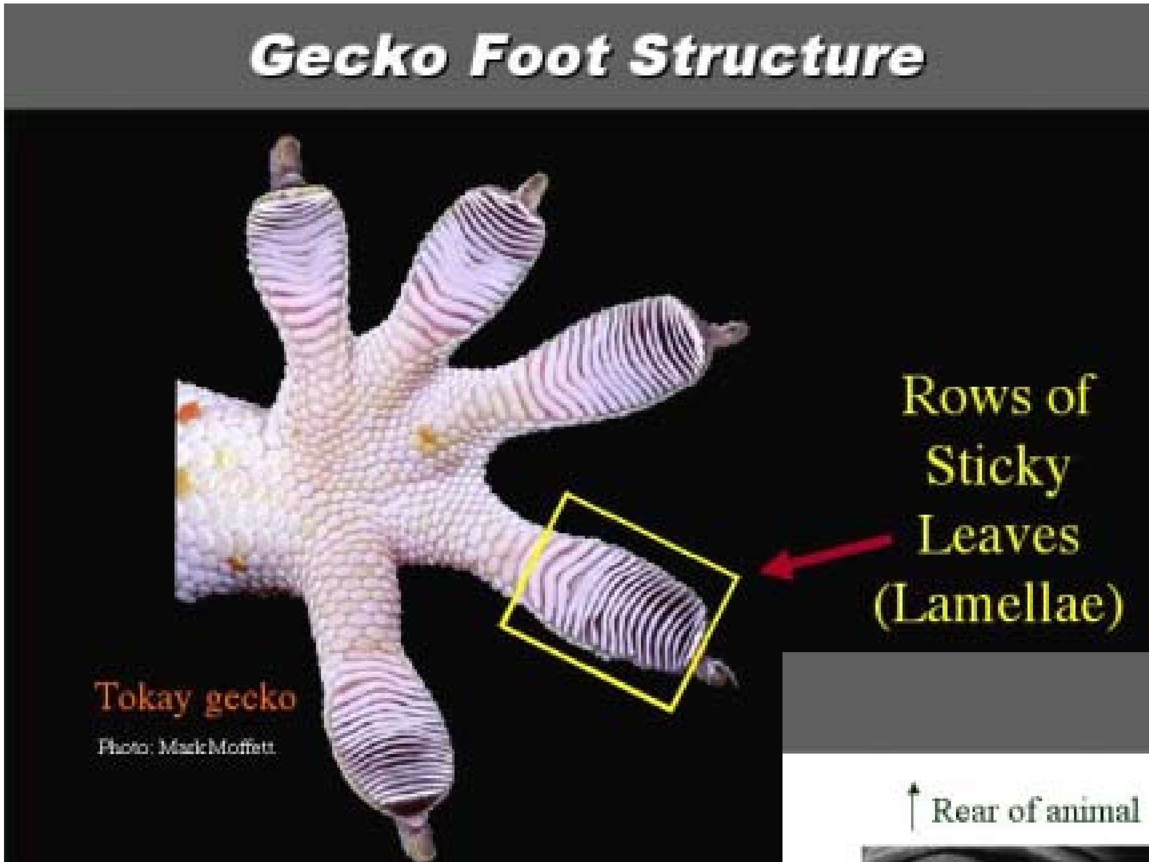
How do gecko feet adhere to surfaces so well?

- *Interlocking devices* →no hooks, can stick on perfectly smooth surface

- *Suction*
- *Extruded goo*
- *Capillary action*
- *Intermolecular forces*
  - *electrostatic attraction*
  - *polar interactions*
  - *van der Waals forces*

How do gecko feet adhere to surfaces so well?

- ***Interlocking devices*** →no hooks, can run on perfectly smooth surface
- ***Suction*** →dead geckos remain stuck to a wall in a vacuum (no pressure difference for suction to function)
- ***Extruded goo*** →no glands in feet, no footprints on surfaces
- ***Capillary action*** →toes are hydrophobic, stick equally well to hydrophobic and hydrophilic surfaces
- ***Intermolecular forces***
  - ***electrostatic attraction*** →works in ionized environment
  - ***polar interactions*** →works on nonpolar surface
  - ***van der Waals forces*** ??????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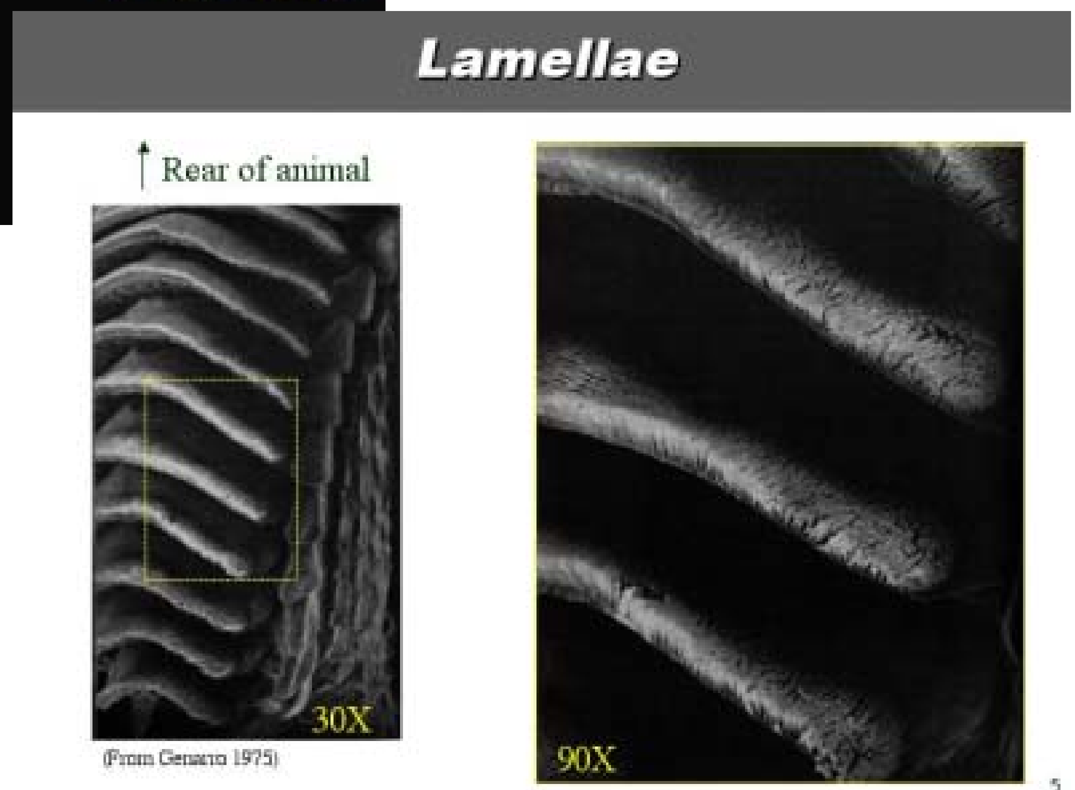
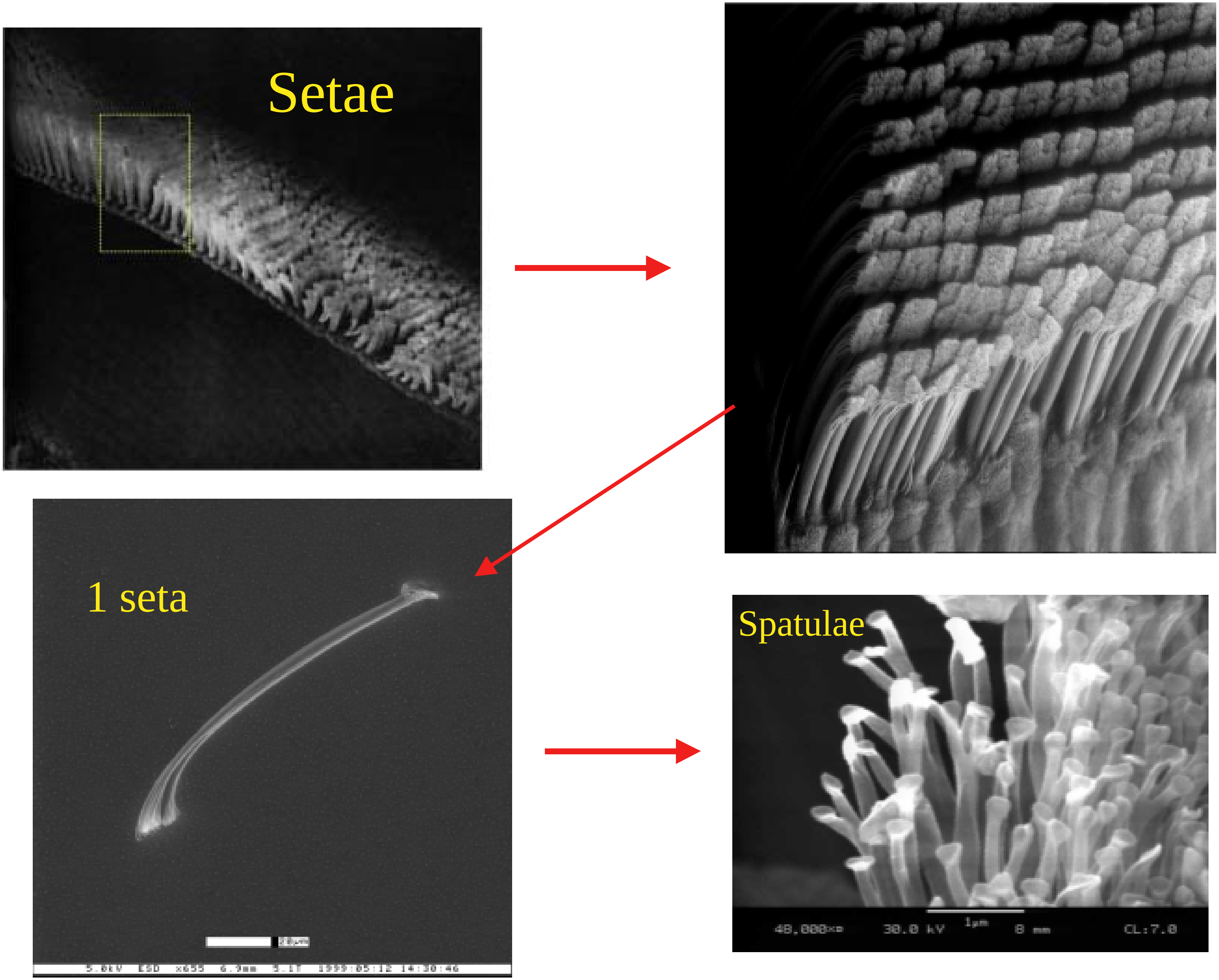

Lamellae
Rear of animal
30X
90X


Setae
1 seta
Spatulae